September 23, 2014

DISCRETE FOURIER TRANSFORM DFT

$$\underset{\sim}{D}^{-1}\underset{\sim}{x} = \underset{\sim}{x}'$$

$$\frac{1}{\sqrt{N}} \sum_{n}^{N} f(n) \left(e^{+i 2\pi n \frac{k}{N}}\right)^{*}$$

$1, \frac{1}{\sqrt{N}}, \frac{1}{N}$

$\frac{k}{N} \rightarrow \xi$

FFT = FAST FOURIER TRANSFORM 1965

32-SAMPLE DFT $\quad N = 2^{m} \quad$ 256, 512, 1024 ...

$x = n \cdot \Delta x$

$$\underset{\sim}{x} \longrightarrow f(n) \longrightarrow f(x)$$

$$\underset{\sim}{x}' = \underset{\sim}{D}^{-1} \underset{\sim}{x}$$

$$\underset{\sim}{x} = \underset{\sim}{D}\, \underset{\sim}{x}'$$

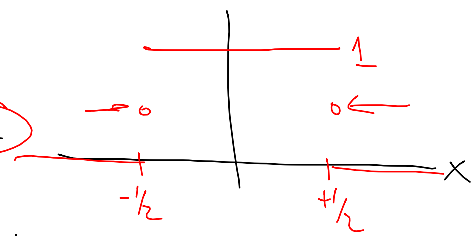
$$\sum_n f(n)\, e^{-i 2\pi n \frac{k}{N}} = F(k)$$

$$\int_{-\infty}^{\infty} f(x)\, e^{-i 2\pi x \xi}\, dx = F(\xi)$$

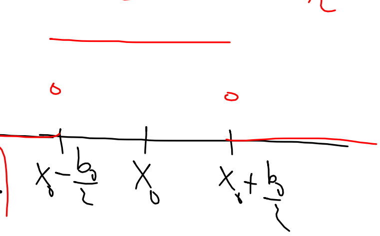
$$f(x) \longrightarrow f(x,y);\ f_r(r,\theta)$$

$$f(x) = \begin{matrix} 1(x) \\ 0(x) \end{matrix}$$

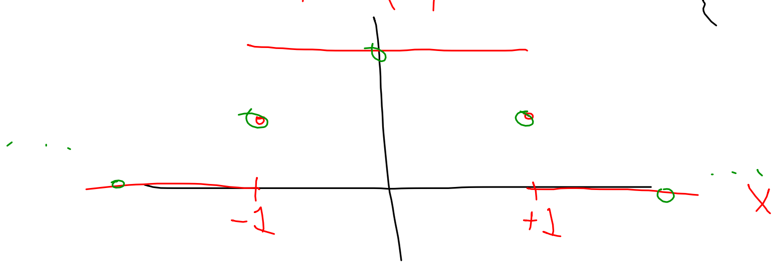
$$\mathrm{RECT}(x) \equiv \begin{cases} 1 & |x| < \frac{1}{2} \\ 1/2 & |x| = 1/2 \\ 0 & |x| > \frac{1}{2} \end{cases}$$

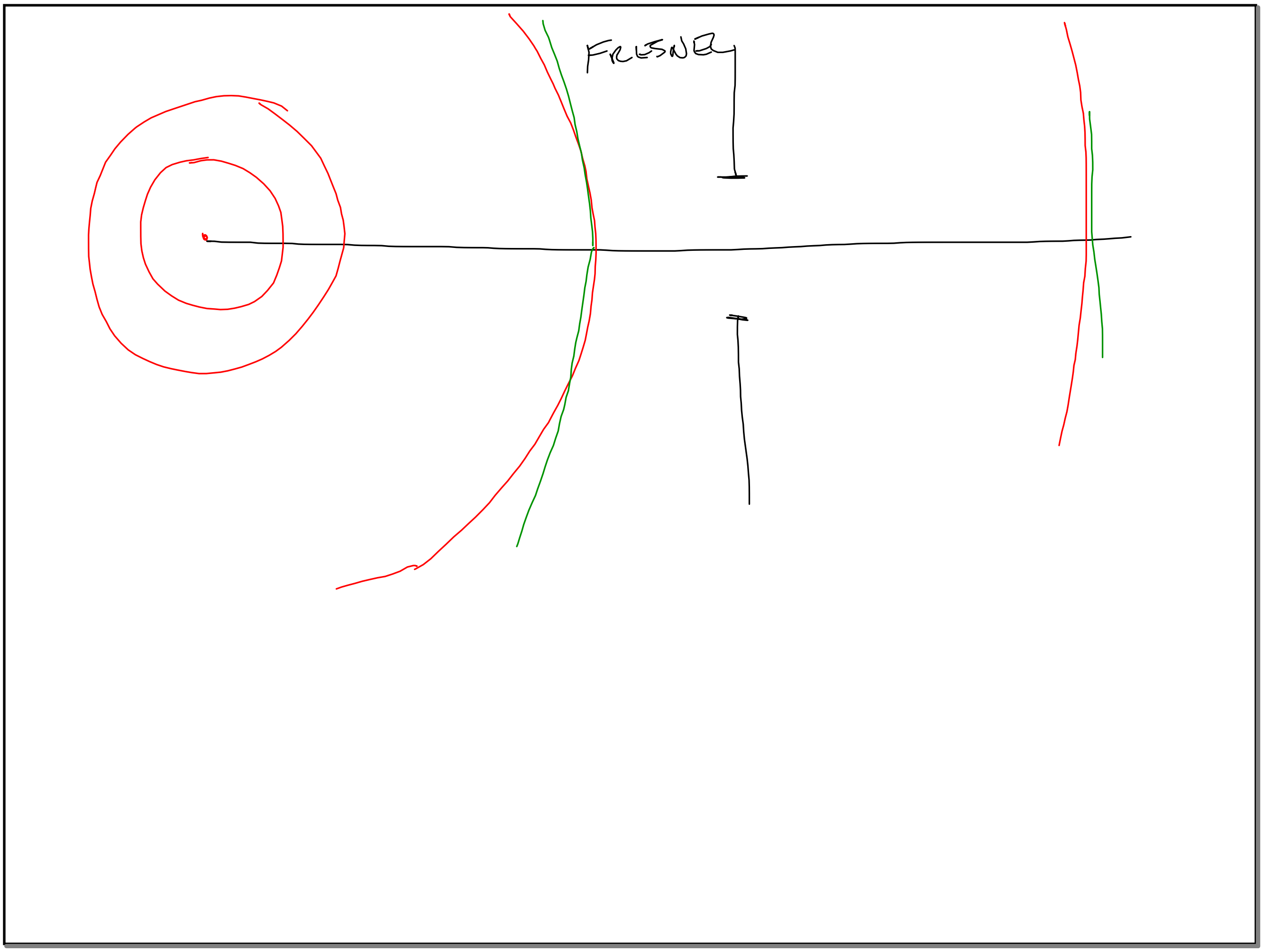


$$\mathrm{RECT}\left(\frac{x - x_0}{b_0}\right)$$

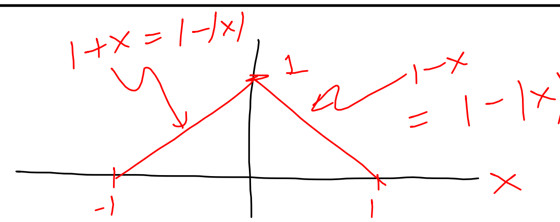


$\mathrm{RECT}\left(\frac{x}{2}\right)$

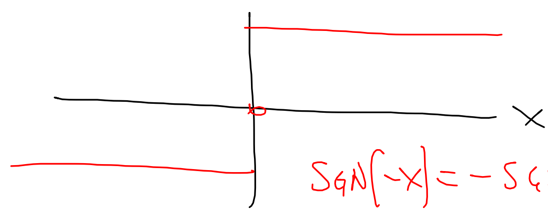



FRESNEL

$$TRI(x) \equiv (1-|x|)\,RECT\left(\frac{x}{2}\right)$$



$$SGN(x) \equiv \begin{cases} 1 & x>0 \\ 0 & x=0 \\ -1 & x<0 \end{cases}$$

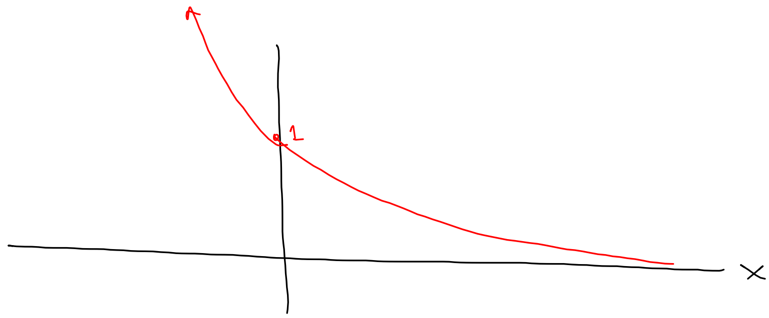


$SGN(-x) = -SGN(x)$

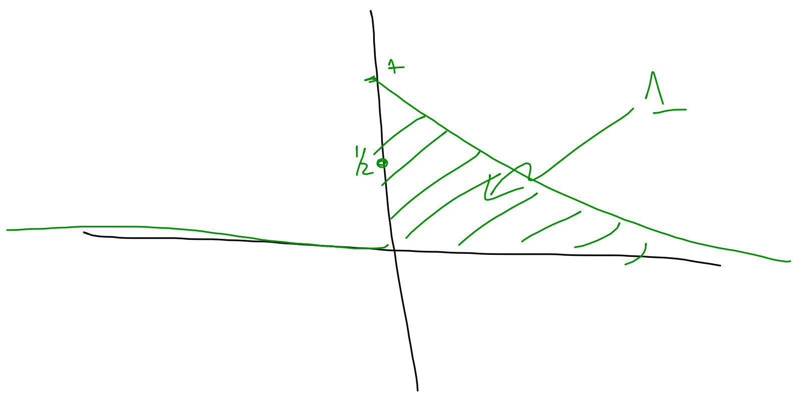
$$STEP(x) \equiv \left.\begin{cases} 1 & x>0 \\ 1/2 & x=0 \\ 0 & x<0 \end{cases}\right\} = \underbrace{\frac{1}{2}\cdot \mathbb{1}(x)}_{\text{even}} + \underbrace{\frac{1}{2}\cdot SGN(x)}_{\text{ODD}}$$

September 23, 2014

e^{-x}

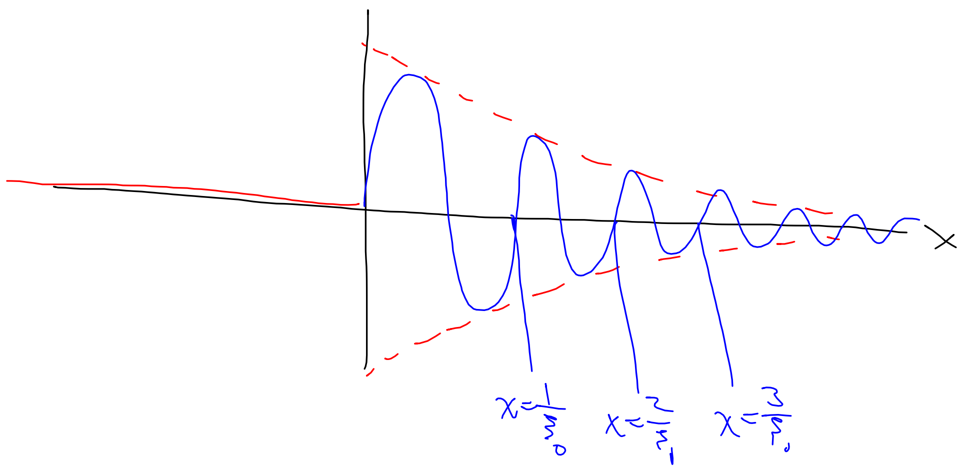


e^{-x} STEP(x) (MODULATE)



September 23, 2014

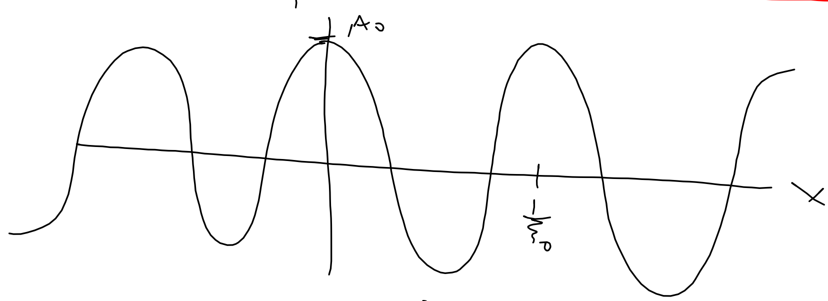
$e^{-x}\,\mathrm{STEP}(x)\cdot \mathrm{SIN}[2\pi\xi_0 x]$



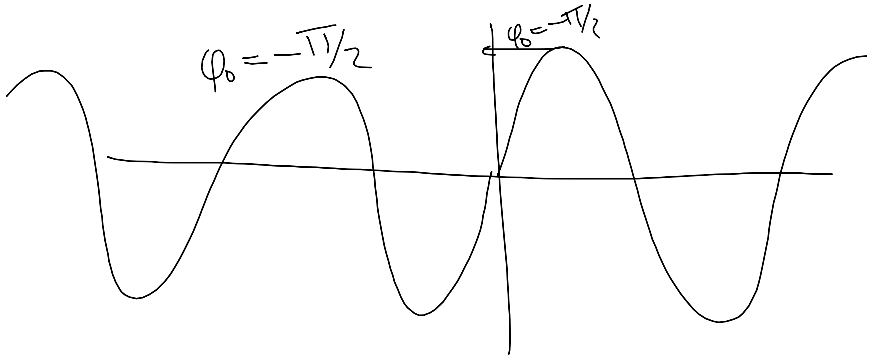
$$A_0 \cos[2\pi \xi_0 x + \varphi_0] = f(x)$$

$$[A_0, \xi_0, \varphi_0]$$

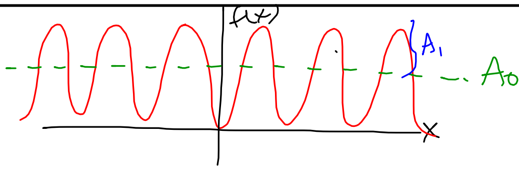
$\varphi_0 = 0$



$$f(x) \text{ s.t. } \frac{d^2}{dx^2} f(x) + \alpha_0^2 f(x) = 0$$



September 23, 2014



$$f_{MAX} = A_0 + A_1 > 0$$

$$f_{MIN} = A_0 - A_1 \geq 0$$

$$f(x) = A_0 + A_1 \cos(2\pi\xi_0 x + \varphi_0) \geq 0$$

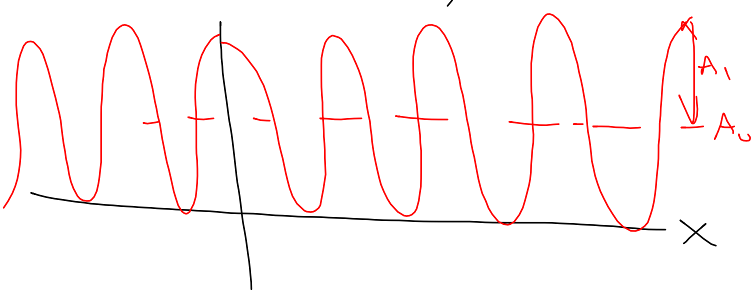
A_0 = CONSTANT PART = "BIAS"

A_1 = AMPLITUDE

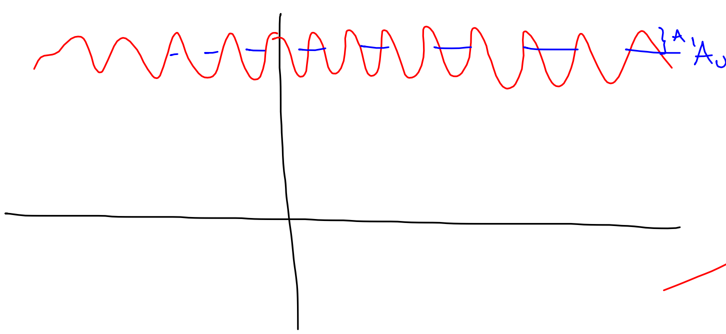
$$A_0 \geq A_1$$

"MODULATION" OF SINUSOID $= m_f \equiv \dfrac{f_{MAX} - f_{MIN}}{f_{MAX} + f_{MIN}}$

$$m_f \equiv \frac{(A_0 + A_1) - (A_0 - A_1)}{(A_0 + A_1) + (A_0 - A_1)} = \frac{2A_1}{2A_0} = \frac{A_1}{A_0} = \frac{\text{AMPLITUDE}}{\text{BIAS}}$$

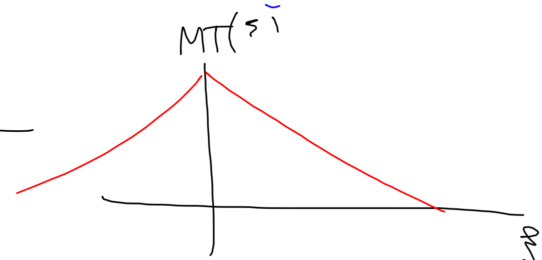


$A_1 = A_0 \Rightarrow m_f = 1$



$$\frac{M_{OUT}(\xi)}{M_{IN}(\xi)} = MT(\xi)$$

$m_f \gtrsim 0$



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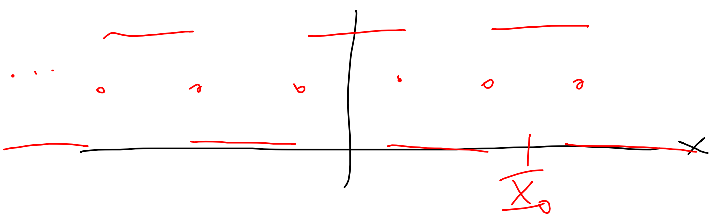
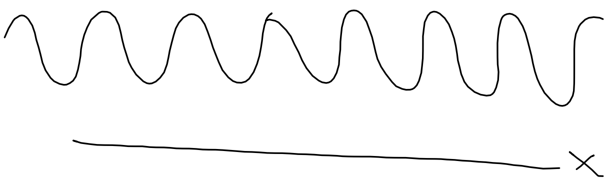
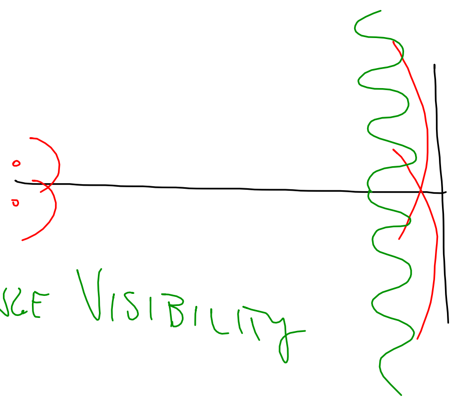


$$\frac{f_{max} - f_{min}}{f_{max} + f_{min}} = \text{'CONTRAST'}$$

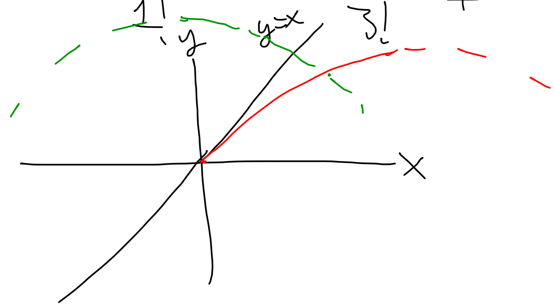




MODULATION $\Longleftrightarrow$ FRINGE VISIBILITY

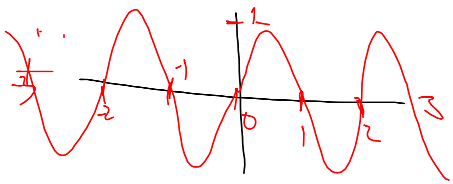
$$\cos\left(2\pi \xi_0 x\right) = 1 - \frac{(2\pi\xi_0 x)^2}{2!} + \frac{(2\pi\xi_0 x)^4}{4!} - \cdots$$

$$\sin\left(2\pi \xi_0 x\right) = \frac{(2\pi\xi_0 x)^1}{1!} - \frac{(2\pi\xi_0 x)^3}{3!} + \cdots$$

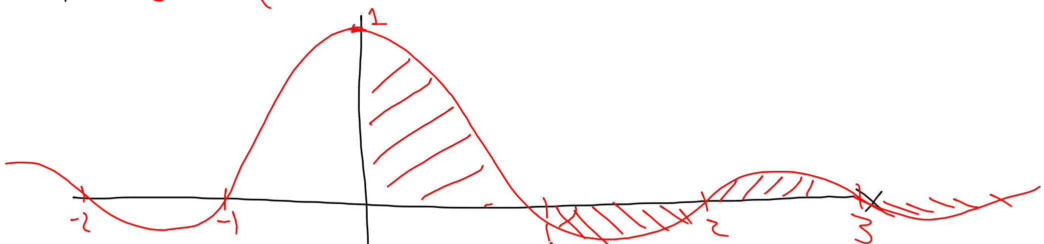


$$\mathrm{SINC}(x) \equiv \frac{\sin \pi x}{\pi x} = \frac{\pi x - \frac{(\pi x)^3}{3!} + \frac{(\pi x)^5}{5!} - \cdots}{\pi x}$$

$$\sin \pi x = \sin\left(2\pi \frac{x}{2}\right)$$



$$= 1 - \frac{(\pi x)^2}{6} + \frac{(\pi x)^4}{120} - \cdots$$



$$\mathrm{SINC}\left(\tfrac{1}{2}\right) = \frac{\sin\left(\frac{\pi}{2}\right)}{\pi/2} = \frac{2}{\pi} \lesssim 2/3$$

$$\mathrm{SINC}\left(\tfrac{3}{2}\right) = \frac{\sin\left(\frac{3\pi}{2}\right)}{3\pi/2} = -\frac{2}{3\pi} \gtrsim 0.2$$

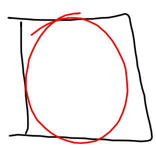
$$\int_{-\infty}^{+\infty} \mathrm{SINC}(x)\,dx = 1$$

$$\mathrm{RECT}(x) \Longleftrightarrow \mathrm{SINC}(\xi)$$

$\mathrm{SINC}^2[x]$, AREA = 1

$\mathrm{TRI}(x) \rightleftarrows \mathrm{SINC}^2[\xi]$

$\ast$

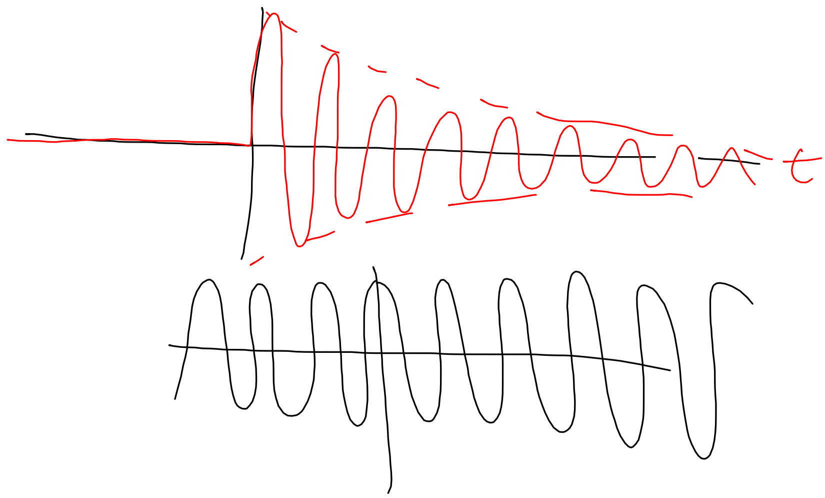


$$e^{-\pi x^2} = \text{GAUSSIAN} \qquad e^{-\pi(x^2+y^2)}$$

$$\int_{-\infty}^{+\infty} e^{-\pi x^2}\,dx = 1$$

$$\text{LOR}[x] \qquad f(x) = \frac{2}{1+(2\pi x)^2} \qquad \text{SPECTRAL LINES}$$

$$e^{-x}\,\text{STEP}(x) \Longleftrightarrow \frac{2}{1+(2\pi\xi)^2}$$


t

BESSEL FUNCTIONS

$$x^2 \frac{d^2}{dx^2} f_\nu(x) + x \frac{d}{dx} f_\nu(x) + (x^2 - \nu^2) f_\nu(x) = 0$$

$$\frac{d^2}{dx^2} f(x) + \alpha_0^2 f(x) = 0 \qquad \text{SINUSOID}$$

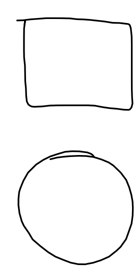
COSINE

$$f_\nu(x) \longrightarrow J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots$$

$$J_1(x) = \frac{x}{2} - \frac{x^3}{2^2 \cdot 4} + \frac{x^5}{2^2 \cdot 4^2 \cdot 6} - \frac{x^7}{2^2 \cdot 4^2 \cdot 6^2 \cdot 8}$$

SINE

$$\mathrm{SINC}(x) = \frac{\sin \pi x}{\pi x} \propto \frac{1}{x}$$



$$\mathrm{SINC}^2(x) \propto \frac{1}{x^2}$$

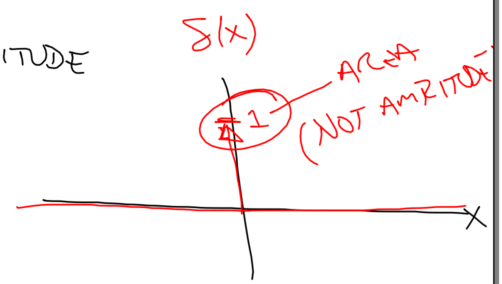
$$J_0(x) \propto \frac{1}{\sqrt{x}}$$

$$\mathrm{BESINC}(x) \propto \frac{J_1(\pi x)}{\pi x} \propto \frac{x^{-1/2}}{x^{-1}} \propto x^{-3/2}$$

DIRAC DELTA FUNCTION: "IMPULSE"

"STAR" INFINITESIMAL AREA

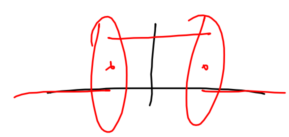
FINITE AMPLITUDE



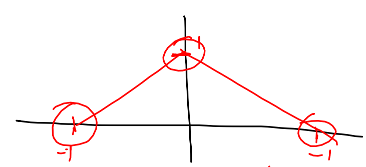
(1) $\delta(x) \equiv 0$ IF $x \neq 0$

(2) $\int_{-\infty}^{+\infty} \delta(x)\,dx \equiv 1$

$$\int_{-\infty}^{+\infty} \lim_{b_0 \to 0} \frac{1}{|b_0|} \mathrm{RECT}\left[\frac{x}{b_0}\right] dx = 1$$



TRI(x)

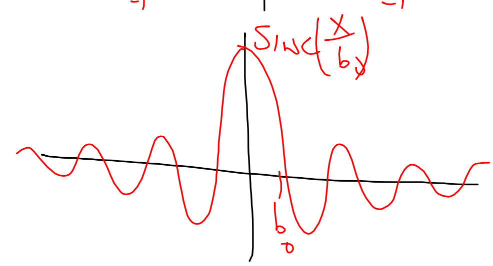


SINC(x)

$\mathrm{SINC}^2(x)$

GAUS

LOR



$$\delta(x) = \lim_{b_0 \to 0} \frac{1}{|b_0|} \mathrm{RECT}\left(\frac{x}{b_0}\right)$$

$$\delta(-x) = \lim_{b_0 \to 0} \frac{1}{|b_0|} \mathrm{RECT}\left(\frac{-x}{b_0}\right) = \delta(+x)$$

$$\delta(-x) = \delta(+x) \Rightarrow \text{EVEN}$$

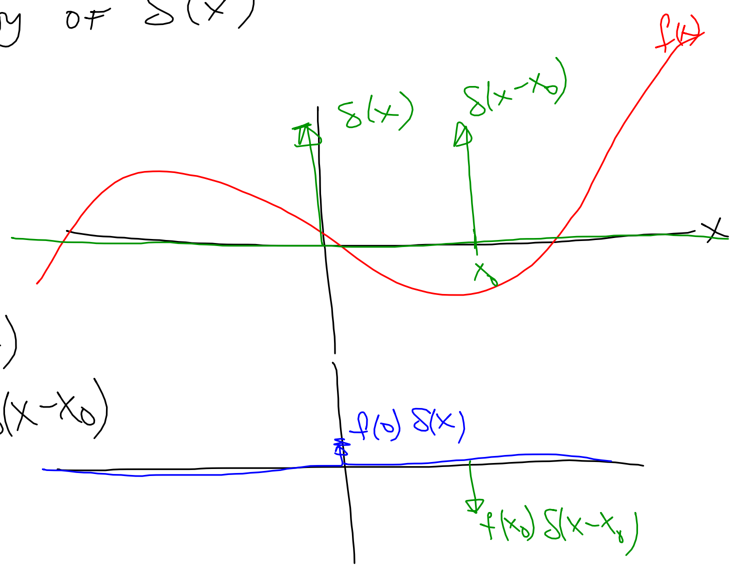
$$\delta\left(\frac{x}{d_0}\right) = \lim_{b_0 \to 0} \frac{1}{|b_0|} \mathrm{RECT}\left(\frac{x/d_0}{b_0}\right) = \lim_{b_0 \to 0} \frac{1}{|b_0|} \mathrm{RECT}\left(\frac{x}{d_0 b_0}\right)$$

AREA $= |d_0|$

$$\boxed{\delta\left(\frac{x}{d_0}\right) = |d_0|\, \delta(x)}$$

Sifting Property of $\delta(x)$

$f(x)\ \delta(x)$

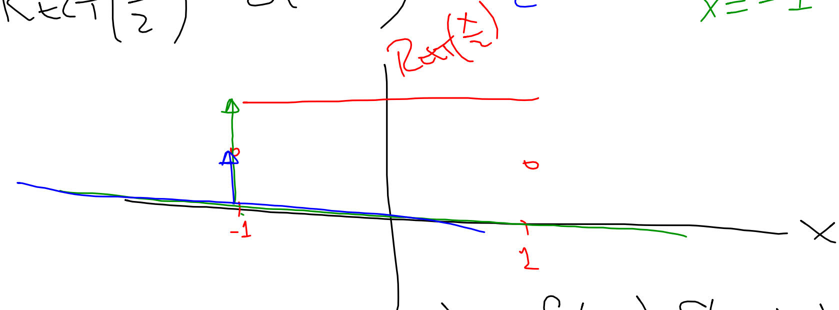


$$f(x)\cdot\delta(x) = f(0)\ \delta(x)$$

$$f(x)\ \delta(x-x_0) = f(x_0)\ \delta(x-x_0)$$

$$\mathrm{Rect}\left(\frac{x}{2}\right) \cdot \delta(x+1) = \frac{1}{2}\,\delta(x+1)$$

$x = -1 \Rightarrow x+1 = 0$



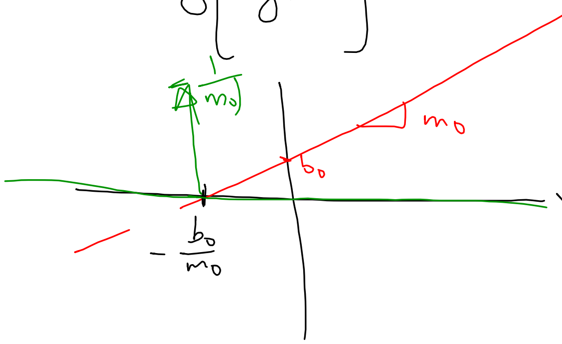
$$f(x)\,\delta(x-x_0) = f(x_0)\,\delta(x-x_0)$$

$$\int_{-\infty}^{+\infty} f(x)\,\delta(x-x_0)\,dx = \int_{-\infty}^{+\infty} f(x_0)\,\delta(x-x_0)\,dx$$

$$\int_{-\infty}^{+\infty} f(x)\,\delta(x-x_0)\,dx = f(x_0) \qquad f(x_0)\underbrace{\int_{-\infty}^{+\infty} \delta(x-x_0)\,dx}_{1}$$

DIRAC DELTA FUNCTION WITH "FUNCTIONAL ARGUMENT"

$$\delta\left[g(x)\right] \qquad \delta\left(\frac{x}{b_0}\right) = |b_0|\,\delta(x)$$



0 IF $m_0x+b_0 \neq 0$

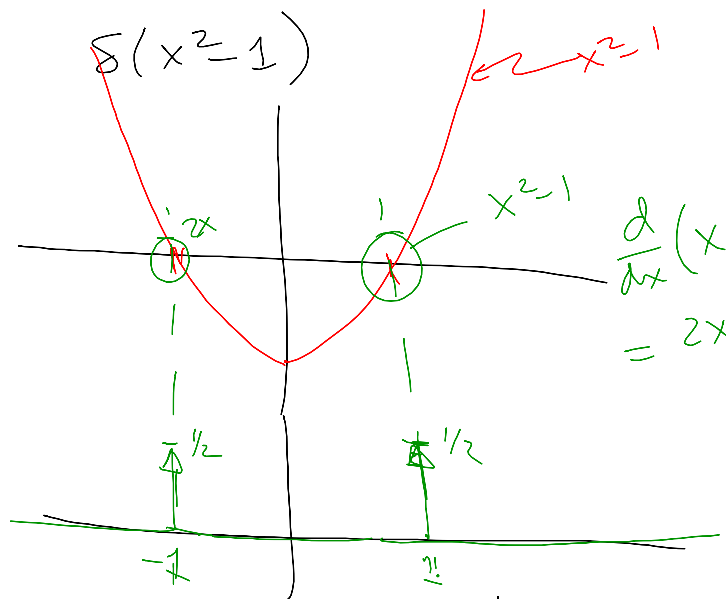
$$\delta(m_0x+b_0) \equiv$$

$$\delta(m_0x+b_0) = \delta\left(\frac{x}{1/m_0}+\frac{\frac{b_0}{m_0}}{1/m_0}\right) = \delta\left(\frac{x+\frac{b_0}{m_0}}{\frac{1}{m_0}}\right)$$

$$= \frac{1}{|m_0|}\,\delta\left(x+\frac{b_0}{m_0}\right)$$

$\delta\left(\frac{x}{b_0}\right) = |b_0|\,\delta(x)$ RECIPROCAL OF SLOPE

$\delta(x^2-1)$



$$\frac{d}{dx}(x^2-1) = 2x\Big|_{x=+1} = 2$$

$$\delta[g(x)] = \frac{1}{|g'(x_0)|}\delta(x-x_0) + \frac{1}{|g'(x_1)|}\delta(x-x_1) + \cdots$$

$$g(x_0) = g(x_1) = \cdots = 0$$