

TWO HOURS, Closed Book, No Notes, TURN OFF ALL PHONES, PAGERS, iPods, etc.; PUT THEM AWAY OUT OF SIGHT

Nonprogrammed calculators OKAY (but no phones as calculators!)

Possibly Useful Information at the end

Show your work. If you solve problems “by inspection”, then write down the thought process that led to your conclusion. You may leave square roots in expressions without evaluating numerically.

SELECT ANY 6 of the 8 problems, equal weight

Staple and submit ONLY your selected problems IN NUMERICAL ORDER

1. We defined the concepts of a linear operator and a shift-invariant operator; a linear operator satisfies the condition:

$$\text{if } \mathcal{O} \{f_n [x]\} = g_n [x], \text{ then } \mathcal{O} \left\{ \sum_{n=1}^N \alpha_n \cdot f_n [x] \right\} = \sum_{n=1}^N \alpha_n \cdot g_n [x]$$

where $\{\alpha_n\}$ are numerical constants that are generally complex valued. A shift-invariant operator satisfies the condition

$$\text{if } \mathcal{O} \{f [x]\} = g [x], \text{ then } \mathcal{O} \{f [x - x_0]\} = g [x - x_0]$$

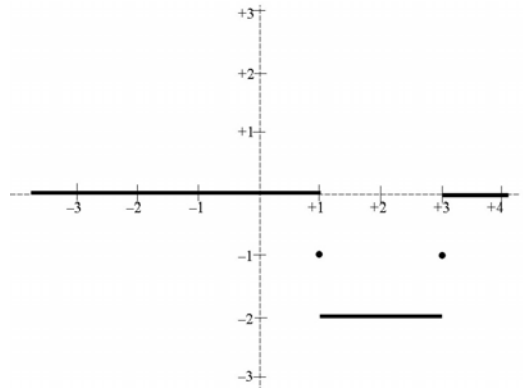
From these definitions, determine whether the Fourier transform operator $\mathcal{F} \{f [x]\}$ satisfies the properties of linearity and of shift invariance. You need to show this, not just write it down.

2. We defined the rectangle function by the expression:

$$RECT [x] = \begin{cases} 0 & \text{if } |x| > \frac{1}{2} \\ \frac{1}{2} & \text{if } |x| = \frac{1}{2} \\ 1 & \text{if } |x| < \frac{1}{2} \end{cases}$$

(a) Sketch $RECT \left[\frac{-x - 3}{2} \right]$ and determine its area

(b) Determine the equation for the function with the following graph:



3. Simplify the expression and sketch the result:

$$g[x] = \int_{x=-\infty}^{x=+\infty} \text{SINC}[x] \cdot \delta[x - 2] dx$$

4. Evaluate the convolution of the pairs of rectangles listed:

(a) $\text{RECT}\left[\frac{x}{2}\right] * \text{RECT}\left[\frac{x}{2}\right]$

(b) $\text{RECT}\left[\frac{x-2}{2}\right] * \text{RECT}\left[\frac{x+1}{2}\right]$

5. Evaluate the discrete Fourier transform of the following four-sample arrays of data indexed from $n = 0$ to $N = 3$:

(a) $f[0] = f[1] = f[2] = f[3] = 1$

(b) $f[0] = f[2] = +1, f[1] = f[3] = -1$

(c) $f[0] = +1, f[1] = 0, f[2] = -1, f[3] = 0$

6. If the continuous Fourier transform of the arbitrary function $f[x]$ is $F[\xi]$, derive (i.e., do not just “write down”) the expression for the Fourier transform of $f\left[x - \frac{3}{2}\right]$ and express it in the two ways, i.e., as real and imaginary parts and as magnitude with phase.

7. For the following functions, sketch them and evaluate their Fourier transforms by direct integration:

(a) $\text{RECT}[x + 1] + \text{RECT}[x - 1]$

(b) $\text{RECT}\left[\frac{x+1}{2}\right] + \text{RECT}\left[\frac{x-1}{2}\right]$

Hint: you may sum the transforms of the component functions.

8. We defined Fourier analysis of the continuous function $f[x]$ to be

$$\int_{x=-\infty}^{x=+\infty} f[x] \cdot (\exp[+i \cdot 2\pi\xi x])^* dx \equiv F[\xi]$$

(a) Write down the corresponding expression for Fourier synthesis of the function $F[\xi]$

(b) Evaluate the expression you wrote down in part (a) for $F[\xi] = \text{RECT}[\xi]$ to find the corresponding space-domain function $f[x]$

Possibly Useful Information:

$$\mathcal{F}\{f[x]\} \equiv \int_{x=-\infty}^{x=+\infty} f[x] \cdot (\exp[+i \cdot 2\pi \cdot \xi x])^* dx$$

$$\text{SINC}[x] \equiv \frac{\sin[\pi x]}{\pi x}$$