IMGS-261: Linear Mathematics for Imaging
Solution #1

1.

$$f[x] = f_{even}[x] + f_{odd}[x] \quad (1)$$

$$f[-x] = f_{even}[-x] + f_{odd}[-x] \xrightarrow[f_{even}[-x]=f_{even}[x]]{f_{odd}[-x]=-f_{odd}[-x]} f_{even}[x] - f_{odd}[x] \quad (2)$$

$$\xrightarrow{(1)+(2)} f[x] + f[-x] = 2f_{even}[x] \Rightarrow \boxed{f_{even}[x] = \frac{f[x]+f[-x]}{2}}$$

We substitute this expression into (1):

$$f_{odd}[x] = f[x] - f_{even}[x] = f[x] - \frac{f[x]+f[-x]}{2} = \boxed{\frac{f[x]-f[-x]}{2} = f_{odd}[x]}$$

2. $\because$ $\cos[A+B] = \cos[A]\cdot\cos[B] - \sin[A]\cdot\sin[B]$

$\therefore$ $f[x] = A_0 + A_1\cos[2\pi\xi_0 x]\cdot\cos[\phi_0] - A_1\sin[2\pi\xi_0 x]\cdot\sin[\phi_0]$

$\because$ The constant A_0 is *even*, cos function part is *even* and sine function part is *odd.*

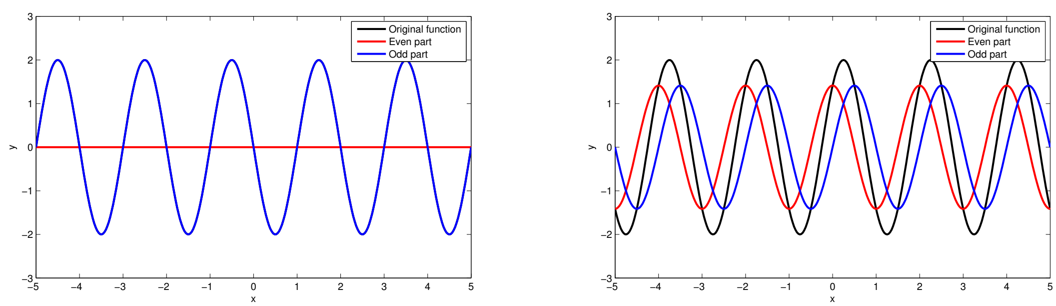
$\therefore$ $\boxed{f_{even}[x] = A_0 + A_1\cos[2\pi\xi_0 x]\cdot\cos[\phi_0],\ f_{odd}[x] = -A_1\sin[2\pi\xi_0 x]\cdot\sin[\phi_0]}$

(a) $f_1[x] = 2\cdot\cos[2\pi\frac{x}{2} + \frac{\pi}{2}] = -2\cdot\sin[2\pi\frac{x}{2}]$

$\Rightarrow f_{even}[x] = 0,\ f_{odd}[x] = -2\cdot\sin[2\pi\frac{x}{2}] = f[x]$

(b) $f_2[x] = 2\cdot\cos[2\pi\frac{x}{2} - \frac{\pi}{4}]$

$\Rightarrow f_{even}[x] = \sqrt{2}\cos[2\pi\frac{x}{2}],\ f_{odd}[x] = \sqrt{2}\sin[2\pi\frac{x}{2}]$



(a) The even(red) and odd(blue) part of $f_1[x]$(black but as the same as odd part)

(b) The even(red) and odd(blue) part of $f_2[x]$(black)

Figure 1: Question #2

3. (a) If $\alpha_0 = 1, \phi_0 = 0$ and $n = 1, f[x] = \cos[\pi x]$.

If $\alpha_0 = 1, \phi_0 = 0$ and $n = 2, f[x] = \cos[\pi x^2]$

(b) Since the argument of a sinusoid is unitless , then $\frac{x}{\alpha_0}$ must be a pure number, so the unit of α_0 must be the same as that of x such that the units are canceled. Since x is a length, then α_0 must also be of a length unit.

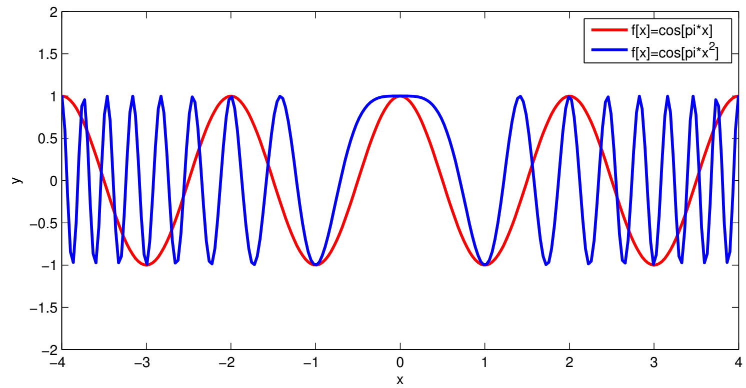
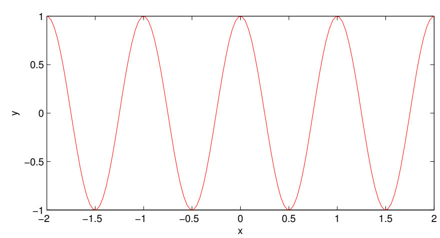
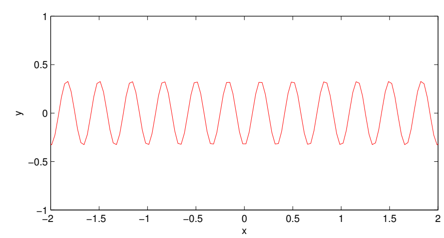


Figure 2: Question #3 $\cos[\pi x]$ in red, $\cos[\pi x^2]$ in blue

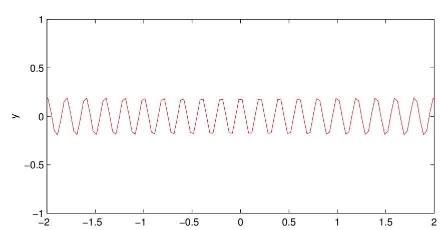
4 .



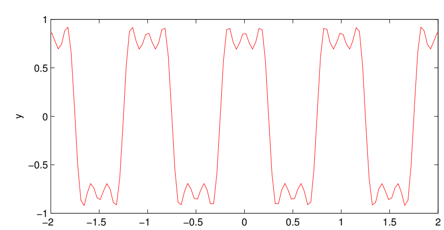
(a) $cos[2\pi x]$



(b) $-\frac{1}{3}cos[2\pi \cdot 3 \cdot x]$



(c) $\frac{1}{5}cos[2\pi \cdot 5 \cdot x]$

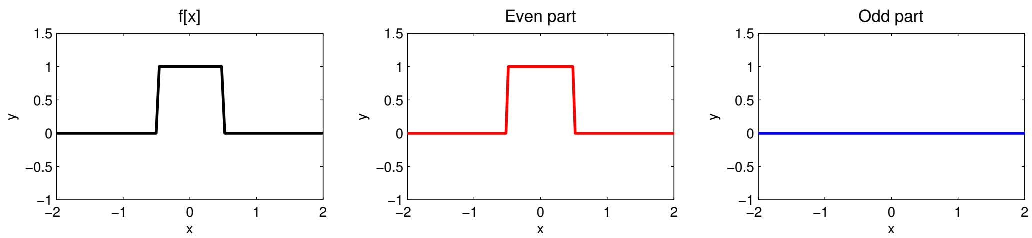


(d) $f_0[x] + f_1[x] + f_2[x]$

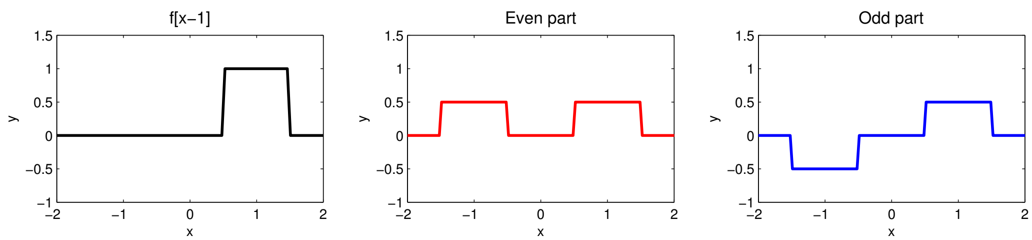
Figure 3: $(a) f_0[x] = cos[2\pi x]$ (b)$f_1[x] = -\frac{1}{3}cos[2\pi \cdot 3 \cdot x](c)f_2[x] = \frac{1}{5}cos[2\pi \cdot 5 \cdot x]$ (d)$f_3[x] = f_0[x] + f_1[x] + f_2[x]$

(e) We will get a square wave with the amplitude of $\pi/4$

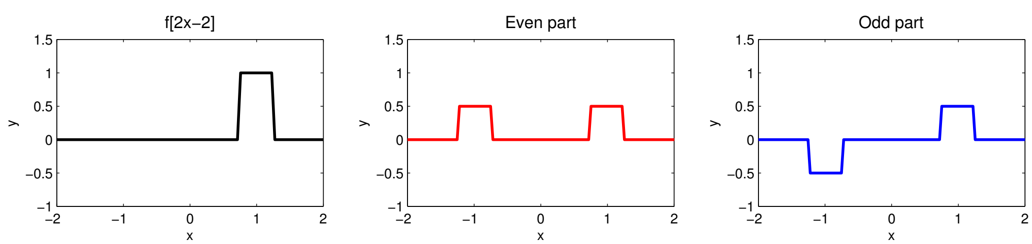
5 .



(a) $f[x]$ and its even and odd part



(b) $f[x-1]$ and its even and odd part



(c) $f[2x-2]$ and its even and odd part