7 OCTOBER 2014

OPERATIONS

$$\mathcal{O}\{f(x)\} = g(x')$$

$$\mathcal{F}\{f(x)\} = g(\xi)$$

LENGTH^{-1}, e.g., $\frac{\text{cycles}}{\text{mm}}$

$[x]$ = LENGTH

$$\mathcal{O}\{f(x)\} = g(x)$$

2 CONSTRAINTS

(1) LINEARITY, ACTION OF $\mathcal{O}$ ON AMPLITUDE f

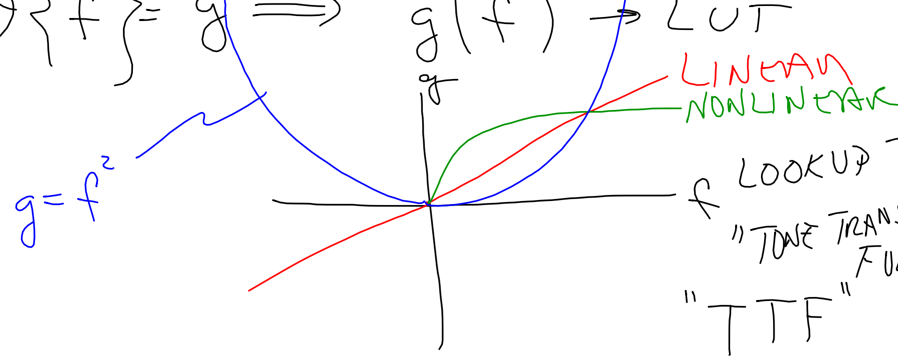
IF $\mathcal{O}\{f_n(x)\} = g_n(x)$

THEN $\mathcal{O}\left\{\sum_n \alpha_n f_n(x)\right\} = \sum_n \alpha_n \mathcal{O}\{f_n(x)\}$

$$= \sum_n \alpha_n g_n(x)$$

NONLINEAR

$\mathcal{O}\{f\} = g \Longrightarrow g(f) \rightarrow$ LUT



LOOKUP TABLE

"TONE TRANSFER FUNCTION"

"TTF"

(2) SHIFT INVARIANCE, ACTION OF $\mathcal{O}$ ON X

IF $\mathcal{O}\left\{f(x)\right\} = g(x)$, THEN (

$$\mathcal{O}\left\{f(x-x_0)\right\} = g(x-x_0)$$

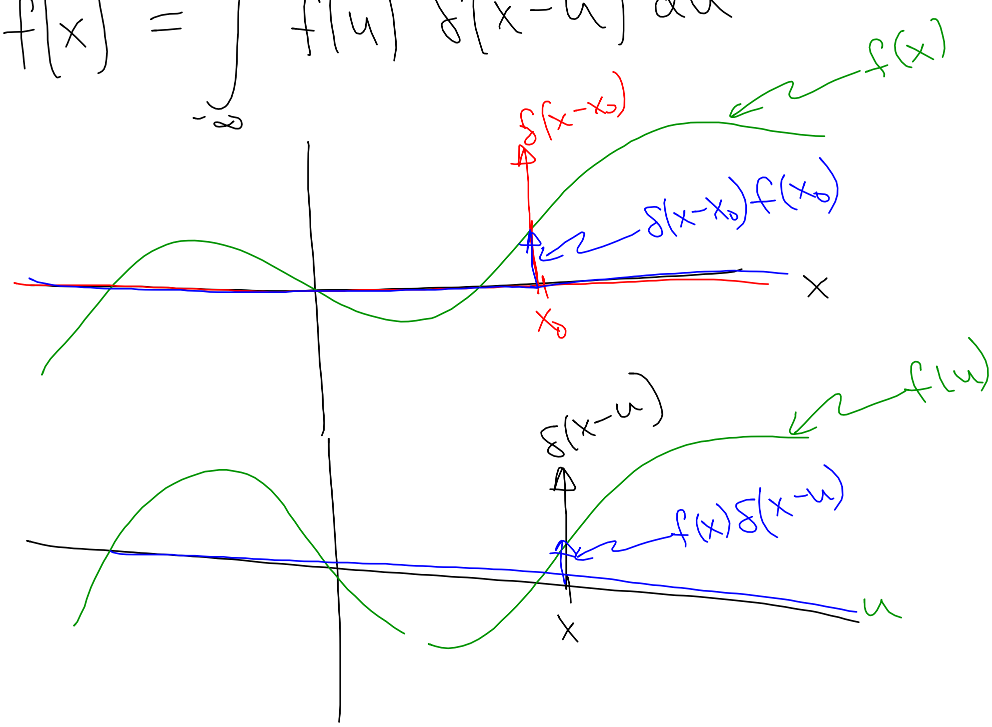
LSI OPERATOR

$$\mathcal{O}\{f(x)\} = g(x)$$

RECALL: $$f(x_0) = \int_{-\infty}^{+\infty} f(x)\, \underbrace{\delta(x_0 - x)}_{\delta(x - x_0)}\, dx$$

$$f(x_0) = \int_{-\infty}^{+\infty} f(u)\, \delta(x_0 - u)\, du$$

$$f(x) = \int_{-\infty}^{\infty} f(u)\, \delta(x - u)\, du$$



$$f(x) = \mathcal{L}\left\{f(x)\right\} = \int_{-\infty}^{+\infty} f(u)\ \delta(x-u)\ du$$

$$\text{LSI } \mathcal{O}\left\{f(x)\right\} = g(x)$$

$$\mathcal{O}\left\{\int_{-\infty}^{+\infty} f(u)\ \delta(x-u)\ du\right\} = g(x)$$

$$g(x) = \int_{-\infty}^{+\infty} \mathcal{O}\left\{f(u)\ \delta(x-u)\right\} du \qquad \text{BY LINEARITY}$$

$\mathcal{O}$ ACTS OF FUNCTIONS OF x

$$g(x) = \int_{-\infty}^{+\infty} f(u)\ \mathcal{O}\left\{\delta(x-u)\right\}\ du \qquad \mathcal{O}\left\{\alpha f(x)\right\} = \alpha\, \mathcal{O}\left\{f(x)\right\}$$

$$\mathcal{O}\left\{\delta(x-u)\right\} \equiv h(x-u), \text{ WHERE } \mathcal{O}\left\{\delta(x)\right\} = h(x)$$

BY SI

IMPULSE

IMPULSE RESPONSE

psf

$$\mathcal{O}\{f(x)\} = \int_{-\infty}^{+\infty} f(u)\, \mathcal{O}\{\delta(x-u)\}\, du$$

$$g(x) \equiv \int_{-\infty}^{+\infty} f(u)\, h(x-u)\, du \quad \text{IF } \mathcal{O} \text{ IS LSI}$$

CONVOLUTION INTEGRAL

ACTION OF $\mathcal{O}$ SPECIFIED COMPLETELY BY $h(x)$

$\mathcal{O} \Longrightarrow \underset{\sim}{A}$ CIRCULANT

$$g(x) = \underbrace{f(x) * h(x)}_{\theta} \equiv \int_{-\infty}^{+\infty} f(\alpha)\, h(x-\alpha)\, d\alpha$$

$$\int_{\alpha=-\infty}^{\alpha=+\infty} f(\alpha)\, h(x-\alpha)\, d\alpha$$

$$v \equiv x-\alpha \Rightarrow \alpha = x - v\,,\quad d\alpha = d(x-v)$$
$$= \cancel{dx} - dv$$
$$= -dv$$

$$\alpha = \pm\infty \Rightarrow v = \mp\infty$$

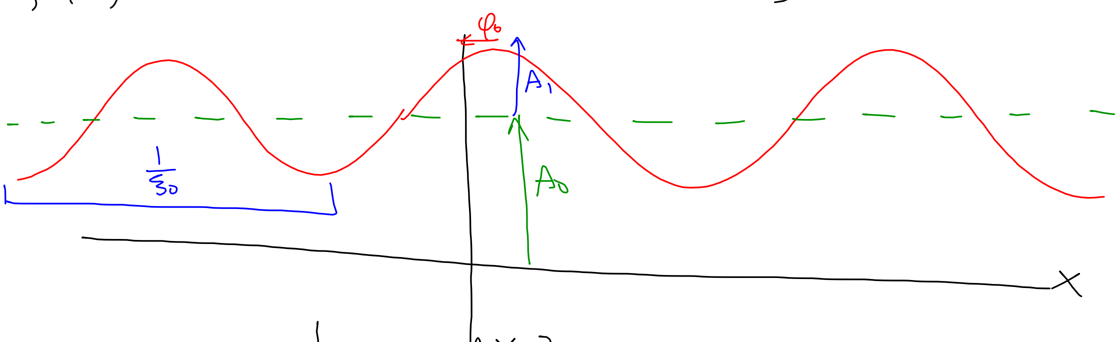
$$g(x) = \int_{v=+\infty}^{v=-\infty} f[x-v]\, h[v]\,(-dv) = \int_{v=-\infty}^{v=+\infty} f(x-v)\, h(v)\, dv$$

$$g(x) = f(x) * h(x) = h(x) * f(x)$$

CONVOLUTION COMMUTES

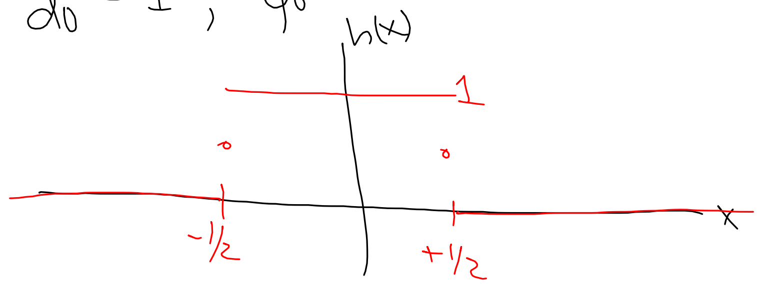
INPUT & IMPULSE RESPONSE MAY SWAP ROLES

$$f(x) = A_0 + A_1 \cos[2\pi \xi_0 x + \varphi_0]$$

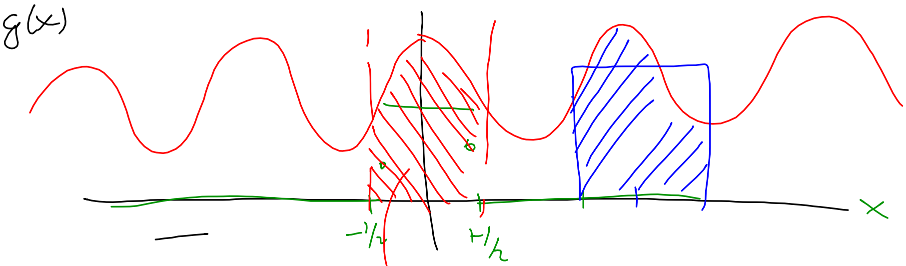


$$h(x) = \frac{1}{|d_0|} \mathrm{RECT}\left[\frac{x}{d_0}\right]; \ \text{AREA} = 1$$

$d_0 = 1$, $\varphi_0 = 0$ RADIANS



$f(x) * h(x) = g(x)$



h(x) IS AN "INTEGRATOR" OVER FINITE LIMITS

$$g(x) = \mathcal{O}\{f(x)\} = \int_{u=-\infty}^{u=+\infty} f(u)\, h(x-u)\, du$$

$$= \int_{u=-\infty}^{u=+\infty} \left(A_0 \cdot 1(u) + A_1 \cos(2\pi\xi_0 u)\right) \mathrm{RECT}[x-u]\, du$$

$$= A_0 \int_{-\infty}^{+\infty} 1(u)\, \mathrm{RECT}(x-u)\, du + A_1 \int_{-\infty}^{+\infty} \cos(2\pi\xi_0 u)\, \mathrm{RECT}(x-u)\, du$$

$$\mathrm{RECT}(x-u) = \begin{cases} 0 & \text{IF } x-u < -1/2 \quad u = x+1/2 \\ 1 & \text{IF } -1/2 < x-u < 1/2 \\ 0 & \text{IF } x-u > +1/2 \quad u = x-1/2 \end{cases}$$

$$= A_0 \int_{u=x-1/2}^{u=x+1/2} 1(u)\, du + A_1 \int_{u=x-1/2}^{x+1/2} \cos(2\pi\xi_0 u) \cdot 1\, du$$

$$A_0 \int_{x-1/2}^{x+1/2} 1 \, du = A_0\left[\left(\cancel{x}+1/2\right) - \left(\cancel{x}-1/2\right)\right] = A_0$$

$$A_1 \int_{x-1/2}^{x+1/2} \cos(2\pi\xi_0 u)\, du = A_1 \left.\frac{\sin(2\pi\xi_0 u)}{2\pi\xi_0}\right|_{u=x-1/2}^{u=x+1/2}$$

$$= \frac{A_1}{2\pi\xi_0}\left[\sin\left(2\pi\xi_0\left(x+1/2\right)\right) - \sin\left(2\pi\xi_0\left(x-1/2\right)\right)\right]$$

$$\left[\left(\cancel{\sin(2\pi\xi_0 x)\cos(\pi\xi_0)} + \cos(2\pi\xi_0 x)\sin(\pi\xi_0)\right) + \left(\cancel{\sin(2\pi\xi_0 x)\cos(\pi\xi_0)} + \cos(2\pi\xi_0 x)\sin(\pi\xi_0)\right)\right]$$

$$= \frac{A_1}{\cancel{2}\pi\xi_0}\cancel{2}\cos(2\pi\xi_0 x)\sin(\pi\xi_0)$$

$$= A_1 \cos(2\pi\xi_0 x)\frac{\sin(\pi\xi_0)}{\pi\xi_0}$$

$$g(x) = \mathcal{O}\{A_0 + A_1 \cos(2\pi \xi_0 x)\}$$

$$= A_0 + A_1 \left[\frac{\sin(\pi \xi_0)}{(\pi \xi_0)}\right] \cos(2\pi \xi_0 x)$$

$\mathrm{SINC}(\xi_0)$

$$g(x) = A_0 + A_1 \,\mathrm{SINC}(\xi_0) \cos(2\pi \xi_0 x)$$

$$f(x) = A_0 + A_1 \cos(2\pi \xi_0 x); \quad m_f = \frac{f_{max} - f_{min}}{f_{max} + f_{min}}$$

$$= \frac{(A_0 + A_1) - (A_0 - A_1)}{(A_0 + A_1) + (A_0 - A_1)} = \frac{A_1}{A_0}$$

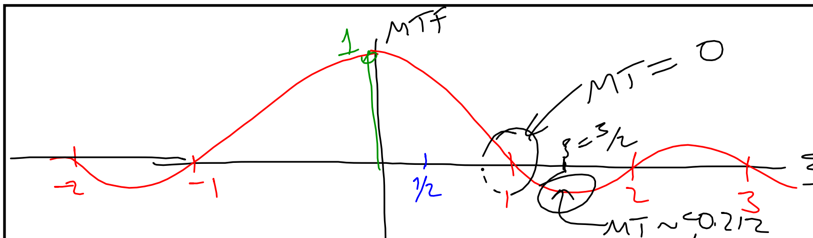
$$m_g = \frac{\left(A_0 + A_1 \,\mathrm{SINC}(\xi_0)\right) - \left(A_0 - A_1 \,\mathrm{SINC}(\xi_0)\right)}{A_0 + A_1 \,\mathrm{SINC}(\xi_0) + A_0 - A_1 \,\mathrm{SINC}(\xi_0)}$$

$$= \frac{A_1}{A_0} \mathrm{SINC}(\xi_0)$$

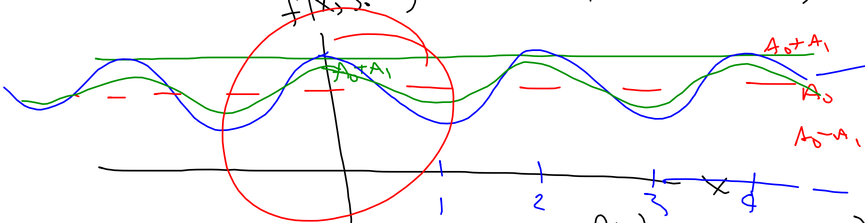
$$\frac{m_g(\xi_0)}{m_f(\xi_0)} = \frac{\frac{A_1}{A_0} \mathrm{SINC}(\xi_0)}{\frac{A_1}{A_0}} = \mathrm{SINC}(\xi_0) \quad MT(\xi_0)$$

$$MT(\xi) = \mathrm{SINC}(\xi) = MTF$$

$$\mathrm{SINC}\left(\tfrac{3}{2}\right) = \frac{\sin \frac{3}{2}\pi}{\frac{3}{2}\pi} = -\frac{2}{3\pi} \simeq -0.212$$



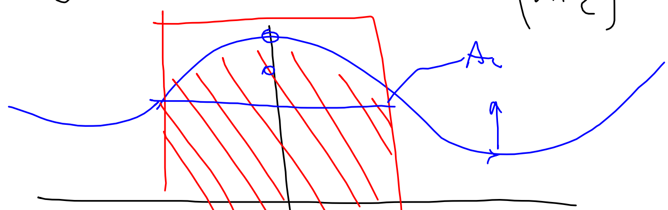
$$f(x; \xi = 0) = A_0 + A_1 \cos(2\pi \cdot 0 \cdot x) = A_0 + A_1$$



$$f(x) = A_0 + A_1 \cos(\pi x)$$

$$\mathrm{SINC}\left(\tfrac{1}{2}\right) = \frac{\sin \frac{\pi}{2}}{\frac{\pi}{2}} = \frac{1}{\pi/2} = \frac{2}{\pi} \simeq 0.637$$

$$g(x) = A_0 + A_1\, \mathrm{SINC}\left[\tfrac{1}{2}\right] \cos\left(2\pi \cdot \tfrac{1}{2} \cdot x\right); \quad \left(2\pi \frac{x}{2}\right)$$



$h(x) = \mathrm{RECT}(x)$ IS A "LOCAL AVERAGER", PUSHES EXTREMA TOWARDS MEAN AMPLITUDE

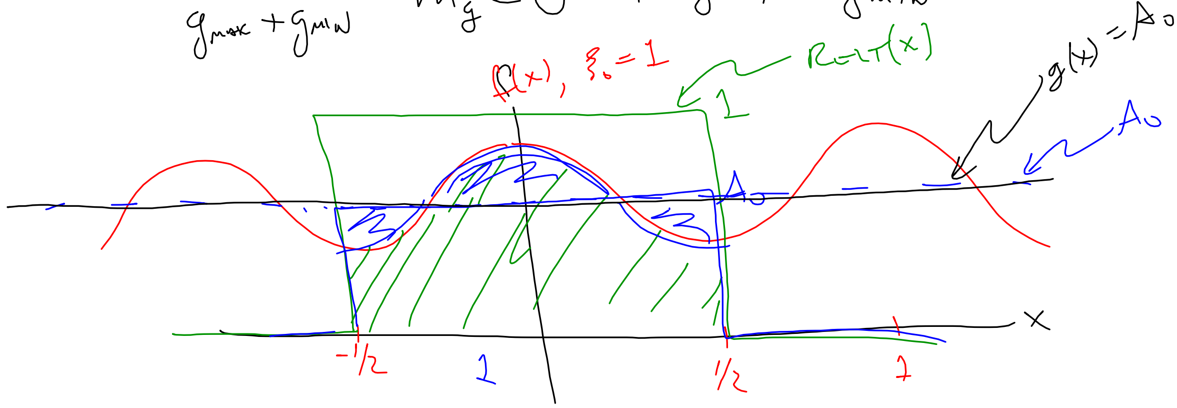
LOCAL AVERAGERS REDUCE "VARIABILITY"

$\xi_0 = 1/2 \quad \mathrm{MTF} = \frac{2}{\pi} \cong 0.637$

AT $\xi_0 = +1$ $\quad MT = 0 = \mathrm{sinc}(1)$

$$\frac{m_g}{m_f} = \frac{\frac{A_1}{A_0}\,\mathrm{sinc}(1)}{\left(\frac{A_1}{A_0}\right)} = 0$$

$$\frac{g_{max} - g_{min}}{g_{max} + g_{min}} = m_g = 0 \Rightarrow g_{max} = g_{min}$$



$$A_0 \cdot 1 = A_0 = g(0) \qquad g(x) = A_0$$

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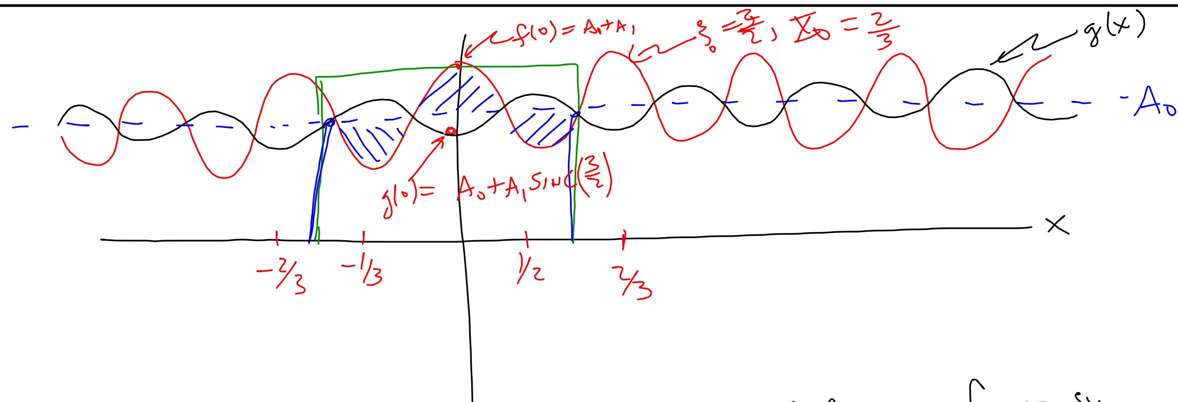
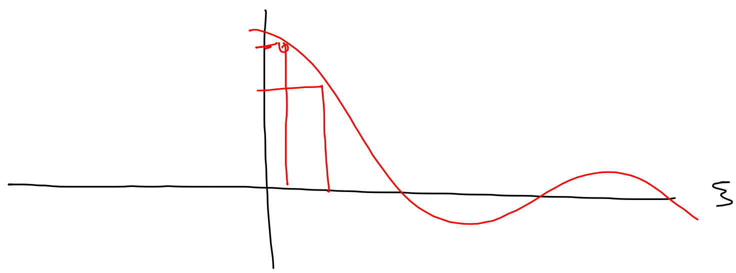


$$A_0 + A_1 \cos(2\pi \xi_0 x) = \underbrace{A_0\, 1(x)}_{\xi = 0} + \frac{A_1}{2}\Big(\underbrace{e^{+i2\pi \xi_0 x}}_{\xi = +\xi_0} + \underbrace{e^{+i2\pi(-\xi_0)x}}_{\xi = -\xi_0}$$

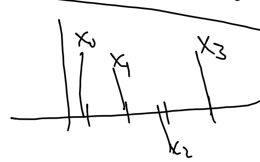


$$f(x) * h(x) = g(x)$$

$$\underset{\sim}{A}\,\underset{\sim}{x} = \underset{\sim}{b}$$

$$\begin{bmatrix} \alpha & \beta & \gamma & \delta \\ \delta & \alpha & \beta & \gamma \\ \gamma & \delta & \alpha & \beta \\ \beta & \gamma & \delta & \alpha \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} ; \quad \underset{\sim}{A}\,\underset{\sim}{x}' = \underset{\sim}{b}'$$

$$\begin{bmatrix} \lambda_0 & & & 0 \\ & \lambda_1 & & \\ & & \lambda_2 & \\ 0 & & & \lambda_3 \end{bmatrix}$$



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$$\underset{}{f(x)} * \overset{\text{psf}}{h(x)} = g(x)$$

$$F(\xi) \cdot \underset{\text{TF}}{H(\xi)} = G(\xi)$$

$$F(\xi) = \frac{G(\xi)}{H(\xi)}$$

$$\underset{\sim}{A}\,\underset{\sim}{x} = \underset{\sim}{b}$$

$$\underset{\sim}{\Lambda}\,\underset{\sim}{x}' = \underset{\sim}{b}'$$