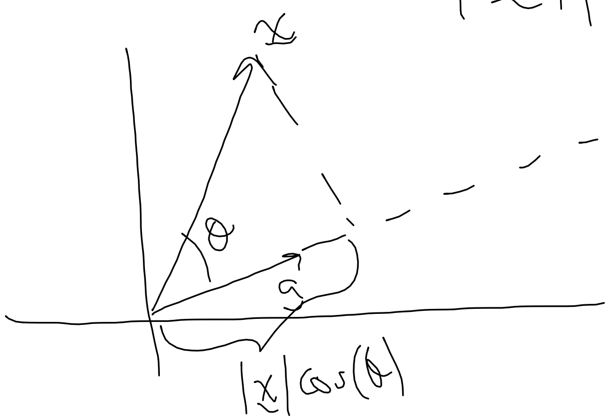
8/28

$$\underset{\sim}{a} \circ \underset{\sim}{x} = b = \sum_n (\underset{\sim}{a})_n (\underset{\sim}{x})_n$$

$$= |\underset{\sim}{a}||\underset{\sim}{x}| \cos(\theta)$$




$$|x| \cos\theta = \frac{\underset{\sim}{a} \circ \underset{\sim}{x}}{|\underset{\sim}{a}|} = \frac{\underset{\sim}{a}}{|\underset{\sim}{a}|} \circ \underset{\sim}{x}$$

$$= \hat{\underset{\sim}{a}} \circ \underset{\sim}{x}$$

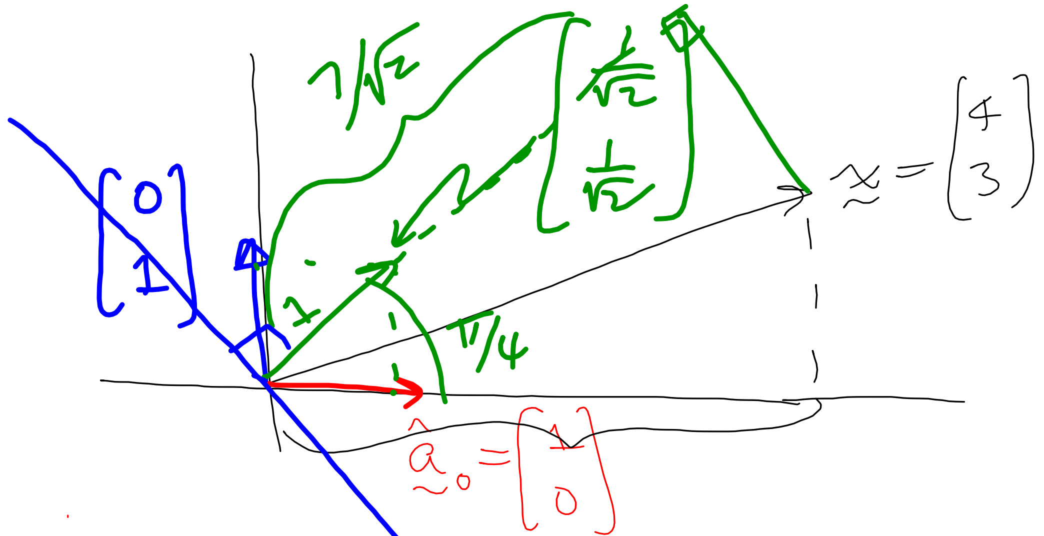
$$|\underset{\sim}{a}| = \sqrt{\underset{\sim}{a} \circ \underset{\sim}{a}} = \sqrt{\sum_n (a_n)^2}$$

$$\underset{\sim}{a} \circ \underset{\sim}{x} = \sum_n (\underset{\sim}{a})_n (\underset{\sim}{x})_n = \underset{\sim}{a}^T \underset{\sim}{x}$$

$$\underset{\sim}{a} = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{N-1} \end{bmatrix} ; \quad \underset{\sim}{a}^T = \begin{bmatrix} a_0 & a_1 & \cdots & a_{N-1} \end{bmatrix}$$

$$\underbrace{\begin{pmatrix} [a_0 \quad a_1 \quad a_{N-1}]_1 \\ [a_0 \quad a_1 \quad \cdots \quad a_{N-1}]_2 \\ \vdots \end{pmatrix}}_{\underset{\sim}{A}} \begin{bmatrix} x_0 \\ x_1 \\ \\ x_{N-1} \end{bmatrix} = a_0 x_0 + a_0 x_1 + \cdots + a_{N-1} x_{N-1}$$

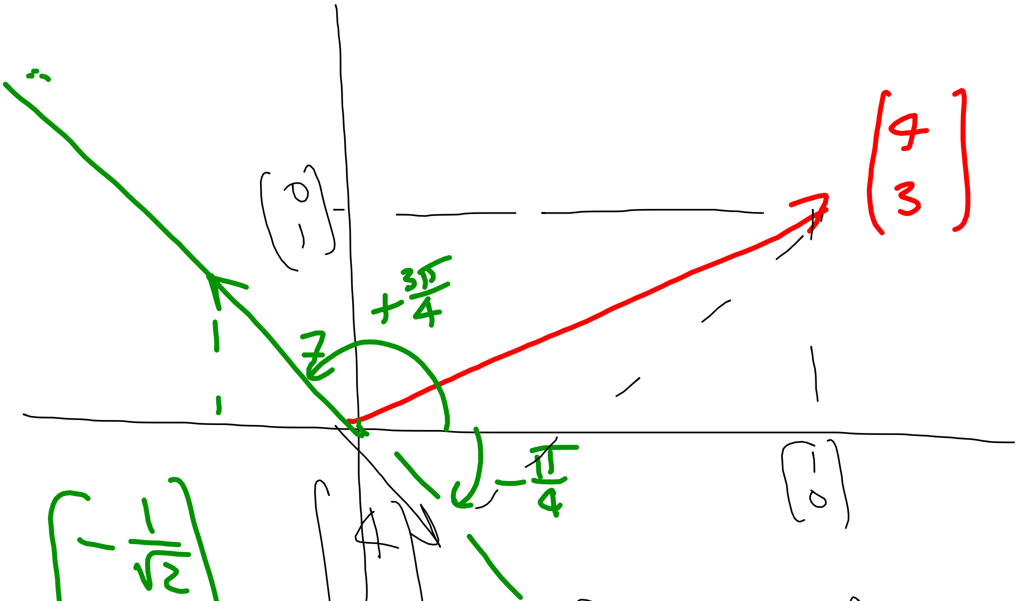
$$= \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{N-} \end{pmatrix}$$




$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

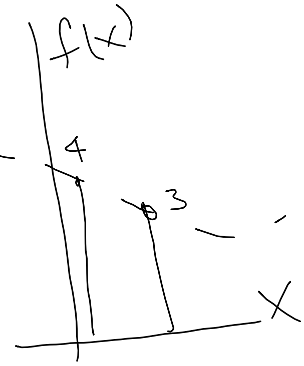
CANONICAL REPRESENTATION

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \frac{7}{\sqrt{2}}$$



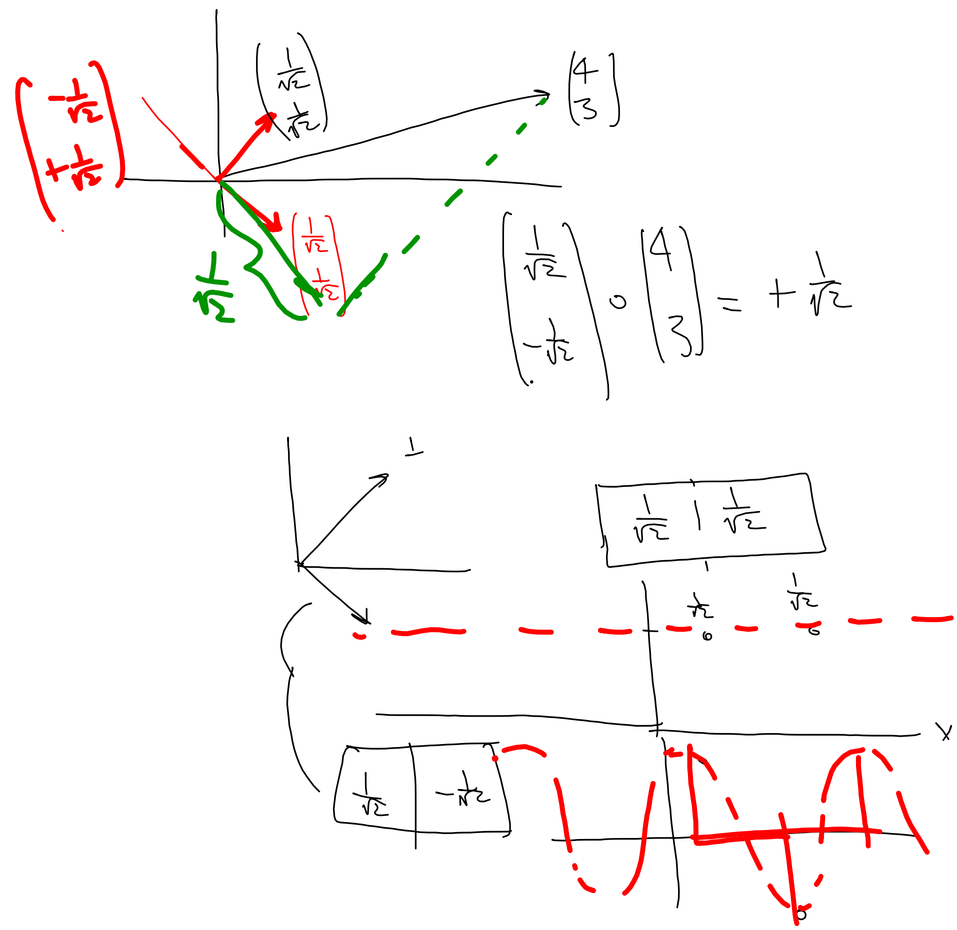

$$\begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \frac{1}{\sqrt{2}}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{7}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$




$$\begin{pmatrix} 4 \\ 3 \end{pmatrix} \Longleftrightarrow \begin{pmatrix} \frac{7}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$f[x]$


$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$
$\begin{pmatrix} 4 \\ 3 \end{pmatrix}$
$\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ +\frac{1}{\sqrt{2}} \end{pmatrix}$
$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$
$\frac{1}{\sqrt{2}}$
$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \circ \begin{pmatrix} 4 \\ 3 \end{pmatrix} = +\frac{1}{\sqrt{2}}$
1
$\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$
$\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$
x

"DC"

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \underbrace{\begin{bmatrix} x_0 \\ x_1 \end{bmatrix}}_{\text{"CANONICAL" REPRESENTATION}} = \underbrace{\frac{1}{\sqrt{2}} \begin{bmatrix} x_0 + x_1 \\ x_0 - x_1 \end{bmatrix}}$$

$\leftarrow$ CONSTANT PART OF $\underset{\sim}{x}$ $\left(f(x)\right)$

$\leftarrow$ "OSCILLATING" PART OF $\underset{\sim}{x}$

"AC"

August 28, 2014

$$
\begin{array}{ccc}
\underset{\sim}{A} & \underset{\sim}{x} & = \underset{\sim}{b} \\
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} &
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} &
= \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}
\end{array}
$$

INVERSE TASK

KNOW $\underset{\sim}{b}$, $\underset{\sim}{A}$; FIND $\underset{\sim}{x}$

$$\underset{\sim}{A}^{-1}\underset{\sim}{b} = \underset{\sim}{x}$$

$$\left\| \begin{pmatrix} \frac{1}{\sqrt{4}} \\ \frac{1}{\sqrt{4}} \\ \frac{1}{\sqrt{4}} \\ \frac{1}{\sqrt{4}} \end{pmatrix} \right\| = \sqrt{\left(\frac{1}{\sqrt{4}}\right)^2 + \left(\frac{1}{\sqrt{4}}\right)^2 + \cdots} = \sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}} = \sqrt{1}$$

August 28, 2014

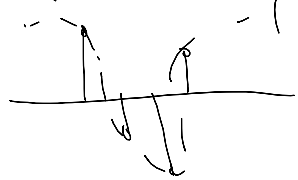
$$\begin{array}{c}\underset{\sim}{A}\\ \begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}\end{array}\begin{bmatrix}x_0\\x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}x_1\\x_2\\x_3\\x_0\end{bmatrix}$$

$$\underset{\sim}{A}^{-1}\ \underset{\sim}{b} = \underset{\sim}{x}$$

$$\begin{bmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_0\end{bmatrix} \quad \begin{bmatrix}x_0\\x_1\\x_2\\x_3\end{bmatrix}$$

$$[x_0\ x_1\ x_2\ x_3]\ x_0$$

$$\underset{\sim}{A}^{-1} = \underset{\sim}{A}^{T}$$



$$\underset{\sim}{A}\;\underset{\sim}{x} = \underset{\sim}{b}$$

$$\begin{bmatrix} -1 & +1 & 0 & 0 \\ 0 & -1 & +1 & 0 \\ 0 & 0 & -1 & +1 \\ +1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_0 \\ x_2 - x_1 \\ x_3 - x_2 \\ x_0 - x_3 \end{bmatrix}$$

CIRCULANT MATRIX $\Rightarrow$ CONVOLUTION

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \underset{\sim}{0}$$

$$\downarrow$$

$$\begin{bmatrix} \; \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} = \underset{\sim}{0}$$

$$f(x) = 1 + \cos(2\pi \xi_0 x)$$

$$f'(x) = 0 + (-2\pi \xi_0)\sin(2\pi \xi_0 x)$$

$$\int f'(x)\,dx = +0 + \cos(2\pi \xi_0 x)$$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{x_0+x_1}{2} \\ \frac{x_2+x_1}{2} \\ \frac{x_3+x_2}{2} \\ \frac{x_0+x_3}{2} \end{bmatrix}$$

$$\underset{\sim}{A}\,\underset{\sim}{x}_N = \underset{\sim}{0}$$

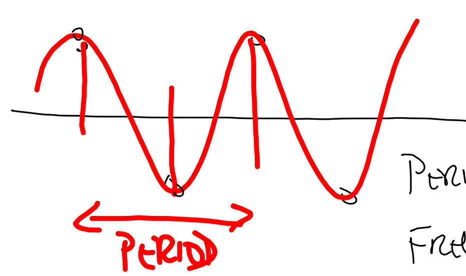
$$\left\{ \underset{\sim}{x}_N \right\} \text{ S.T. } \underset{\sim}{A}\,\underset{\sim}{x}_N = \underset{\sim}{0}$$

$$\Longrightarrow \underset{\sim}{A}^{-1} \text{ DOES NOT EXIST}$$

$$\underset{\sim}{A}$$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} +1 \\ -1 \\ +1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \underset{\sim}{0}$$

$$\begin{bmatrix} +1 & -1 & +1 & -1 \end{bmatrix}$$




$$\text{PERIOD} \Rightarrow 2 \ \frac{\text{SAMPLES}}{\text{CYCLE}}$$

$$\text{FREQ} \Rightarrow \frac{1}{2} \ \frac{\text{CYCLE}}{\text{SAMPLE}}$$

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

EIGENVECTOR OF $\underset{\sim}{A}$

$$\underset{\sim}{A} \; \underset{\sim}{x} = \underset{\sim}{b} = \underset{\sim}{x}$$

IF $\underset{\sim}{A}\,\underset{\sim}{x} = \lambda\,\underset{\sim}{x}$, THEN $\underset{\sim}{x}$ IS AN EIGENVECTOR OF $\underset{\sim}{A}$

SCALAR    $\lambda$ IS EIGENVALUE

$$\begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \underset{\sim}{0} = 0 \cdot \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$\lambda \downarrow$$

$$\begin{pmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{pmatrix}\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} = (a+b+c+d)\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$