IMGS-261: Linear Mathematics for Imaging
Solution #5

1. (a)(b)

$$\hat{\underline{\mathbf{x}}}_{\mathbf{1}} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot\frac{0}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix};$$

$$\hat{\underline{\mathbf{x}}}_{\mathbf{2}} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot\frac{1}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000+0.0000i \\ 0.9239+0.3827i \\ 0.7071+0.7071i \\ 0.3827+0.9239i \\ 0.0000+1.0000i \\ -0.3827+0.9239i \\ -0.7071+0.7071i \\ -0.9239+0.3827i \\ -1.0000+0.0000i \\ -0.9239-0.3827i \\ -0.7071-0.7071i \\ -0.3827-0.9239i \\ -0.0000-1.0000i \\ 0.3827-0.9239i \\ 0.7071-0.7071i \\ 0.9239-0.3827i \end{bmatrix};$$

$$\hat{\underline{\mathbf{x}}}_{\mathbf{3}} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot\frac{2}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000+0.0000i \\ 0.7071+0.7071i \\ 0.0000+1.0000i \\ -0.7071+0.7071i \\ -1.0000+0.0000i \\ -0.7071-0.7071i \\ -0.0000-1.0000i \\ 0.7071-0.7071i \\ 1.0000-0.0000i \\ 0.7071+0.7071i \\ 0.0000+1.0000i \\ -0.7071+0.7071i \\ -1.0000+0.0000i \\ -0.7071-0.7071i \\ -0.0000-1.0000i \\ 0.7071-0.7071i \end{bmatrix};$$

$$\underline{\hat{\mathbf{x}}_4} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot \frac{3}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000+0.0000i \\ 0.3827+0.9239i \\ -0.7071+0.7071i \\ -0.9239-0.3827i \\ -0.0000-1.0000i \\ 0.9239-0.3827i \\ 0.7071+0.7071i \\ -0.3827+0.9239i \\ -1.0000+0.0000i \\ -0.3827-0.9239i \\ 0.7071-0.7071i \\ 0.9239+0.3827i \\ 0.0000+1.0000i \\ -0.9239+0.3827i \\ -0.7071-0.7071i \\ 0.3827-0.9239i \end{bmatrix};$$

$$\underline{\hat{\mathbf{x}}_5} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot \frac{5}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000+0.0000i \\ 0.0000+1.0000i \\ -1.0000+0.0000i \\ -0.0000-1.0000i \\ 1.0000-0.0000i \\ 0.0000+1.0000i \\ -1.0000+0.0000i \\ -0.0000-1.0000i \\ 1.0000-0.0000i \\ 0.0000+1.0000i \\ -1.0000+0.0000i \\ -0.0000-1.0000i \\ 1.0000-0.0000i \\ -0.0000+1.0000i \\ -1.0000+0.0000i \\ -0.0000-1.0000i \end{bmatrix};$$

$$\underline{\hat{\mathbf{x}}_6} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot \frac{6}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000+0.0000i \\ -0.3827+0.9239i \\ -0.7071-0.7071i \\ 0.9239-0.3827i \\ 0.0000+1.0000i \\ -0.9239-0.3827i \\ 0.7071-0.7071i \\ 0.3827+0.9239i \\ -1.0000+0.0000i \\ 0.3827-0.9239i \\ 0.7071+0.7071i \\ -0.9239+0.3827i \\ -0.0000-1.0000i \\ 0.9239+0.3827i \\ -0.7071+0.7071i \\ -0.3827-0.9239i \end{bmatrix};$$

$$\underline{\hat{\mathbf{x}}}_{\mathbf{7}} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot \frac{7}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000 + 0.0000i \\ -0.7071 + 0.7071i \\ -0.0000 - 1.0000i \\ 0.7071 + 0.7071i \\ -1.0000 + 0.0000i \\ 0.7071 - 0.7071i \\ 0.0000 + 1.0000i \\ -0.7071 - 0.7071i \\ 1.0000 - 0.0000i \\ -0.7071 + 0.7071i \\ -0.0000 - 1.0000i \\ 0.7071 + 0.7071i \\ -1.0000 + 0.0000i \\ 0.7071 - 0.7071i \\ -0.0000 + 1.0000i \\ -0.7071 - 0.7071i \end{bmatrix};$$

$$\underline{\hat{\mathbf{x}}}_{\mathbf{8}} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot \frac{8}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000 + 0.0000i \\ -0.9239 + 0.3827i \\ 0.7071 - 0.7071i \\ -0.3827 + 0.9239i \\ -0.0000 - 1.0000i \\ 0.3827 + 0.9239i \\ -0.7071 - 0.7071i \\ 0.9239 + 0.3827i \\ -1.0000 + 0.0000i \\ 0.9239 - 0.3827i \\ -0.7071 + 0.7071i \\ 0.3827 - 0.9239i \\ -0.0000 + 1.0000i \\ -0.3827 - 0.9239i \\ 0.7071 + 0.7071i \\ -0.9239 - 0.3827i \end{bmatrix};$$

$$\underline{\hat{\mathbf{x}}}_{\mathbf{9}} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot \frac{9}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000 + 0.0000i \\ -1.0000 + 0.0000i \\ 1.0000 - 0.0000i \\ -1.0000 + 0.0000i \\ 1.0000 - 0.0000i \\ -1.0000 + 0.0000i \\ 1.0000 - 0.0000i \\ -1.0000 + 0.0000i \\ 1.0000 - 0.0000i \\ -1.0000 + 0.0000i \\ 1.0000 - 0.0000i \\ -1.0000 + 0.0000i \\ 1.0000 - 0.0000i \\ -1.0000 - 0.0000i \\ 1.0000 - 0.0000i \\ -1.0000 + 0.0000i \end{bmatrix};$$

$$\underline{\hat{\mathbf{x}}}_{\mathbf{10}} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot\frac{10}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000+0.0000i \\ -0.9239-0.3827i \\ 0.7071+0.7071i \\ -0.3827-0.9239i \\ 0.0000+1.0000i \\ 0.3827-0.9239i \\ -0.7071+0.7071i \\ 0.9239-0.3827i \\ -1.0000+0.0000i \\ 0.9239+0.3827i \\ -0.7071-0.7071i \\ 0.3827+0.9239i \\ -0.0000-1.0000i \\ -0.3827+0.9239i \\ 0.7071-0.7071i \\ -0.9239+0.3827i \end{bmatrix};$$

$$\underline{\hat{\mathbf{x}}}_{\mathbf{11}} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot\frac{11}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000+0.0000i \\ -0.7071-0.7071i \\ 0.0000+1.0000i \\ 0.7071-0.7071i \\ -1.0000+0.0000i \\ 0.7071+0.7071i \\ -0.0000-1.0000i \\ -0.7071+0.7071i \\ 1.0000-0.0000i \\ -0.7071-0.7071i \\ -0.0000+1.0000i \\ 0.7071-0.7071i \\ -1.0000+0.0000i \\ 0.7071+0.7071i \\ -0.0000-1.0000i \\ -0.7071+0.7071i \end{bmatrix};$$

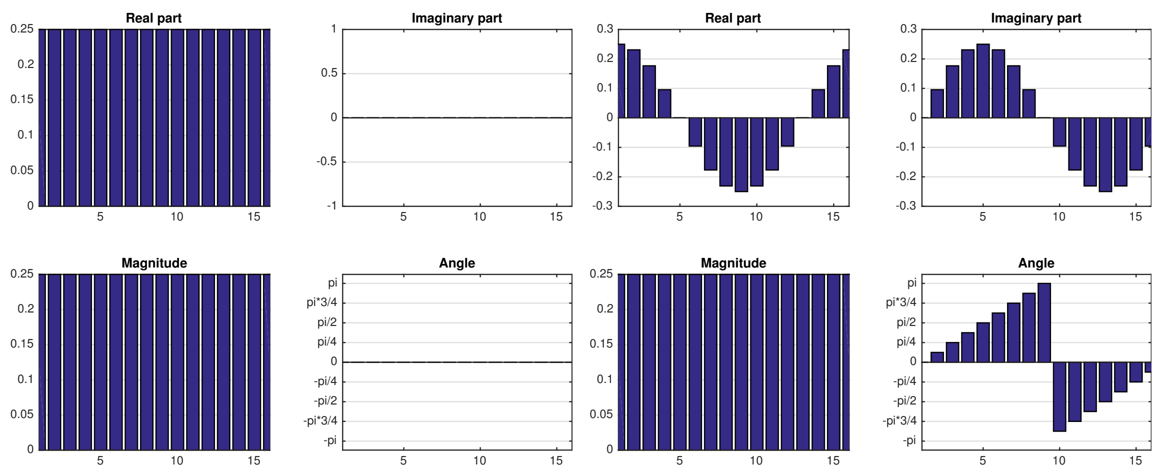
$$\underline{\hat{\mathbf{x}}}_{\mathbf{12}} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot\frac{12}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000+0.0000i \\ -0.3827-0.9239i \\ -0.7071+0.7071i \\ 0.9239+0.3827i \\ -0.0000-1.0000i \\ -0.9239+0.3827i \\ 0.7071+0.7071i \\ 0.3827-0.9239i \\ -1.0000+0.0000i \\ 0.3827+0.9239i \\ 0.7071-0.7071i \\ -0.9239-0.3827i \\ 0.0000+1.0000i \\ 0.9239-0.3827i \\ -0.7071-0.7071i \\ -0.3827+0.9239i \end{bmatrix};$$

$$\underline{\hat{\mathbf{x}}}_{\mathbf{13}} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot \frac{13}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000 + 0.0000i \\ -0.0000 - 1.0000i \\ -1.0000 + 0.0000i \\ 0.0000 + 1.0000i \\ 1.0000 - 0.0000i \\ -0.0000 - 1.0000i \\ -1.0000 + 0.0000i \\ -0.0000 + 1.0000i \\ 1.0000 - 0.0000i \\ -0.0000 - 1.0000i \\ -1.0000 + 0.0000i \\ 0.0000 + 1.0000i \\ 1.0000 - 0.0000i \\ -0.0000 - 1.0000i \\ -1.0000 - 0.0000i \\ 0.0000 + 1.0000i \end{bmatrix};$$

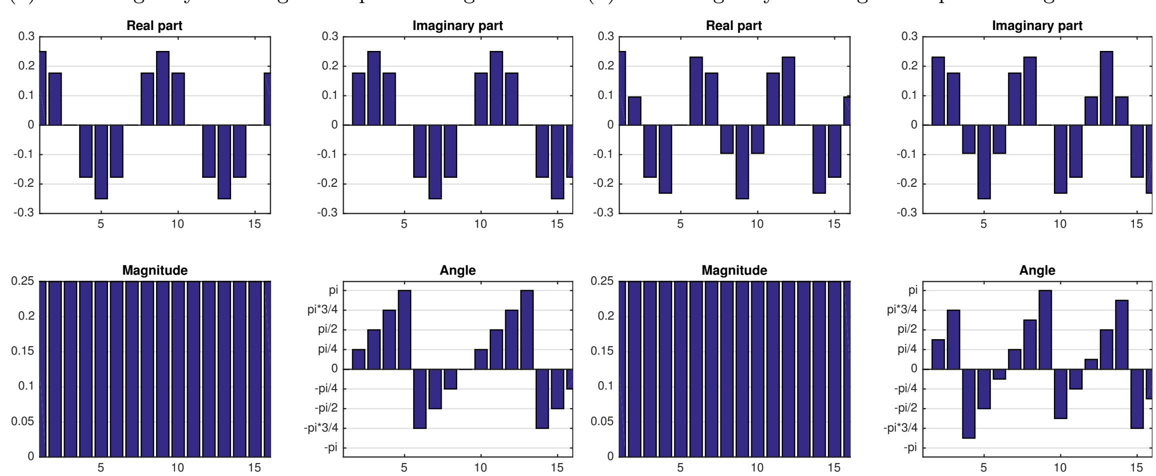
$$\underline{\hat{\mathbf{x}}}_{\mathbf{14}} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot \frac{14}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000 + 0.0000i \\ 0.3827 - 0.9239i \\ -0.7071 - 0.7071i \\ -0.9239 + 0.3827i \\ -0.0000 + 1.0000i \\ 0.9239 + 0.3827i \\ 0.7071 - 0.7071i \\ -0.3827 - 0.9239i \\ -1.0000 - 0.0000i \\ -0.3827 + 0.9239i \\ 0.7071 + 0.7071i \\ 0.9239 - 0.3827i \\ -0.0000 - 1.0000i \\ -0.9239 - 0.3827i \\ -0.7071 + 0.7071i \\ 0.3827 + 0.9239i \end{bmatrix};$$

$$\underline{\hat{\mathbf{x}}}_{\mathbf{15}} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot \frac{15}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000 + 0.0000i \\ 0.7071 - 0.7071i \\ -0.0000 - 1.0000i \\ -0.7071 - 0.7071i \\ -1.0000 + 0.0000i \\ -0.7071 + 0.7071i \\ -0.0000 + 1.0000i \\ 0.7071 + 0.7071i \\ 1.0000 - 0.0000i \\ 0.7071 - 0.7071i \\ -0.0000 - 1.0000i \\ -0.7071 - 0.7071i \\ -1.0000 - 0.0000i \\ -0.7071 + 0.7071i \\ 0.0000 + 1.0000i \\ 0.7071 + 0.7071i \end{bmatrix};$$

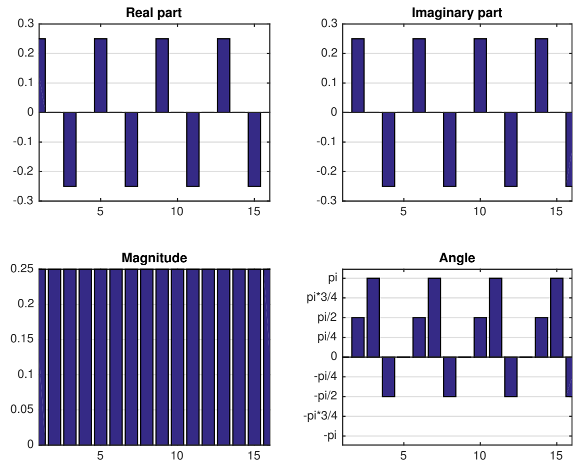
$$\hat{\underline{\mathbf{x}}}_{\mathbf{16}} = \frac{1}{4}\begin{bmatrix} \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 0] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 1] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 2] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 3] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 4] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 5] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 6] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 7] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 8] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 9] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 10] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 11] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 12] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 13] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 14] \\ \exp[+i\cdot 2\pi\cdot \frac{16}{16}\cdot 15] \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1.0000 + 0.0000i \\ 0.9239 - 0.3827i \\ 0.7071 - 0.7071i \\ 0.3827 - 0.9239i \\ -0.0000 - 1.0000i \\ -0.3827 - 0.9239i \\ -0.7071 - 0.7071i \\ -0.9239 - 0.3827i \\ -1.0000 + 0.0000i \\ -0.9239 + 0.3827i \\ -0.7071 + 0.7071i \\ -0.3827 + 0.9239i \\ 0.0000 + 1.0000i \\ 0.3827 + 0.9239i \\ 0.7071 + 0.7071i \\ 0.9239 + 0.3827i \end{bmatrix};$$



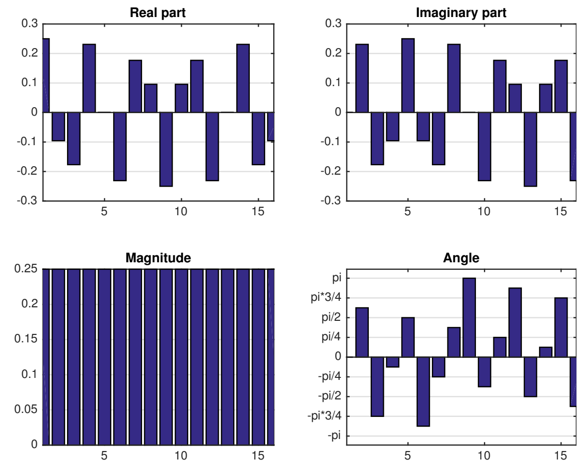
(a) Real-imaginary and magnitude-phase of eigenvector 1 (b) Real-imaginary and magnitude-phase of eigenvector 2



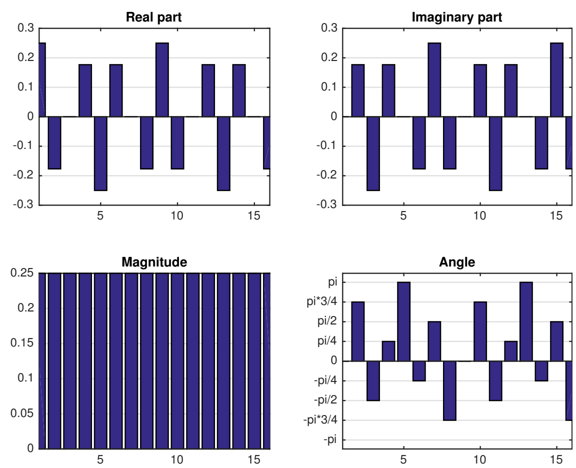
(c) Real-imaginary and magnitude-phase of eigenvector 3 (d) Real-imaginary and magnitude-phase of eigenvector 4



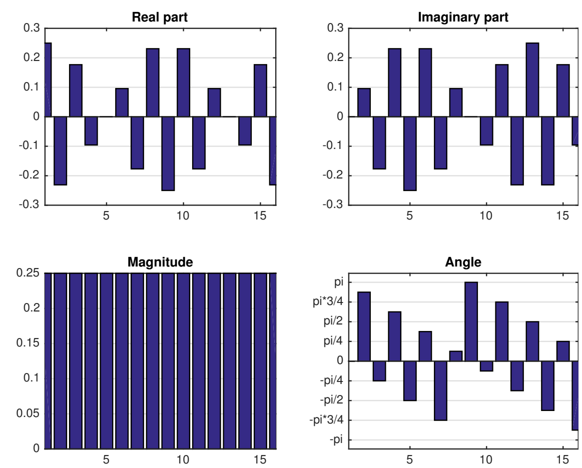
(e) Real-imaginary and magnitude-phase of eigenvector 5



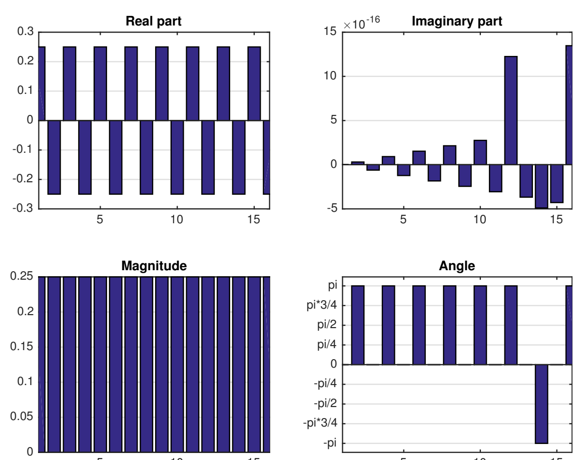
(f) Real-imaginary and magnitude-phase of eigenvector 6



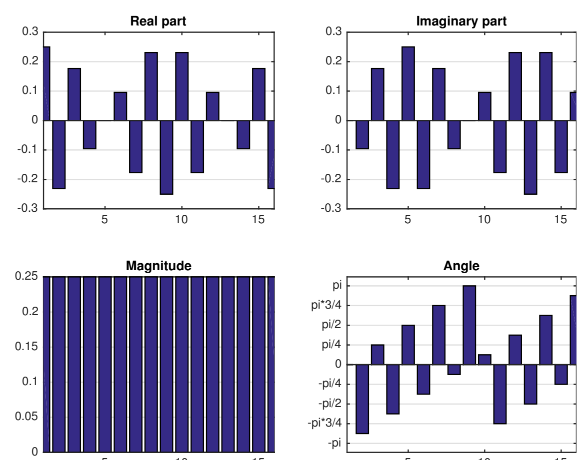
(g) Real-imaginary and magnitude-phase of eigenvector 7



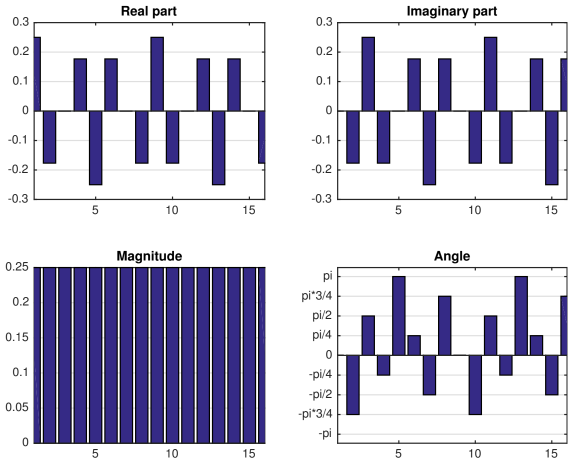
(h) Real-imaginary and magnitude-phase of eigenvector 8



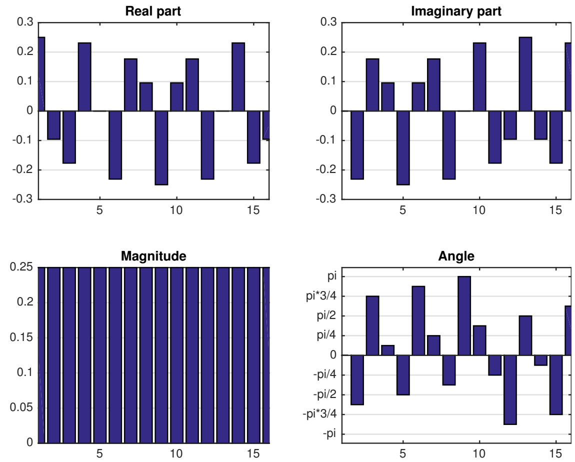
(i) Real-imaginary and magnitude-phase of eigenvector 9



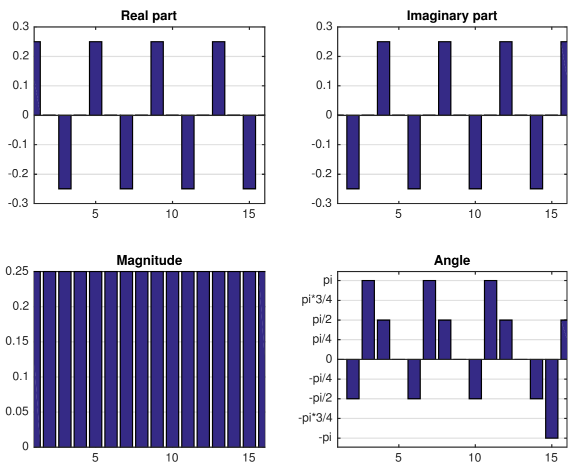
(j) Real-imaginary and magnitude-phase of eigenvector 10



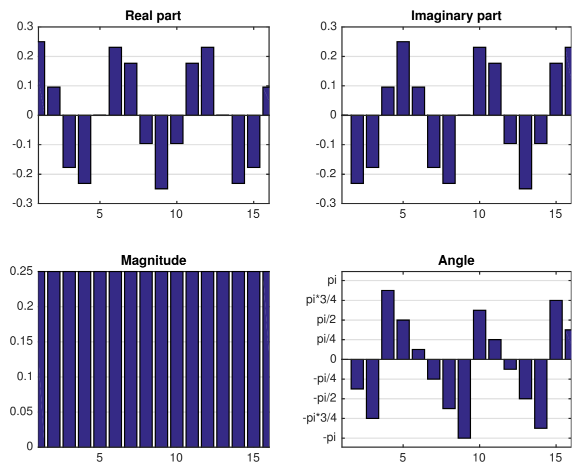
(k) Real-imaginary and magnitude-phase of eigenvector 11



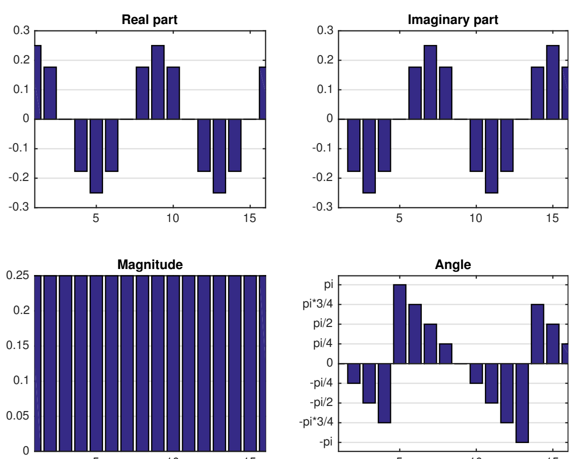
(l) Real-imaginary and magnitude-phase of eigenvector 12



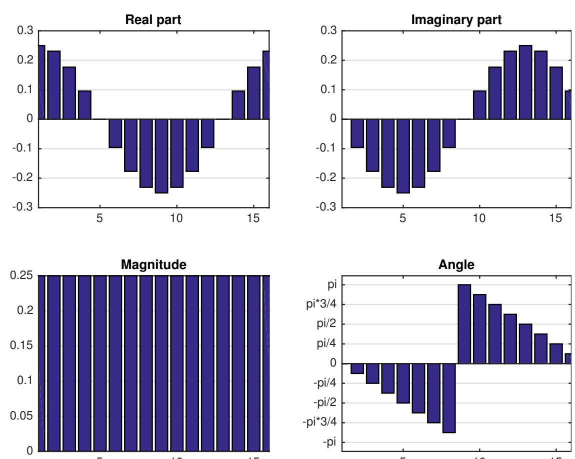
(m) Real-imaginary and magnitude-phase of eigenvector 13



(n) Real-imaginary and magnitude-phase of eigenvector 14



(o) Real-imaginary and magnitude-phase of eigenvector 15



(p) Real-imaginary and magnitude-phase of eigenvector 16

Figure 1: Figure of 1(b)

2. (a)

$$|A_1 - I\lambda| = \begin{vmatrix} -1-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = -(1+\lambda)(2-\lambda) = 0 \Rightarrow \lambda_1 = -1,\ \lambda_2 = 2$$

$$\Lambda_1 = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \quad \Lambda_1^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

for $\lambda_1 = -1$, $A\hat{\underline{x}}_1 = \lambda_1 \hat{\underline{x}}_1$:

$$\begin{bmatrix} -1 & 0 \\ 0 & +2 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} = -1 \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} \Rightarrow \begin{cases} -x_{11} = -x_{11} \\ 2x_{12} = -1\, x_{12} \end{cases} \Rightarrow \begin{cases} x_{11} = 1 \text{ (for narmarlization)} \\ x_{12} = 0. \end{cases}$$

$$\hat{\underline{x}}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

for $\lambda_2 = 2$, $A\hat{\underline{x}}_2 = \lambda \hat{\underline{x}}_2$:

$$\begin{bmatrix} -1 & 0 \\ 0 & +2 \end{bmatrix}\begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} = 2 \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} \Rightarrow \begin{cases} -x_{21} = 2x_{21} \\ 2x_{22} = 2x_{22} \end{cases} \Rightarrow \begin{cases} x_{21} = 0 \\ x_{22} = 1 \text{ (for narmalization)} \end{cases}$$

(b)

$$|A_2 - I\lambda| = \begin{vmatrix} -1-\lambda & 0 & 0 \\ 0 & -2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = (1+\lambda)(2+\lambda)(3-\lambda) = 0 \Rightarrow \lambda_1 = -1,\ \lambda_2 = -2,\ \lambda_3 = 3$$

$$\Lambda_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad \Lambda_2^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix}$$

for $\lambda_1 = -1$. $A\hat{\underline{x}}_1 = \lambda_1 \hat{\underline{x}}_1$:

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} = -1 \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} \Rightarrow \begin{cases} -x_{11} = -x_{11} \\ -2x_{12} = -x_{12} \\ 3x_{13} = -x_{13} \end{cases} \Rightarrow \begin{cases} x_{11} = 1 \\ x_{12} = 0 \\ x_{13} = 0 \end{cases} \Rightarrow \hat{\underline{x}}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Similarly, we could get $\hat{\underline{x}}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ for $\lambda_2 = -2$, $\hat{\underline{x}}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ for $\lambda_3 = 3$

(c) For 2x2 circulant matrix, there are two normalized eigenvectors :

$\hat{\underline{x}}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\hat{\underline{x}}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Corresponding eigenvalues $\begin{cases} \lambda_1 = 0 + 1 = 1 \\ \lambda_2 = (0 + 1 \cdot e^{2\pi i/2}) = -1 \end{cases}$

$$\Lambda_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \Lambda_3^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(d) For 2x2 circulant matrix.

$\hat{\underline{x}}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\hat{\underline{x}}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Corresponding eigenvalues $\begin{cases} \lambda_1 = \frac{1}{2} + \frac{1}{2} = 1 \\ \lambda_2 = \frac{1}{2} + \frac{1}{2} \cdot e^{2\pi i/2} = 0 \end{cases}$

$\Lambda_4 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ Λ_4^{-1} doesn't exist.

(e) For 3×3 circulant matrix

$$\hat{X}_1 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix},\quad \hat{X}_2 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\-\frac{1}{2}+\frac{\sqrt{3}}{2}i\\-\frac{1}{2}-\frac{\sqrt{3}}{2}i\end{bmatrix}\quad \hat{X}_3 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\-\frac{1}{2}-\frac{\sqrt{3}}{2}i\\-\frac{1}{2}+\frac{\sqrt{3}}{2}i\end{bmatrix}$$

Corresponding eigenvalues are

$$\begin{cases}\lambda_1 = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1\\ \lambda_2 = \frac{1}{3}+\frac{1}{3}\cdot(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)+\frac{1}{3}(-\frac{1}{2}-\frac{\sqrt{3}}{2}i) = 0\\ \lambda_3 = \frac{1}{3}+\frac{1}{3}(-\frac{1}{2}-\frac{\sqrt{3}}{2}i)+\frac{1}{3}(-\frac{1}{2}+\frac{\sqrt{3}}{2}i) = 0\end{cases}$$

$\Lambda_5 = \begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}$ Λ_5^{-1} doesn't exist.

3. (c) For each case, write down the 8×8 matrix $\underline{A}$ (assume $\underline{X_n} = \begin{bmatrix}X_1\\X_2\\X_3\\X_4\\X_5\\X_6\\X_7\\X_8\end{bmatrix}$).

$$A_1\cdot\begin{bmatrix}X_1\\X_2\\X_3\\X_4\\X_5\\X_6\\X_7\\X_8\end{bmatrix} = \begin{bmatrix}X_2-X_8\\X_3-X_1\\X_4-X_2\\X_5-X_3\\X_6-X_4\\X_7-X_5\\X_8-X_6\\X_1-X_7\end{bmatrix} \Rightarrow A_1 = \begin{bmatrix}0&1&0&0&0&0&0&-1\\-1&0&1&0&0&0&0&0\\0&-1&0&1&0&0&0&0\\0&0&-1&0&1&0&0&0\\0&0&0&-1&0&1&0&0\\0&0&0&0&-1&0&1&0\\0&0&0&0&0&-1&0&1\\1&0&0&0&0&0&-1&0\end{bmatrix}$$

$$A_2\cdot\begin{bmatrix}X_1\\X_2\\X_3\\X_4\\X_5\\X_6\\X_7\\X_8\end{bmatrix} = \begin{bmatrix}(X_3-X_1)-(X_2-X_8)\\(X_4-X_2)-(X_3-X_1)\\(X_5-X_3)-(X_4-X_2)\\(X_6-X_4)-(X_5-X_3)\\(X_7-X_5)-(X_6-X_4)\\(X_8-X_6)-(X_7-X_5)\\(X_1-X_7)-(X_8-X_6)\\(X_2-X_8)-(X_1-X_7)\end{bmatrix} = \begin{bmatrix}-X_1-X_2+X_3+X_8\\-X_2-X_3+X_4+X_1\\-X_3-X_4+X_5+X_2\\-X_4-X_5+X_6+X_3\\-X_5-X_6+X_7+X_4\\-X_6-X_7+X_8+X_5\\-X_7-X_8+X_1+X_6\\-X_8-X_1+X_2+X_7\end{bmatrix} \Rightarrow A_2 = \begin{bmatrix}-1&-1&1&0&0&0&0&1\\1&-1&-1&1&0&0&0&0\\0&1&-1&-1&1&0&0&0\\0&0&1&-1&-1&1&0&0\\0&0&0&1&-1&-1&1&0\\0&0&0&0&1&-1&-1&1\\1&0&0&0&0&1&-1&-1\\-1&1&0&0&0&0&1&-1\end{bmatrix}$$

$$A_3\cdot\begin{bmatrix}X_1\\X_2\\X_3\\X_4\\X_5\\X_6\\X_7\\X_8\end{bmatrix} = \begin{bmatrix}X_1-\frac{1}{2}(X_2+X_8)\\X_2-\frac{1}{2}(X_3+X_1)\\X_3-\frac{1}{2}(X_4+X_2)\\X_4-\frac{1}{2}(X_5+X_3)\\X_5-\frac{1}{2}(X_6+X_4)\\X_6-\frac{1}{2}(X_7+X_5)\\X_7-\frac{1}{2}(X_8+X_6)\\X_8-\frac{1}{2}(X_1+X_7)\end{bmatrix} = \begin{bmatrix}X_1-\frac{1}{2}X_2-\frac{1}{2}X_8\\X_2-\frac{1}{2}X_3-\frac{1}{2}X_1\\X_3-\frac{1}{2}X_4-\frac{1}{2}X_2\\X_4-\frac{1}{2}X_5-\frac{1}{2}X_3\\X_5-\frac{1}{2}X_6-\frac{1}{2}X_4\\X_6-\frac{1}{2}X_7-\frac{1}{2}X_5\\X_7-\frac{1}{2}X_8-\frac{1}{2}X_6\\X_8-\frac{1}{2}X_1-\frac{1}{2}X_7\end{bmatrix} \Rightarrow A_3 = \begin{bmatrix}1&-\frac{1}{2}&0&0&0&0&0&-\frac{1}{2}\\-\frac{1}{2}&1&-\frac{1}{2}&0&0&0&0&0\\0&-\frac{1}{2}&1&-\frac{1}{2}&0&0&0&0\\0&0&-\frac{1}{2}&1&-\frac{1}{2}&0&0&0\\0&0&0&-\frac{1}{2}&1&-\frac{1}{2}&0&0\\0&0&0&0&-\frac{1}{2}&1&-\frac{1}{2}&0\\0&0&0&0&0&-\frac{1}{2}&1&-\frac{1}{2}\\-\frac{1}{2}&0&0&0&0&0&-\frac{1}{2}&1\end{bmatrix}$$

(According to the description of A_3. $g_3[n] = f[n] - \frac{1}{2}\cdot(f[n+1] \oplus f[n-1])$)

↓ not −

If using $g[n] = f[n] - \frac{1}{2}(f[n+1] - f[n-1])$

$$A_4 = \begin{bmatrix}1&-\frac{1}{2}&0&0&0&0&0&\frac{1}{2}\\-\frac{1}{2}&1&\frac{1}{2}&0&0&0&0&0\\0&-\frac{1}{2}&1&\frac{1}{2}&0&0&0&0\end{bmatrix}$$

(a) The eigenvectors for all A_n:

$$\hat{\underline{X}}_1 = \frac{1}{2\sqrt{2}}\begin{bmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{bmatrix} \qquad \hat{\underline{X}}_2 = \frac{1}{2\sqrt{2}}\begin{bmatrix}1\\ e^{i2\pi\cdot\frac{1}{8}}\\ e^{i2\pi\cdot\frac{2}{8}}\\ e^{i2\pi\cdot\frac{3}{8}}\\ e^{i2\pi\cdot\frac{4}{8}}\\ e^{i2\pi\cdot\frac{5}{8}}\\ e^{i2\pi\cdot\frac{6}{8}}\\ e^{i2\pi\cdot\frac{7}{8}}\end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix}1\\ \frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\\ i\\ -\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\\ -1\\ -\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\\ -i\\ \frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\end{bmatrix} \qquad \hat{\underline{X}}_3 = \frac{1}{2\sqrt{2}}\begin{bmatrix}1\\ e^{i2\pi\frac{2}{8}}\\ e^{i2\pi\frac{2}{8}\cdot 2}\\ e^{i2\pi\frac{2}{8}\cdot 3}\\ e^{i2\pi\frac{2}{8}\cdot 4}\\ e^{i2\pi\frac{2}{8}\cdot 5}\\ e^{i2\pi\frac{2}{8}\cdot 6}\\ e^{i2\pi\frac{2}{8}\cdot 7}\end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix}1\\ i\\ -1\\ -i\\ 1\\ i\\ -1\\ -i\end{bmatrix}$$

$$\hat{\underline{X}}_4 = \frac{1}{2\sqrt{2}}\begin{bmatrix}1\\ e^{i2\pi\cdot\frac{3}{8}}\\ e^{i2\pi\cdot\frac{3}{8}\cdot 2}\\ e^{i2\pi\frac{3}{8}\cdot 3}\\ e^{i2\pi\cdot\frac{3}{8}\cdot 4}\\ e^{i2\pi\frac{3}{8}\cdot 5}\\ e^{i2\pi\frac{3}{8}\cdot 6}\\ e^{i2\pi\frac{3}{8}\cdot 7}\end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix}1\\ -\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\\ -i\\ \frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\\ -1\\ \frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\\ i\\ -\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\end{bmatrix} \qquad \hat{\underline{X}}_5 = \frac{1}{2\sqrt{2}}\begin{bmatrix}1\\ e^{i2\pi\cdot\frac{4}{8}}\\ e^{i2\pi\cdot\frac{4}{8}\cdot 2}\\ e^{i2\pi\cdot\frac{4}{8}\cdot 3}\\ e^{i2\pi\cdot\frac{4}{8}\cdot 4}\\ e^{i2\pi\frac{4}{8}\cdot 5}\\ e^{i2\pi\frac{4}{8}\cdot 6}\\ e^{i2\pi\frac{4}{8}\cdot 7}\end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix}1\\ -1\\ 1\\ -1\\ 1\\ -1\\ 1\\ -1\end{bmatrix}$$

$$\hat{\underline{X}}_6 = \frac{1}{2\sqrt{2}}\begin{bmatrix}1\\ -\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\\ i\\ \frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\\ -1\\ \frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\\ -i\\ -\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\end{bmatrix} \qquad \hat{\underline{X}}_7 = \frac{1}{2\sqrt{2}}\begin{bmatrix}1\\ -i\\ -1\\ i\\ 1\\ -i\\ -1\\ i\end{bmatrix} \qquad \hat{\underline{X}}_8 = \frac{1}{2\sqrt{2}}\begin{bmatrix}1\\ \frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\\ -i\\ -\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\\ -1\\ -\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\\ i\\ \frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\end{bmatrix}$$

(b)

$$D = [\,\hat{\underline{X}}_1\ \hat{\underline{X}}_2\ \hat{\underline{X}}_3\ \hat{\underline{X}}_4\ \hat{\underline{X}}_5\ \hat{\underline{X}}_6\ \hat{\underline{X}}_7\ \hat{\underline{X}}_8\,]$$

$$= \frac{1}{2\sqrt{2}}\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
1 & \frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2} & i & -\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2} & -1 & -\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2} & -i & \frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\\
1 & i & -1 & -i & 1 & i & -1 & -i\\
1 & -\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2} & -i & \frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2} & -1 & \frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2} & i & -\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1\\
1 & -\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2} & i & \frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2} & -1 & \frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2} & -i & -\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\\
1 & -i & -1 & i & 1 & -i & -1 & i\\
1 & \frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2} & -i & -\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2} & -1 & -\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2} & i & \frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}
\end{bmatrix}$$

(d) For A_1.

$$\Lambda_1 = \begin{bmatrix} 0 & & & & & & & \\ & 1.4142i & & & & & & \\ & & 2i & & & & 0 & \\ & & & 1.4142i & & & & \\ & & & & 0 & & & \\ & 0 & & & & -1.4142i & & \\ & & & & & & -2i & \\ & & & & & & & -1.4142i \end{bmatrix}$$

$$\Lambda_2 = \begin{bmatrix} 0 & & & & & & & \\ & -1-0.4142i & & & & & 0 & \\ & & -2-2i & & & & & \\ & & & -1-2.4142i & & & & \\ & & & & 0 & & & \\ & 0 & & & & -1+2.4142i & & \\ & & & & & & -2+2i & \\ & & & & & & & -1+0.4142i \end{bmatrix}$$

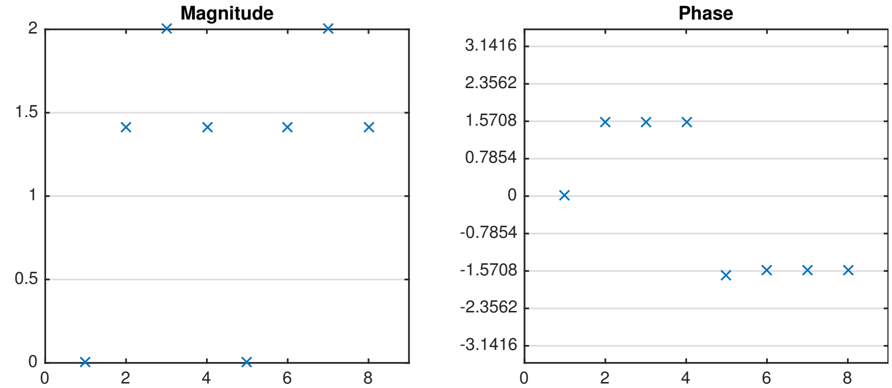
$$\Lambda_3 = \begin{bmatrix} 0 & & & & & & & \\ & 0.2929 & & & & 0 & & \\ & & 1 & & & & & \\ & & & 1.7071 & & & & \\ & & & & 2 & & & \\ & 0 & & & & 1.7071 & & \\ & & & & & & 1 & \\ & & & & & & & 0.2929 \end{bmatrix}$$

$$\Lambda_4 = \begin{bmatrix} 1 & & & & & & & \\ & 1-0.7071i & & & & & & \\ & & 1-i & & & & & \\ & & & 1-0.7071i & & & & \\ & & & & 1 & & & \\ & & & & & 1+0.7071i & & \\ & & & & & & 1+i & \\ & & & & & & & 1+0.7071i \end{bmatrix}$$

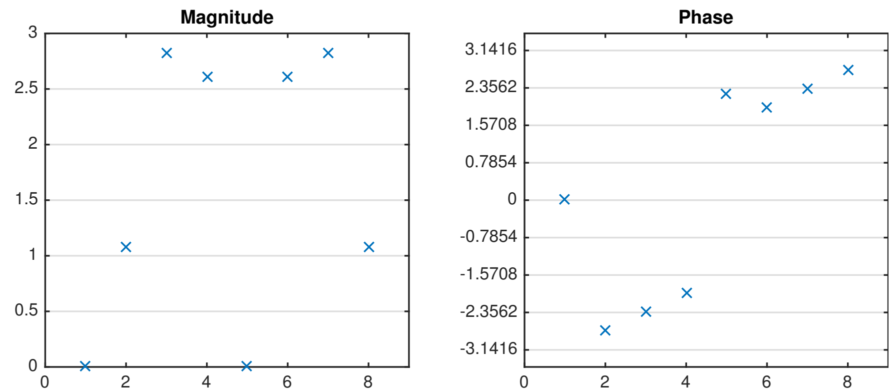
(e) Next page

(f) $\det(A_1) = \det(A_2) = \det(A_3) = 0$ $\therefore$ A_1, A_2, A_3 are noninvertible

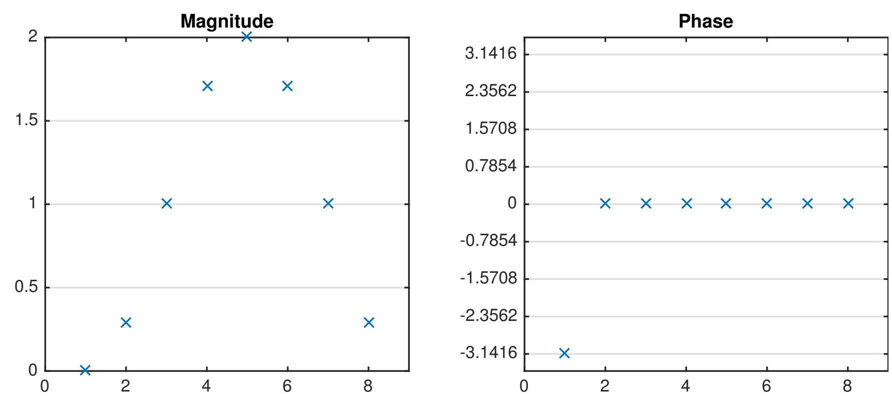
$\det(A_4) \approx 4.5$ $\therefore$ A_4 is invertible



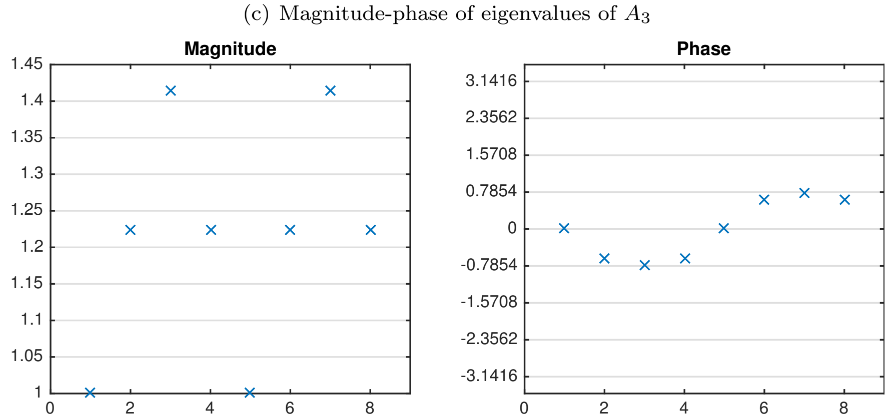
(a) Magnitude-phase of eigenvalues of A_1



(b) Magnitude-phase of eigenvalues of A_2



(c) Magnitude-phase of eigenvalues of A_3



(d) Magnitude-phase of eigenvalues of A_4

Figure 2: Figure of 3(e)