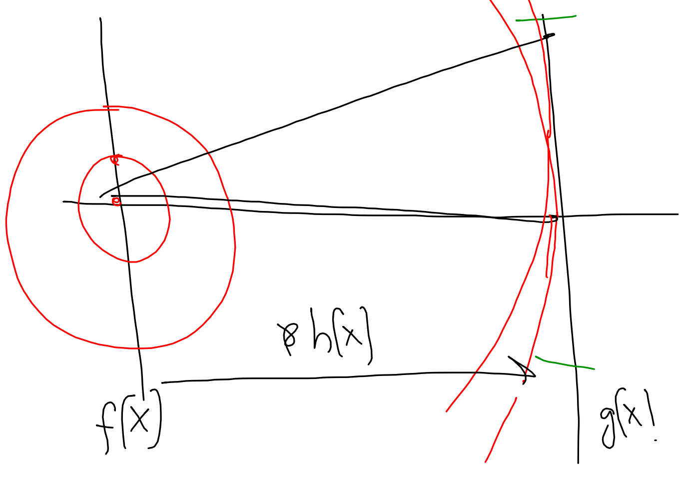
16 OCTOBER

$$\mathrm{SINC}\left[\frac{x}{b_0}\right] \otimes \mathrm{SINC}\left[\frac{x}{d_0}\right] = g(x)$$

$$f(x) \otimes h(x) = g(x)$$



October 16, 2014

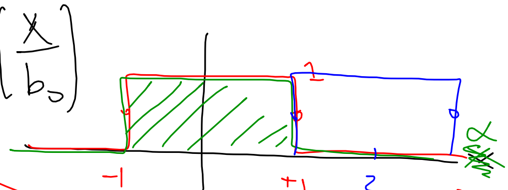
$$f(x) * h(x) = g(x)$$

$$\mathrm{RECT}(x) * \mathrm{RECT}(x) = \mathrm{TRI}(x)$$

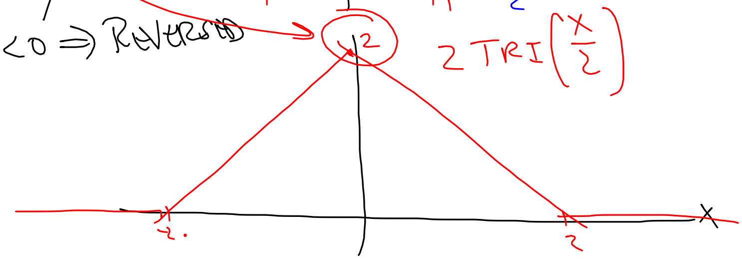
$$\mathrm{RECT}\left(\frac{x}{2}\right) * \mathrm{RECT}\left(\frac{x}{2}\right) =$$

$$f\left(\frac{x}{b_0}\right) * h\left(\frac{x}{b_0}\right) = |b_0|\, g\left(\frac{x}{b_0}\right)$$

· $b_0 < 0 \Rightarrow$ REVERSED



$2\,\mathrm{TRI}\left(\frac{x}{2}\right)$



CORRELATION

$$f(x) \otimes h(x) = \int f(\alpha)\, h(x-\alpha)\, d\alpha$$

$$m(\alpha)$$

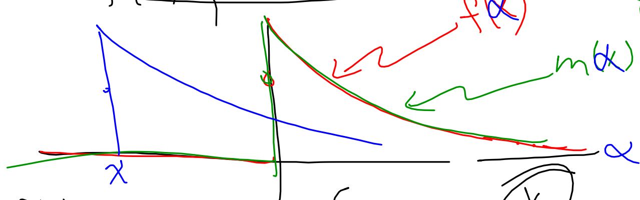
$$f(x) \star \underbrace{m(x)} = g(x) = \int_{-\infty}^{\infty} f(\alpha)\, m(\alpha \pm x)\, d\alpha$$

$$\alpha - x$$

"MATCHING" FUNCTION

$f(x)$ & $m(x)$ ARE REAL

e^{-x} STEP(x)



AUTOCORRELATION

$$g(0) = \int f(\alpha)\, f^{*}[\alpha - 0]\, d\alpha$$

$$\int \left| f(\alpha) \right|^2 d\alpha \quad \text{IF } X = 0$$

$$\underline{f(x) \star f(x)} = \int_{-\infty}^{+\infty} f(\alpha)\, f^*(\alpha - x)\, d\alpha$$

$$= \underline{f(x) * f^*(-x)}$$

CROSSCORRELATION

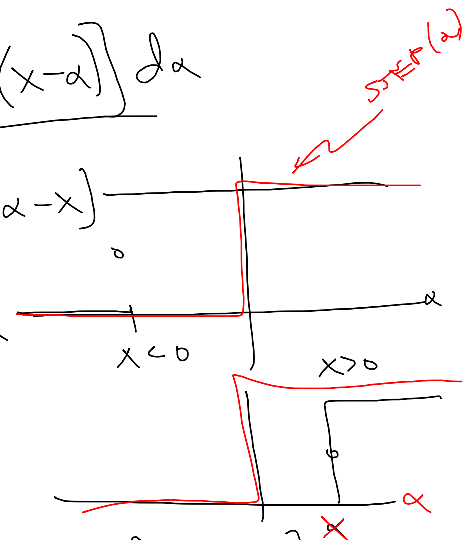
$$f(x) \star m(x) = f(x) * m^*(-x)$$

$$e^{-x}\,\mathrm{STEP}[x] \star e^{-x}\,\mathrm{STEP}[x] = e^{-x}\,\mathrm{STEP}[x] * e^{-(-x)}\,\mathrm{STEP}[-x]$$

$$f[x] \star f[x] = f[x] * f^{*}[-x]$$

$$e^{-x}\,\mathrm{STEP}[x] \;\alpha\; e^{+x}\,\mathrm{STEP}[-x]$$

$$\int_{-\infty}^{+\infty} e^{-\alpha}\,\underline{\mathrm{STEP}[\alpha]} \cdot e^{x-\alpha}\,\underline{\mathrm{STEP}[-(x-\alpha)]}\,d\alpha$$



$$\int e^{-\alpha}\, e^{x-\alpha}\,\mathrm{STEP}(\alpha)\,\mathrm{STEP}[\alpha - x]\,d\alpha$$

$x<0$

$$x<0 \quad e^{x}\int_{0}^{\infty} e^{-2\alpha}\,d\alpha = e^{x}\left.\frac{e^{-2\alpha}}{-2}\right|_{\alpha=0}^{\alpha=\infty} = e^{x}\cdot\left[0-\frac{1}{-2}\right] = \frac{1}{2}e^{x}$$

$x<0$

$$x>0 \quad e^{x}\int_{x}^{\infty} e^{-2\alpha}\,d\alpha = e^{x}\left.\frac{e^{-2\alpha}}{-2}\right|_{\alpha=x}^{\alpha=\infty} = e^{x}\left(0-\frac{e^{-2x}}{-2}\right)$$

$$= e^{x}\,\frac{e^{-2x}}{+2} = \frac{e^{-x}}{2} \quad x>0$$

$$\left\{\begin{array}{lll} \frac{e^{x}}{2} & x<0 & \frac{e^{-|x|}}{2} \\ \frac{e^{-x}}{2} & x>0 & \frac{e^{-|x|}}{2} \end{array}\right\} \frac{e^{-|x|}}{2}$$

$$e^{+i\pi x^2} \star e^{+i\pi x^2} = e^{+i\pi x^2} * \left(e^{+i\pi(-x)^2}\right)^*$$

$$f(x) * f^*(-x)$$

$$= e^{+i\pi x^2} * e^{-i\pi x^2} = \delta(x)$$

"STOCHASTIC" FUNCTIONS = "NOISE"

Fourier Transforms

Analysis $\underset{\sim}{x} \longrightarrow \underset{\sim}{x}' = \underset{\sim}{D}^{-1}\underset{\sim}{x} = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}(\underset{\sim}{x})_n\left(e^{+i2\pi n\cdot\frac{k}{N}}\right)^*$

Synthesis $\underset{\sim}{x}' \longrightarrow \underset{\sim}{x} = \underset{\sim}{D}\,\underset{\sim}{x}' = \frac{1}{\sqrt{N}}\sum_{k=0}^{N}\left(\underset{\sim}{x}'\right)_k e^{+i2\pi\frac{k}{N}\cdot n}$

$$\mathcal{F}\{f(x)\} = F(\xi) = \int_{-\infty}^{+\infty} f(x)\cdot\left(e^{+i2\pi\xi x}\right)^* dx$$

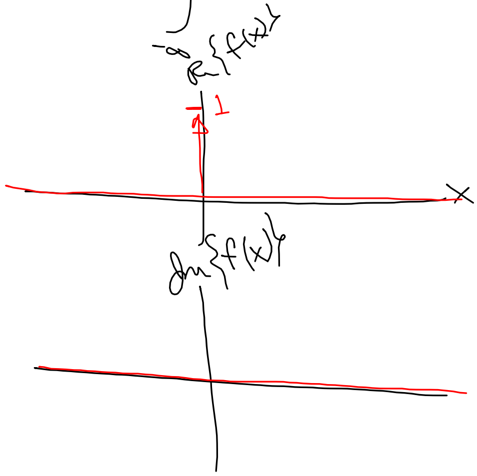
$$= \int_{-\infty}^{+\infty} f(x)\, e^{-i2\pi\xi x}\, dx$$

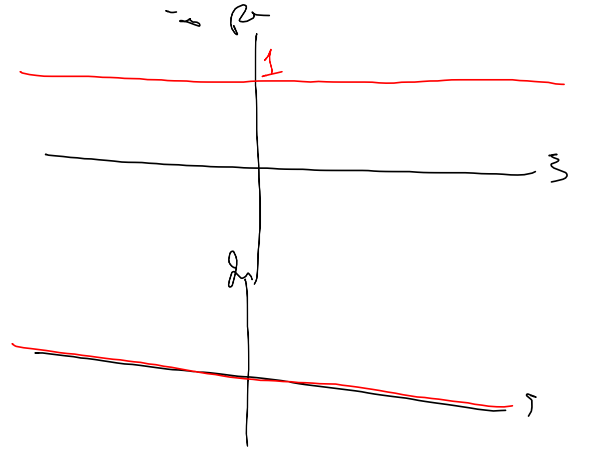
$$\mathcal{F}^{-1}\{F(\xi)\} = f(x) = \int_{-\infty}^{+\infty} F(\xi)\, e^{+i2\pi\xi x}\, d\xi$$

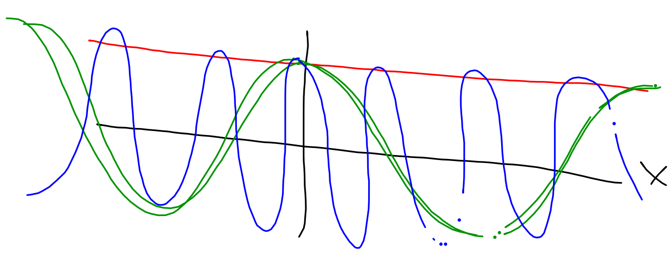
$$f(x) = \delta(x) \qquad \mathcal{F}\{\delta(x)\} = \int_{-\infty}^{+\infty} \delta(x)\, e^{-i2\pi\xi x}\, dx$$

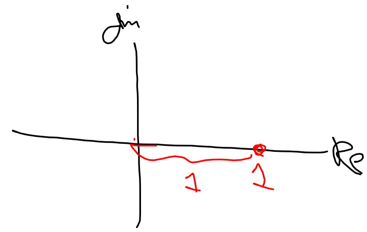
$$f(x)\,\delta(x-x_0) = f(x_0)\,\underline{\underline{\delta(x-x_0)}}$$

$$\int_{-\infty}^{+\infty} \delta(x-0)\, e^{-i2\pi\xi\cdot 0}\, dx = \int_{-\infty}^{+\infty} \delta(x)\cdot 1\, dx = \underline{1}[\xi]$$









$$\delta(x-x_0)$$

$$\mathcal{F}\{\delta(x-x_0)\} = \int_{-\infty}^{+\infty} \delta(x-x_0)\, e^{-i2\pi\xi x}\, dx$$

$$= \int_{-\infty}^{+\infty} \delta(x-x_0)\, e^{-i2\pi\xi x_0}\, dx$$

$$= e^{-i2\pi\xi x_0} \underbrace{\int_{-\infty}^{+\infty} \delta(x-x_0)\, dx}_{1}$$

$$\mathcal{F}\{\delta(x-x_0)\} = e^{-i2\pi\xi x_0} \cdot \underbrace{1(\xi)}$$

$$\Phi\{F(\xi)\} = -2\pi\xi x_0; \quad |F(\xi)| = 1(\xi)$$

$$\mathcal{F}\{Rect(x)\} = \int_{-\infty}^{+\infty} Rect(x)\, e^{-i2\pi\xi x}\, dx$$

$$= \int_{-1/2}^{+1/2} 1\; e^{-i2\pi\xi x}\, dx = \left. \frac{e^{-i2\pi\xi x}}{-i2\pi\xi} \right|_{x=-1/2}^{x+1/2}$$

$$= \frac{e^{-i2\pi\xi\cdot(+1/2)} - e^{-i2\pi\xi(-1/2)}}{-i2\pi\xi}$$

$$= \frac{e^{+i\pi\xi} - e^{-i\pi\xi}}{2i\,(\pi\xi)} = \frac{\sin(\pi\xi)}{\pi\xi} \equiv \mathrm{Sinc}(\xi)$$