IMGS-261 – Linear Mathematics for Imaging (52271)

(Edited 19 January 2015)

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Office Hours M 10:30am-11:30am, T+Th 11am-12n, W 2pm-3pm, and by appointment

Course Webpage: http://www.cis.rit.edu/class/simg320/index.html

Teaching Assistant: TBD (if any)

Meeting Rooms/Times: TTh, 9:00am – 10:45am , Room 76-1235

Additional times may be scheduled (perhaps weekly) for *OPTIONAL* problem sessions.

Prerequisites:

Calculus

Details:

Homework will be assigned, and is to be handed in **on time** (adjustments will be considered **in advance,** in cases of unforeseeable emergency, adjustments may be considered after the fact). Scores for assignments handed in late without appropriate excuse will be penalized heavily and homework will not be accepted after solutions have been posted. Problems will be graded and solutions to all problems will be handed back as quickly as possible after the homework is due.

Homework **–** 30% (Assignments usually given Tuesdays, due 1 week later in class.)

Midterm Exams (anticipated dates: Th 3/5/2015, Th 4/16/2015) **–** 30% (15% each)

Final Exam (cumulative) **–** 40% (Tuesday 19 May 2015, 8am – 10am, 76-1235 (I will be requesting a three-hour block for the final exam)

My philosophy on exams: they are intended to test *understanding* of material, which is the ability to assimilate concepts and synthesize useful results for applications. This is *not* the same as the ability to parrot discussions of concepts or replicate the solutions to homework problems. I particularly like problems that appear to be difficult but actually are easy if you see the connection. My exams seem to have a reputation among students.

Time If you are having problems learning the material, ***DON'T WAIT***; ASK for help ***EARLY*** – in class and/or out of class. Though I keep my office door closed, PLEASE knock. If I am not doing something urgent, I will set aside time to help – working with students individually or in small groups is the most enjoyable part of my job.

Grading:

The approximate mapping of numerical to letter grades will likely be:

Numerical Score	Letter Grade
88 - 100	A
76.0 - 87.9	B
68.0 - 75.9	C
60.0 - 67.9	D
< 60	F

but I reserve the right for some flexibility. Note that the histogram bins are a bit wider for higher scores.

I do not use *MyCourses*, so you will have to keep track of your own grade average if desired.

A word of warning: it is not wise to ask no questions, in class or otherwise, until *just* before some deadline (the beginning of class when homework is due or just before an exam), when the questions are suddenly *urgent*. By then, it may be too late. My advice is to your homework and exam studying early so that you can ask questions in time to have an positive impact on your understanding.

Because the classroom **DOES NOT HAVE** a Smart Board and video capability, **you have to attend the class to be certain that you will see the material.**

Subject:

This course introduces some of the mathematical tools for describing imaging systems, specifically those that are useful to describe linear discrete (sampled) operations. In other words, the course develops mathematical models of imaging systems and applies them to problems relevant in imaging. This course introduces and makes use of material that often is considered in courses on matrix algebra, linear algebra, and complex analysis. However, the treatment is very different from the "slant" generally encountered in those courses. We will describe the input to a linear imaging system as a vector $\underline{\mathbf{x}}$ and the system as a matrix $\underline{\mathbf{A}}$. The output (the "image" will be a vector $\underline{\mathbf{b}}$ so that:

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{x}} = \underline{\mathbf{b}}$$

Our goal generally is to solve the *inverse problem* where we know the output vector $\underline{\mathbf{b}}$ and the system $\underline{\mathbf{A}}$ and wish to find the input vector (the object) $\underline{\mathbf{x}}$. To do so for the specific type of imaging system of interest, it will be very convenient to represent the known entities in a different coordinate system (similar to changing from Cartesian to polar coordinates). TheThe tool we will develop is the *Fourier transform*. We shall approach the discrete case (sampled functions that may be described as vectors), so to derive the *discrete Fourier transform* (DFT). The DFT has a twin sibling, the *fast Fourier transform* (FFT) that is readily available in many computational software packages (e.g., MATLAB®, Mathcad, Mathematica®, etc.). The DFT is a weighted summation (what mathematicians call a *linear combination*) of the components of a vector *f with N* components to generate a new (and equivalent) vector *F* via:

$$F[k] = c \cdot \sum_{n=0}^{N-1} f[n] \cdot \exp\left[-2\pi i \frac{nk}{N}\right]$$

where $i \equiv \sqrt{-1}$ (hence the need to consider complex analysis), *n* is the index that specifies the component of the input vector, and *k* is the index for the output vector. The constant *c* is used to normalize the result and varies with application. We shall have to go through a number of background steps before arriving at this result.

If time allows, we shall also introduce the continuous case, which is an integral applied to an input function $f[x]$ to generate a new (and equivalent) function $F[\xi]$ via:

$$F[\xi] = \int_{-\infty}^{+\infty} f[x] \cdot \exp[-2\pi i \xi x] \, dx$$

Many (if not most) students find this material to be challenging, but it is *essential* to attain some level of mastery of Fourier theory (also called *linear systems theory*) if you plan to work in any area of imaging. Fourier transforms appear in many (if not all) aspects of imaging, especially to describe and/or predict the behavior of optical imaging systems and to filter digital or analog images. If you are not very familiar with the subject (and even if you are), you should expect to devote a significant amount of time to the subject outside of class. A rule of thumb for academic classes is that you will spend 2-3 hours outside of class *per hour* of class time, hence 8-12 hours per week **outside of** class. You should use this material as a springboard to your specific subject of interest. **You would be wise to plan to do outside reading on the subject throughout the term**.

Text materials:

Fourier Methods in Imaging, R.L. Easton, Jr., Wiley, 2010, ISBN 978-0-470-68983-7, list price $165 from Wiley (Adobe e-book format for the same price). The book is available from the usual suspects: the RIT Bookstore (B&N@RIT, http://rit.bncollege.com/ and from Amazon.com). It may be available from the Society for Imaging Science and Technology at a substantial discount of $120 (including shipping). The book considers continuous and discrete imaging systems and optical imaging, and thus is relevant to other courses. Note that MANY other books also are available that touch on aspects of mathematical models of imaging systems and parameters. You should peruse these and (at least) be aware of the treatments (see the bibliography list). Though my book may be considered to be expensive at $165 list, it is cheaper than the alternatives listed below AND it is useful for at least two additional courses: digital image processing and physical (wave) optics.

This subject is covered in other books. Note that I have many of these on my shelf and they are available for loan (list below). Among the many possibly useful web resources, Derek Walvoord (a former student) recommends a set of free video lectures on linear algebra by Gilbert Strang from MIT, who has written several excellent textbooks on the subject. Note that the notation Strang uses differs in some respects from that we shall use, e.g., Strang references the complex conjugate of the number z as $\overline{z}$, whereas we shall use the notation z^*.

Computing Resources:

Signals:

For some specific applications in 1-D linear systems, my DOS program for PCs ("***SIGNALS***") may be useful. It was written with the intent of being easy to use and may be downloaded from

http://www.cis.rit.edu/people/faculty/easton/signals/signals.zip

The user manual also is available online at:

http://www.cis.rit.edu/resources/software/sig_manual/index.html

The program runs XP and earlier versions of Windows that retain the capability to run DOS programs, but it will not run in Win7 (and probably not in Vista). In the new versions of Windows (or in Macintosh OSX and Linux), the program may be run in DOSBox:

http://www.dosbox.com/

SignalShow:

The Java counterpart of ***Signals***, called ***SignalShow***, which was written as a Senior Project by Juliet Bernstein Fiss, illustrates both 1-D and 2-D examples of Fourier mathematics and imaging. The beta releases for the three primary computing platforms (Windows, Macintosh OSX, and Linux) are available online at:

Download Jar: http://homes.cs.washington.edu/~juliet/photography/SignalShow.jar

Source Code (Dropbox link): https://www.dropbox.com/sh/h3c07v15kf1ptao/PQ1_7n8nEa

Videos that illustrate the use of the program are posted on YouTube. You may find this program very helpful in your quest to visualize the concepts in this course.

Useful References from Magazines and Journals: (with links to pdf copies)

1. "The Fourier Transform," R.N. Bracewell, in ***Scientific American***, June 1989, pp.86-95
2. Numerical Transforms," R.N. Bracewell, in ***Science***, **v.248**, 11 May 1990, pp.697-704
3. "Fourier Analysis Using a Spreadsheet," R.A. Dory and J.H. Harris, in ***Computers in Physics***, **Vol. 2,** Nov.-Dec. 1988, pp. 83-86
4. "A Plain Man's *(sic)* Guide to the FFT," P. Kraniauskas, in ***IEEE Signal Processing Magazine***, v.**11**, April 1994, pp. 24-35
5. "Tom, Dick, and Mary Discover the DFT," J.R. Deller, Jr., in ***IEEE Signal Processing Magazine***, v.**11** April 1994, pp. 36-50
6. SIGNALS, Interactive Software for One-Dimensional Signal Processing," R.L. Easton, Jr., in ***Computer Applications in Engineering Education***, v.**1**, December 1993, pp.489-501
7. "Fast Fourier Transforms for Fun and Profit," W.M. Gentleman and G. Sande, in *Proceedings - Fall Joint Computer Conference*, 1966, pp.563-578

Mathematical Foundations of Linear Systems:

1. For review, Schaum's Outlines on Calculus, Linear Algebra, Vector Analysis, Matrices, and Complex Variables and Schaum's *Mathematical Handbook*
2. *Advanced Mathematical Methods for Engineering and Science Students*, G. Stephenson, P.M.Radmore, Cambridge, 1990 (§2 on special functions, §7 on Fourier transforms).
3. *Linear Algebra and its Applications* (4th Edition), Gilbert Strang, Harcourt,Brace,Jovanovitch, 2009, (Chapters on orthogonal projections, eigenvectors, change of bases).
4. Any of several books on mathematical physics, *e.g*., Kreysig, Arfken, Byron and Fuller, ...

Fourier Transforms in Mathematics:

1. *The Fourier Integral and Certain of its Applications*, N.Wiener, Dover Publications, 1958 (first published in 1933 -- *tediously mathematical*), QA404.W47.
2. *An Introduction to the Theory of Fourier's Series and Integrals*, H.S. Carslaw, Dover Publications, 1950 (first published in 1930 -- *also mathematical, but easier to read than Wiener)* QA404.C32
3. *A Handbook of Fourier Theorems*, D.C. Champeney, Cambridge, 1987, (*best of the three*) QA403.5.C47.

Fourier Transforms in Physics/Engineering:

1. *Fourier Series and Boundary-Value Problems*, James Brown and R.V.Churchill, McGraw-Hill, 8th Edition, 2011, (*classic text with lots of physical applications*), QA404.C6.
2. *A First Course in Fourier Analysis*, D.M. Kammler, Prentice-Hall, 2000, (useful discussions of mathematical and computational aspects), QA403.5.K36.
3. *Fourier Transforms and their Physical Applications*, D.C.Champeney, Academic Press, 1973, *(excellent)*, QA403.5.C46.
4. *Fourier methods for mathematicians, scientists, and engineers*, M.Cartwright, Ellis Horwood, 1990, *(paperback, introductory, lots of physical applications)*, QA403.5.C37.
5. *The Fourier Transform and Its Applications* (Second Edition, Revised), R.N.Bracewell, McGraw-Hill, 1986, *(the standard reference on 1-D Fourier, good discussion of discrete transforms and applications)*, QA403.5.B7
6. *Fourier Transforms, An Introduction for Engineers*, R.M.Gray and J.W.Goodman, Kluwer Academic Publishers, 1995, *(aimed at discrete transform, not as useful as I expected)*, TK5102.9.G73.

7. *A student's guide to Fourier transforms*, J.F.James, Cambridge, 1995, QC20.7.F67J36, (*thin, cheap, useful)*
8. *The Fourier Integral and its Applications*, A. Papoulis, McGraw-Hill, 1962, *(old - preFFT, though good mix of mathematical theory and practical applications)*, QA404.P32.
9. *Fourier Transforms*, I.N.Sneddon, Dover Publications, 1995 (first published in 1951), *(similar comments to Papoulis)*, QA404.S53.
10. *Fourier Analysis*, T.W.Körner, Cambridge, 1988, *(potpourri of Fourier from nonconventional point of view -- historically driven)*, QA403.5.K67.
11. *Exercises for Fourier Analysis*, T.W.Körner, Cambridge, 1993, *(see comment above)* ,QA403.5.K66.
12. *Integral Transforms in Science and Engineering*, K.B.Wolf, Plenum, 1979, *(mathematical reference)*, QA432.W64.
13. *Probability, Statistical Optics, and Data Testing*, 2nd Ed. B.R.Frieden, Springer-Verlag, 1991 (particularly §4 on Fourier methods, *(excellent discussion of applications of statistical principles to many types of imaging problems, not just optics)*, QA273.F89.
14. *Statistical Optics*, J.W. Goodman, Wiley, 1985, *(applications of Fourier theory to statistics, particularly in optics)*, QC355.2.G66.
15. *Who is Fourier? A Mathematical Adventure*, Transnational College of LEX, Language Research Foundation, 1995. *($25 paperback translated from Japanese, very introductory, lots of pictorial examples. usefulness limited by lack of index).*
16. *The Hartley Transform*, R.N.Bracewell, Oxford, 1986, (*special case of Fourier transform, a favorite of the author)* QA403.5.B73.

Discrete Fourier Transforms:

1. *The FFT, Fundamentals and Concepts*, R.W.Ramirez, Prentice-Hall, 1985, *(graphical introduction to discrete Fourier transform)*.QA403.5.R36.
2. *The Fast Fourier Transform and its Applications*, E.O.Brigham, Prentice-Hall, 1988, *(excellent)*, QA403.B75.
3. Fast Fourier Transforms, J.S.Walker, 2nd Edition, CRC Press, 1996, *(new edition, includes software)*, QA403.W33.
4. *Multidimensional Digital Signal Processing*, D.E.Dudgeon and R.M.Mersereau, Prentice-Hall, 1984 (§1-2), *(written for EEs, but good discussion of 2-D discrete transform,* TK5102.5.D83.
5. *Digital Image Processing*, K.R.Castleman, Prentice-Hall, 1996 (§1-2,§9-16), *(excellent for lots of imaging problems, demonstrates relationship of linear systems to optical systems)*,TA1632.C37.

Linear Systems and Optical Imaging:

1. *Introduction to Fourier Optics*, J.W.Goodman, (2nd Edition), McGraw-Hill, 1996, *(updated classic)*, QC355.G65.
2. *Fourier Optics, An Introduction* (2nd Edition), E.G.Steward, Wiley, 1987, *(useful introduction, lower level than Goodman)*, QC454.F7S83.
3. *Introduction to the Optical Transfer Function*, C.S.Williams and O.A.Becklund, Wiley, 1989, *(specialized topic of linear systems in optics)*, QC367.W55.
4. *Systems and Transforms with Applications in Optics*, A.Papoulis, McGraw-Hill, 1968, *(another classic, showing its age a bit)*, QC383.P23.
5. *Applications of Optical Fourier Transforms*, H.Stark, ed., Academic Press, 1982, *(as implied, discussions of specific applications)*, TA1632.A68.

6. *Quantitative Coherent Imaging: Theory, Methods, and Some Applications*, J.M.Blackledge, Academic Press, 1989, *(nice desciption of subject, unusual notation/spellings)*, QC476.C6.B553
7. *The New Physical Optics Notebook*, Reynolds, DeVelis, Parrent, and Thompson, SPIE Press, 1989, *(applications of linear systems to optics/holography)*, QC395.2.N48.
8. *Fourier Series and Optical Transform Techniques in Contemporary Optics*, Raymond Wilson, John Wiley & Sons, Inc, 1995. QC454.F7 W55 (ISBN 0-471-30357-7)

Image Recovery:

1. *Image Restoration and Reconstruction*, R.H.T.Bates and M.J.McDonnell, Oxford University Press, 1986, *(application of linear systems to imaging)*, TA1632.B36.
2. *Image Recovery, Theory and Application*, (H.Stark, ed.), Academic Press, 1987, *(similar to Bates but more applications, multiple authors, fragmented)*, TA1632.I4824.

Other books containing useful discussions of imaging subjects:

1. *Principles of Digital Image Synthesis*, Andrew Glassner, Morgan-Kauffman, 1995 (two volumes), *(very nice discussion of broad range of imaging topics, relevant material in §4-5,§8-10*), T385.G585.
2. *Image Reconstruction in Radiology*, J. Anthony Parker, CRC Press, 1990, (*of much more general application than the title indicates; written for medical students and radiologists, does not require a "high" level of mathematical knowledge, useful intuitive discussions of imaging principles*) RC78.7.D53 P36.
3. *Radiological Imaging*, H.H. Barrett and W.Swindell, Academic Press, 1981, *(terrific book, also much more general than indicated by its title),* RC78.B337, (§2, §4 on Linear Systems, §3 on Random Processes, §7 on Computed Tomography)