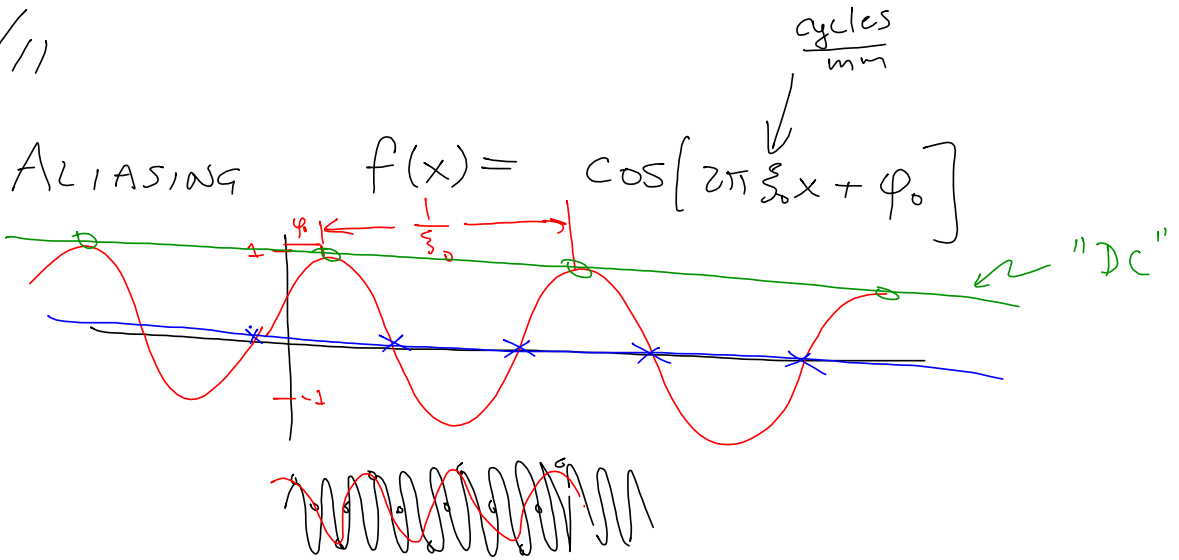


9/11



$$\underline{\underline{D}} = \left[\begin{array}{c|c|c} \hat{x}_0 & \hat{x}_1 & \dots & \hat{x}_{N-1} \end{array} \right]; \quad \underline{\underline{D}}^T = \underline{\underline{D}}$$

$\underline{\underline{A}}$ IS $N \times N$ CIRCULANT \Rightarrow $\hat{x}_{\sim k} = \left(\frac{1}{\sqrt{N}} e^{+i2\pi n \cdot \frac{k}{N}} \right)$

$\frac{k}{N} =$ # OF CYCLES IN N SAMPLES

$0 \leq n \leq N-1$
 $0 \leq k \leq N-1$

$N=4$

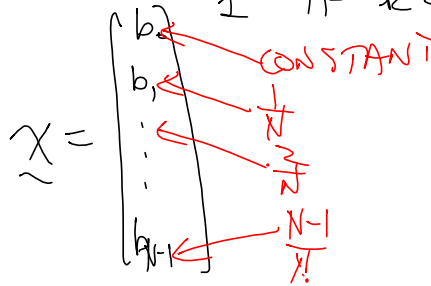
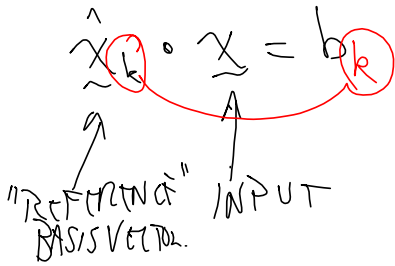
$$\hat{x}_{\sim 0} = \frac{1}{\sqrt{4}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}; \quad \hat{x}_{\sim 1} = \frac{1}{2} \begin{pmatrix} 1 \\ +i \\ -1 \\ -i \end{pmatrix}; \quad \hat{x}_{\sim 2} = \frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}; \quad \hat{x}_{\sim 3} = \frac{1}{2} \begin{pmatrix} 1 \\ -i \\ -1 \\ +i \end{pmatrix}$$

\uparrow
 $k=0$

\tilde{D} VECTORS \hat{x}_k ARE "ORTHONORMAL"

$$\hat{x}_k \cdot \hat{x}_l = 0 \quad \text{IF } k \neq l$$

$$1 \quad \text{IF } k = l$$



$$\underbrace{\tilde{x}}_{\text{CANONICAL}} \rightarrow \underbrace{\tilde{x}'}_{\text{FREE}} = \underbrace{\tilde{D}^{-1}}_{\text{"PROJECTION" OPERATOR}} \tilde{x} = \tilde{D}^* \tilde{x}$$

$$\tilde{x}' \rightarrow \tilde{x} = \tilde{D} \tilde{x}' ; \text{ SYNTHESIZE } \tilde{x} \text{ FROM FREE}$$

$$\begin{pmatrix} \tilde{x}' \\ \tilde{x} \end{pmatrix}_0 \text{ "ZERO" COMPONENT OF } \tilde{x}' = \text{CONSTANT PART OF } \tilde{x}$$

$$\tilde{x} = \tilde{D} \tilde{x}' \rightarrow \text{SYNTHESIS}$$

$$\tilde{D} = \frac{1}{\sqrt{N}} \left(e^{+i2\pi n \cdot \frac{k}{N}} \right)$$

$$\tilde{D}^{-1} = \tilde{D}^* = \frac{1}{\sqrt{N}} e^{-i2\pi n \cdot \frac{k}{N}}$$

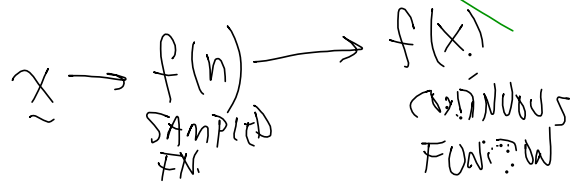
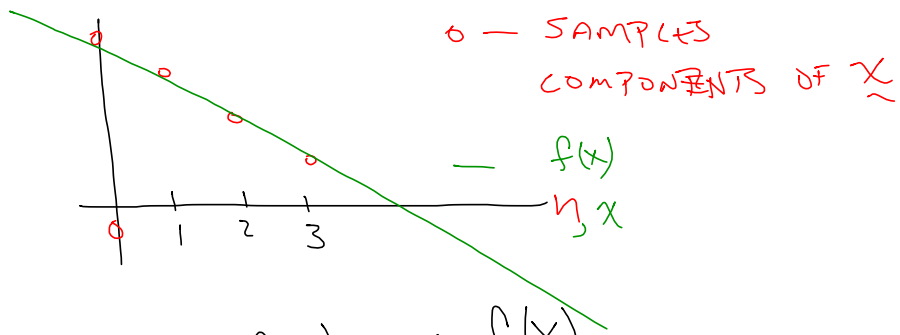
$$\tilde{X}' = \tilde{D}^{-1} \tilde{X} = \sum_{n=0}^{N-1} \left(\frac{1}{\sqrt{N}} e^{-i2\pi \frac{nk}{N}} \right) \left(\tilde{X} \right)_n \quad \text{DFT ANALYSIS}$$

$$\tilde{X} = \tilde{D} \tilde{X}' = \sum_{k=0}^{N-1} \frac{1}{\sqrt{N}} e^{+i2\pi \frac{nk}{N}} \cdot \left(\tilde{X}' \right)_k \quad \text{IDFT SYNTHESIS}$$

INPUT

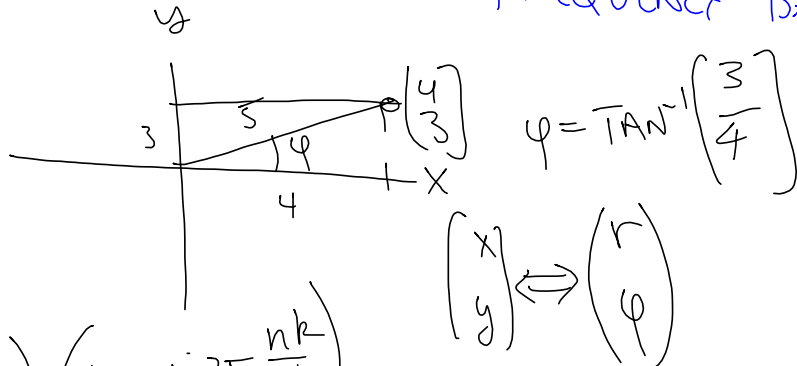
$$\tilde{x} \rightarrow \tilde{x}' = \tilde{D}^{-1} \tilde{x}$$

$\begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}$



$$\frac{1}{N} \sum_{n=0}^{N-1} f(n) \left(e^{+i2\pi \frac{nk}{N}} \right) = F(k)$$

FREQUENCY BASIS



$$\sum_{k=0}^{N-1} F(k) \left(\frac{1}{N} e^{+i2\pi \frac{nk}{N}} \right) = f(n)$$

$$\tilde{x} \rightarrow \sum_{-\infty}^{+\infty} f(n) \rightarrow f(x)$$

$$[\xi] = \frac{\text{cycles}}{\text{mm}} \\ \text{ANALYSIS}$$

$$\int_{-\infty}^{+\infty} f(x) \left(e^{+i 2\pi x \xi} \right) dx \equiv F(\xi)$$

$$\int_{-\infty}^{+\infty} F(\xi) e^{+i 2\pi x \xi} d\xi = f(x) \quad \text{SYNTHESIS}$$

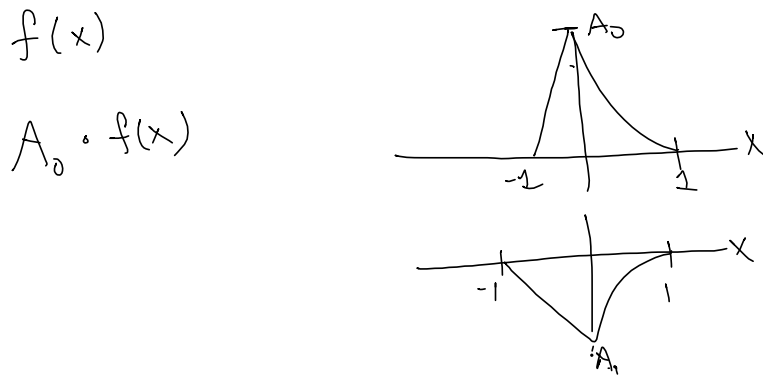
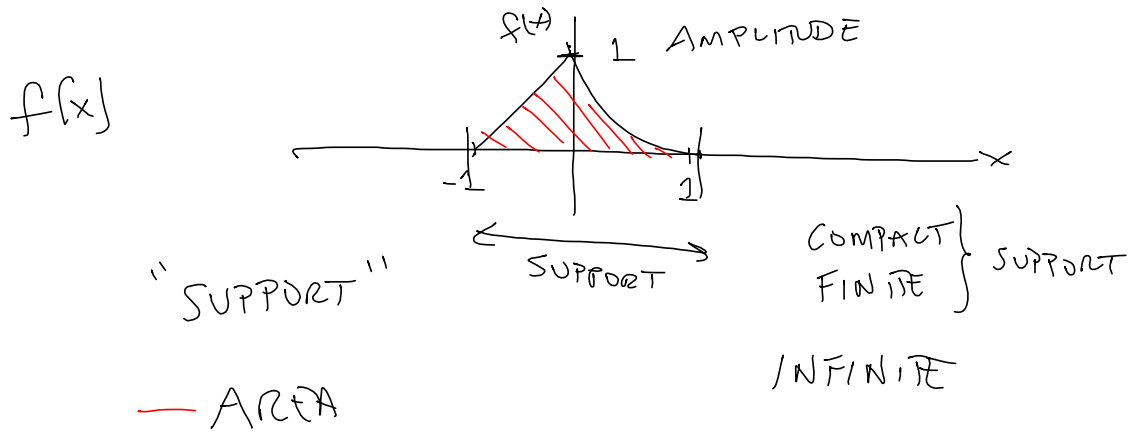
$$f(x) \Leftrightarrow F(\xi)$$

$f(x)$

"SPECIAL" FUNCTIONS §6

$$f(x), \mathcal{O}, g(x); \quad \mathcal{O}\{f(x)\} = g(x)$$

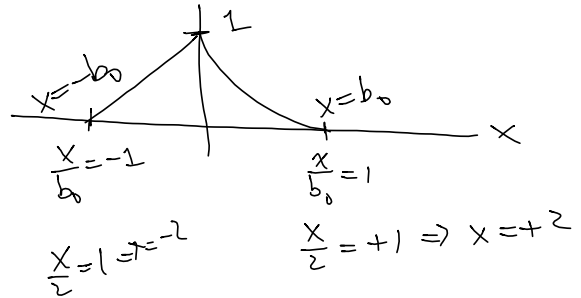
$$\mathcal{O}\{f(x, y, z, t, \lambda)\} = g(x, y) \quad \underbrace{A}_{\sim} \underbrace{x}_{\sim} = \underbrace{b}_{\sim}$$



$$f\left(\frac{x}{b_0}\right)$$

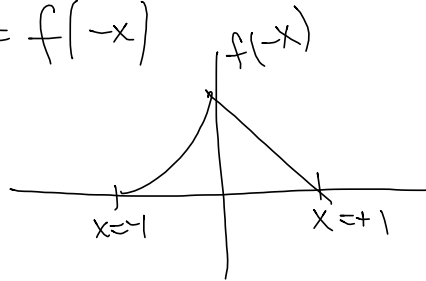
$$b_0 = 2 \Rightarrow f\left(\frac{x}{2}\right)$$

SCALING



$$b_0 = -1 ; f\left(\frac{x}{-1}\right) = f(-x)$$

REVERSING $f(x)$
 SUPPORT UNCHANGED
 AREA UNCHANGED



$$f(-2x) = f\left(\frac{x}{-1/2}\right)$$

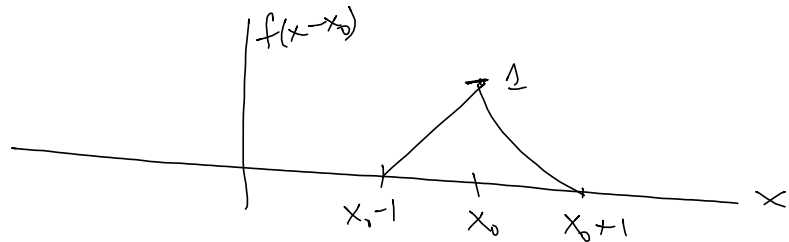
$$b_0 = 0: f\left(\frac{x}{0}\right) ?$$

$$b_0 = \infty \quad f\left(\frac{x}{\infty}\right) = f(0)$$

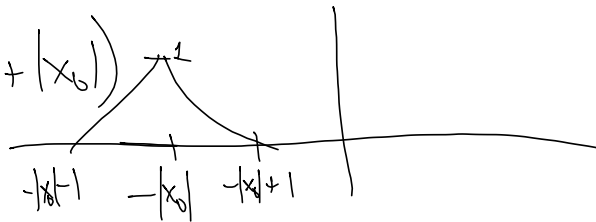
TRANSLATION

$$f(x - x_0)$$

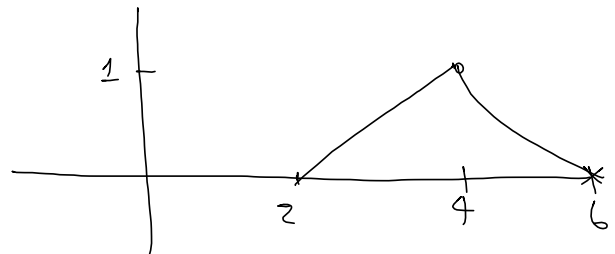
$$f(0) \rightarrow x_0$$



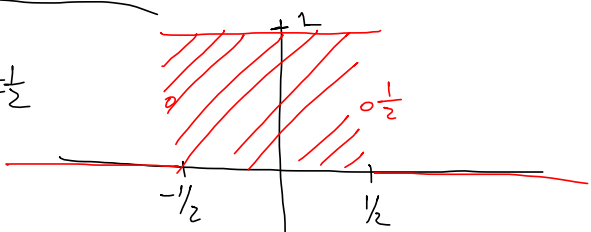
$$f(x - (-|x_0|)) = f(x + |x_0|)$$



$$f\left(\frac{x - x_0}{b_0}\right); \quad f\left(\frac{x - 4}{2}\right)$$



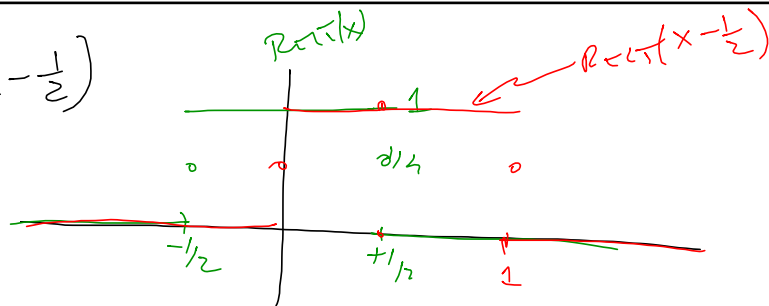
$$\begin{aligned} & -f\left(\frac{x}{2}\right) \\ & f\left(\frac{x}{-2}\right) \end{aligned}$$

$$\text{RECT}(x) \equiv \begin{cases} 1 & |x| < \frac{1}{2} \\ \frac{1}{2} & |x| = \frac{1}{2} \Rightarrow x = \pm \frac{1}{2} \\ 0 & |x| > \frac{1}{2} \end{cases}$$


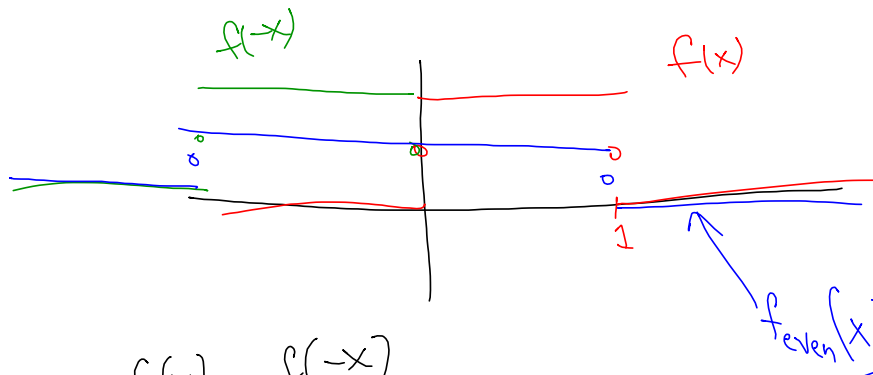
$$\text{RECT}(x) = \text{RECT}(-x); \text{ "EVEN" } \quad \text{"SYMMETRIC"} \quad \text{IF } f(-x) = f(+x)$$

$$\text{IF } f(-x) = -f(x) \Rightarrow f(x) = -f(-x); \text{ "ODD" }, \text{ "ANTI SYMMETRIC"}$$

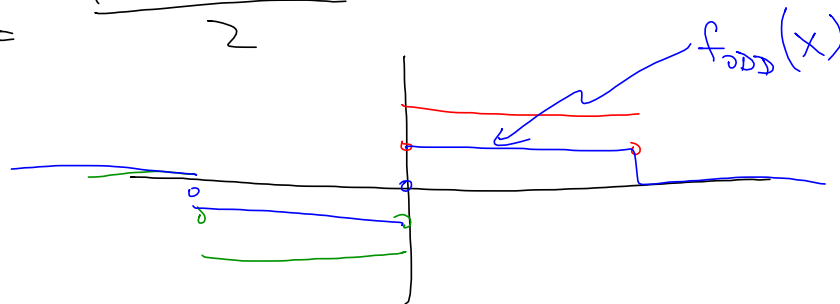
$$\text{Reet}(x - \frac{1}{2})$$



$$f_{\text{even}}(x) = \frac{f(x) + f(-x)}{2} \quad ; \quad f(-x) = \frac{f(-x) + f(-(-x))}{2}$$



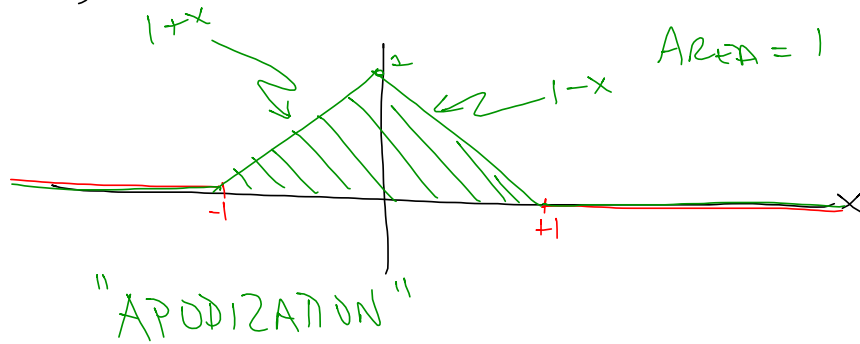
$$f_{\text{odd}}(x) = \frac{f(x) - f(-x)}{2}$$

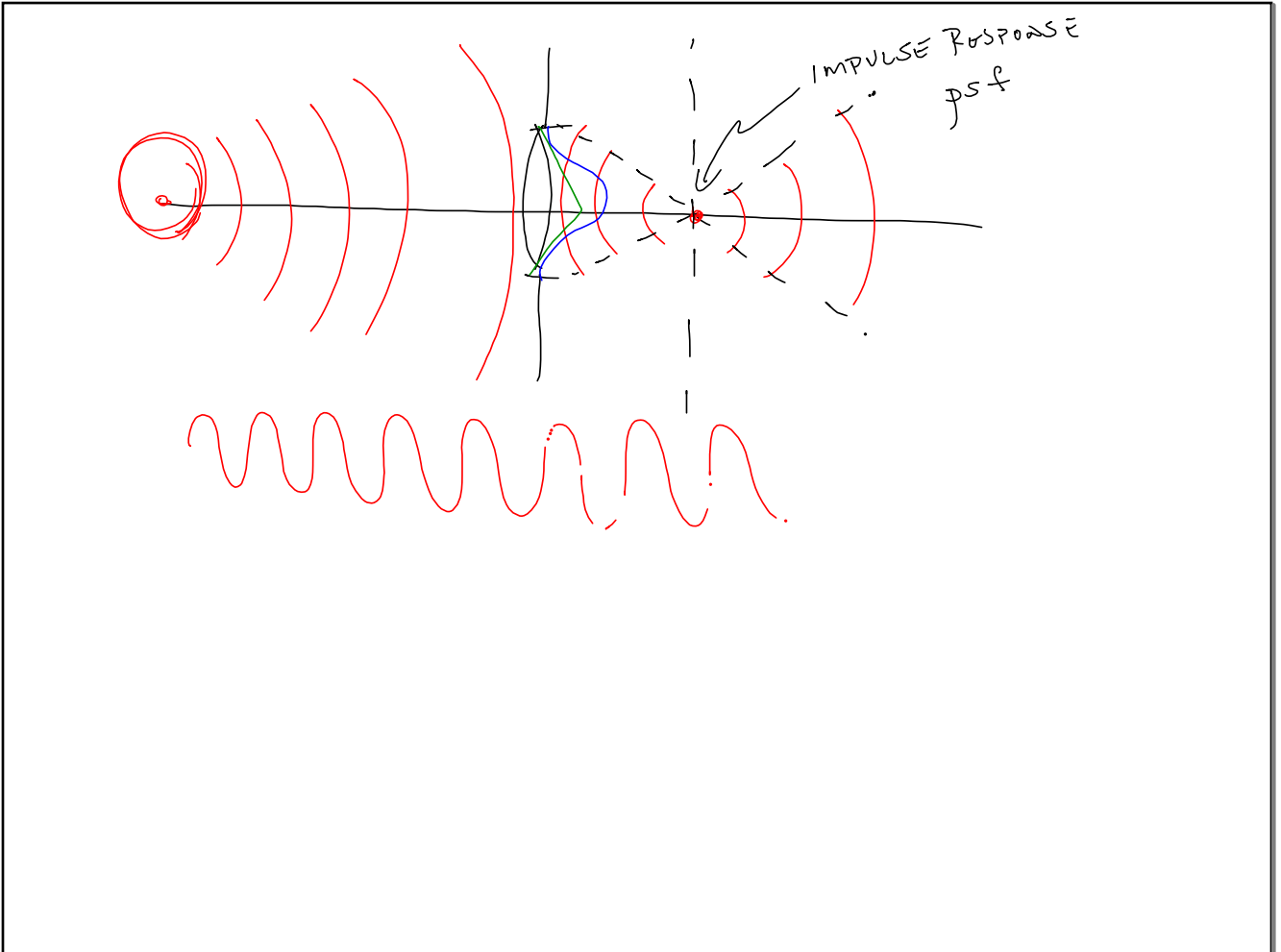


$$\begin{aligned} f(x) &= f_{\text{even}}(x) + f_{\text{odd}}(x) \\ &= \frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2} = f(x) \end{aligned}$$

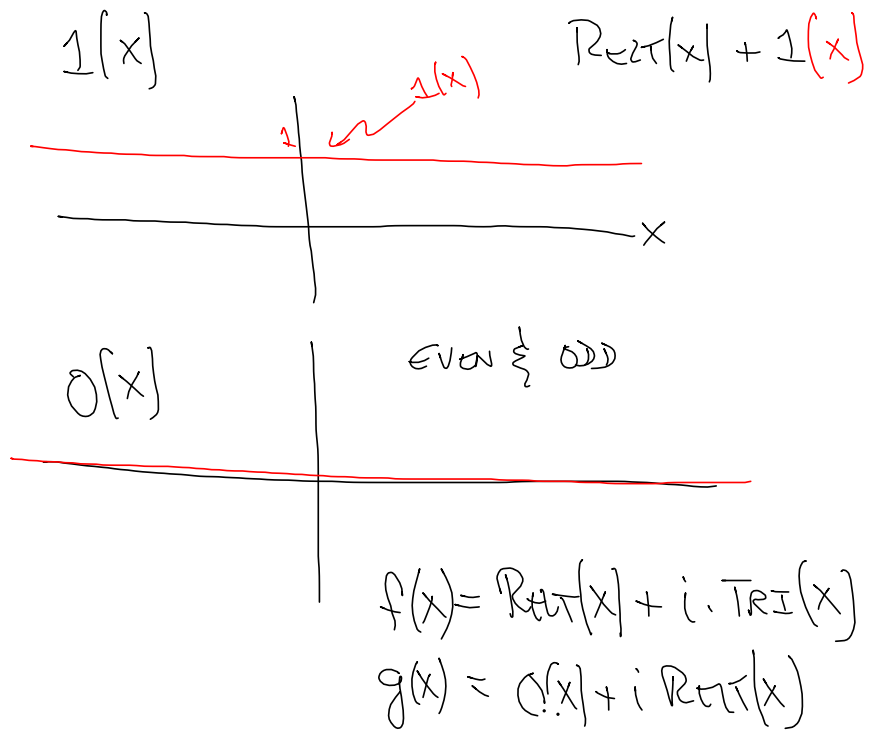
$$f(x) \rightarrow F(\xi)$$

$$\text{TrI}[x] \equiv (1-|x|) \text{Re} \left[\frac{x}{z} \right]$$





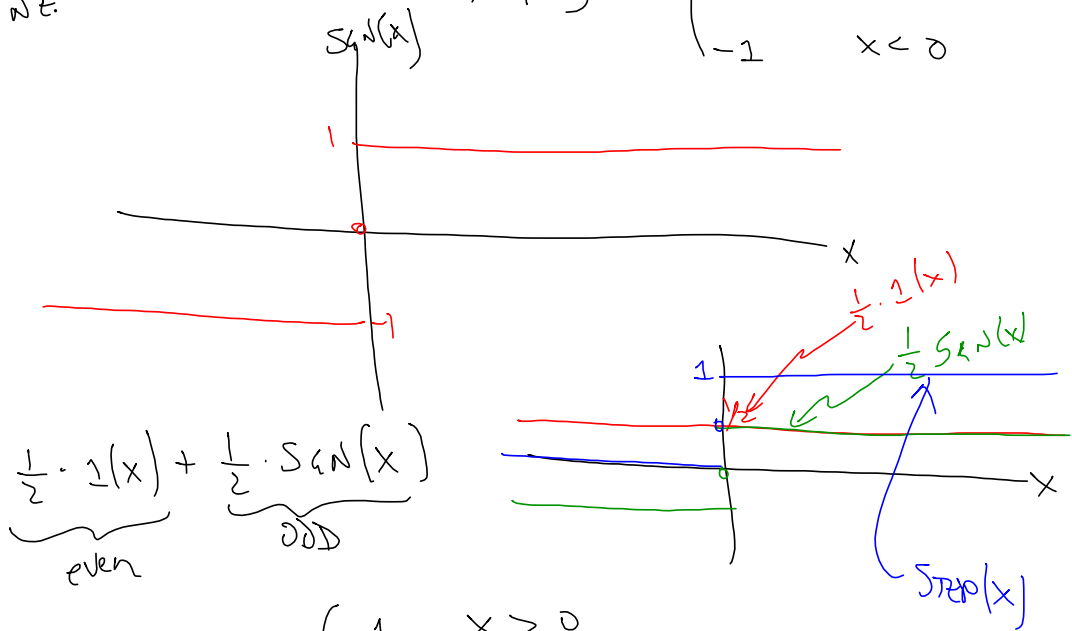
CONSTANTS: "PLACEHOLDERS"



"SIGNUM" FUNCTION

SIGN FUNCTION
SINE

$$\text{SIGN}(x) \equiv \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$



$$\underbrace{\frac{1}{2} \cdot \text{SIGN}(x)}_{\text{even}} + \underbrace{\frac{1}{2} \cdot \text{SIGN}(x)}_{\text{odd}}$$

$$\text{STEP}(x) = \begin{cases} 1 & x > 0 \\ \frac{1}{2} & x = 0 \\ 0 & x < 0 \end{cases}$$

$$\cos\left(2\pi\xi_0 x - \frac{\pi}{2}\right) \cdot \text{STEP}(x)$$

$$\left(\cancel{\cos(2\pi\xi_0 x)} \overset{0}{\cos\left(\frac{\pi}{2}\right)} + \cancel{\sin(2\pi\xi_0 x)} \overset{1}{\sin\left(\frac{\pi}{2}\right)} \right) \cdot \text{STEP}(x)$$

$$\sin(2\pi\xi_0 x) \cdot \text{STEP}(x)$$

