

Chapter 7

Propagation of Light Waves

7.1 Wavefronts

7.1.1 Plane Waves

The form of any wave (matter or electromagnetic) is determined by its source and described by the shape of its wavefront, i.e., the locus of points of constant phase. If a traveling wave is emitted by a planar source, then the points of constant phase form a plane surface parallel to the face of the source. Such a wave is called a *plane wave*, and travels in one direction (ideally). Since energy is conserved, the total energy in the wave must equal the energy emitted by the source, and therefore the *energy density* (the energy passing through a unit area), is constant for a plane wave. Recall that in a wave of amplitude A and frequency ω , the energy $\mathcal{E} \propto A^2\omega^2$. Therefore, for a plane wave, the amplitude is constant; the wave does not attenuate.

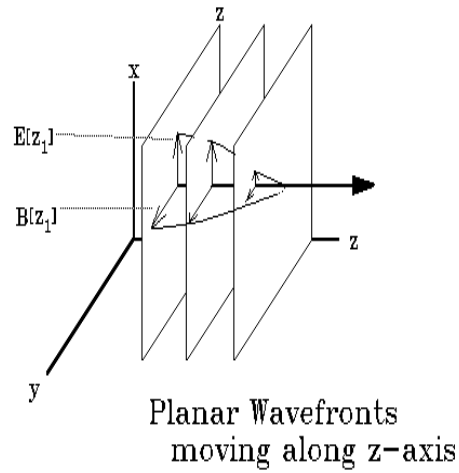
Plane wave toward $z = +\infty$ at velocity $v_\phi = \frac{\omega}{k}$, wavelength $\lambda = \frac{2\pi}{k}$, frequency $\nu = \frac{\omega}{2\pi}$, amplitude A_0 :

$$f[x, y, z, t] = A_0 \cos [kz \mp \omega t]$$

(n.b., no variation in y or z)

General 3-D plane wave traveling in a direction $\underline{\mathbf{k}} = [k_x, k_y, k_z]$, $\underline{\mathbf{r}} = [x, y, z]$ and the definition of the scalar product (dot product):

$$f[\underline{\mathbf{r}}, t] = A_0 \cos [\underline{\mathbf{k}} \bullet \underline{\mathbf{r}} - \omega t] \implies \underline{\mathbf{k}} \bullet \underline{\mathbf{r}} = k_x x + k_y y + k_z z$$



7.1.2 Cylindrical Waves

If a wave is emitted from a line source, the wavefronts are *cylindrical*. Since the wave expands to fill a cylinder of radius r_0 , the wavefront crosses a cylindrical area that grows as $Area = 2\pi r h \propto r$. Therefore, since energy is conserved, the energy per unit area must decrease as r increases:

$$\frac{\mathcal{E}}{Area} = constant = \frac{\mathcal{E}}{2\pi r h} \propto \frac{\mathcal{E}}{r} \propto \frac{A_0^2}{r} = constant$$

$$\Rightarrow amplitude \propto \frac{A_0}{\sqrt{r}}$$

The equation for a cylindrical wavefront emerging from (or collapsing into) a line source is:

$$f[x, y, z, t] = A[r] \cos [kr \mp \omega t]$$

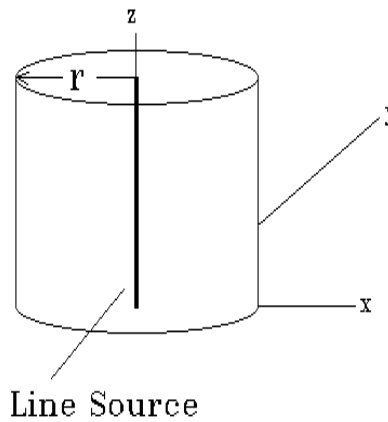
$$= \frac{A_0}{\sqrt{r}} \cos [kr \mp \omega t]$$

$$r = \sqrt{x^2 + y^2} > 0$$

“ - ” \Rightarrow *emerging*

“ + ” \Rightarrow *collapsing*

$A_0 =$ *amplitude at $r = 0$*



Cylindrical waves expanding from a line source.

7.1.3 Spherical Waves

The wavefront emerging from (or collapsing into) a point is spherical. The area the wave must cross increases as $x^2 + y^2 + z^2 = r^2$ (area of sphere is $4\pi r^2$). Therefore the energy density drops as r^2 and the amplitude of the wave must decrease as $\frac{1}{r}$. The equation for a spherical wave is

$$\begin{aligned} f[x, y, z, t] &= f[\mathbf{r}, t] = A[r] \cos[kr \mp \omega t] \\ &= \frac{A_0}{r} \cos[kr \mp \omega t], \text{ where } r > 0 \end{aligned}$$

“ - ” \implies *emerging*

“ + ” \implies *collapsing*

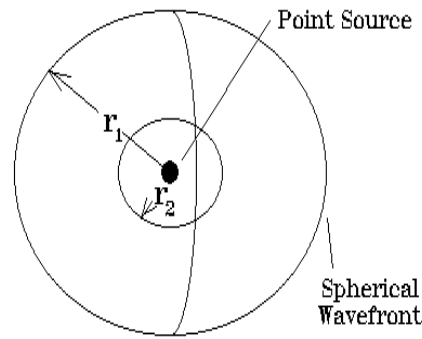
$A_0 = \text{amplitude at } r = 0$

Note the pattern for the amplitude of plane, cylindrical, and spherical waves:

plane wave \implies 2-D source (plane) \implies amplitude $A[r] \propto r^{-0} = 1$

cylindrical wave \implies 1-D source (line) $\implies A[r] \propto r^{-\frac{1}{2}}$

spherical wave \implies 0-D source (point) $\implies A[r] \propto r^{-1}$



Spherical waves expanding from a point source.

7.2 Huygens' Principle

I, §1, §3

The spherical wave is the basic wave for light propagation using Huygens' principle. In 1678, Christiaan Huygens theorized a model for light propagation that claimed that each point on a propagating wavefront (regardless of "shape") could be assumed to be a source of a new spherical wave. The sum of these secondary spherical "wavelets" produced the subsequent wavefronts. Huygens' principle had the glaring disadvantage that these secondary spherical wavefronts propagated "backwards" as well as forwards. This problem was later solved by Fresnel and Kirchhoff in the 19th century. With that correction, the Huygens' model provides a very useful model for light propagation that naturally leads to expressions for "diffracted" light.