

1. For $f_1[x] = \text{SINC}[x] \equiv \frac{\sin[\pi x]}{\pi x}$, use the “modulation theorem” to evaluate and sketch the Fourier transform of $f_2[x] = f_1[x] \cdot f_1[x] = (f_1[x])^2$
2. Use the result of #1 to sketch the form of the Fourier transform of $f_3[x] = f_1[x] \cdot f_1[x] \cdot f_1[x] = (f_1[x])^3$; you may evaluate the functional form first, but you only “have” to sketch the result.
3. Use the result of #2 to sketch the form of the Fourier transform of $f_3[x] = f_1[x] \cdot f_1[x] \cdot f_1[x] \cdot f_1[x] = (f_1[x])^4$.
4. Determine the conditions that must be satisfied for the following expression to be valid:

$$\cos[2\pi\xi_0 x] * \text{SINC}\left[\frac{x}{b_0}\right] = A_0 \cos[2\pi\xi_0 x]$$

5. Determine the conditions that must be satisfied for the following expression to be valid:

$$\cos[2\pi\xi_0 x] * \text{SINC}^2\left[\frac{x}{b_0}\right] = B_0 \cos[2\pi\xi_0 x]$$

6. Consider the convolution of 100 scaled *SINC* functions with width parameters that “start small” and grow for succeeding terms:

$$g[x] = \frac{1}{1} \text{SINC}\left[\frac{x}{1}\right] * \frac{1}{2} \text{SINC}\left[\frac{x}{2}\right] * \frac{1}{3} \text{SINC}\left[\frac{x}{3}\right] * \dots * \frac{1}{n} \text{SINC}\left[\frac{x}{n}\right] * \dots * \frac{1}{100} \text{SINC}\left[\frac{x}{100}\right]$$

- (a) Sketch the first three *SINC* functions on the same graph, including both the amplitudes and “width” parameters.
 - (b) Evaluate the functional form of and sketch $g[x]$.
7. Consider the convolution of a *SINC* function with width b_0 and a *SINC*² function with width d_0 :

$$\text{SINC}\left[\frac{x}{b_0}\right] * \text{SINC}^2\left[\frac{x}{d_0}\right] = g[x] = A_0 \cdot \text{SINC}^2\left[\frac{x}{d_0}\right]$$

- (a) Find the conditions on the width parameters b_0 , d_0 , and A_0 that must be satisfied for this expression to be valid.
- (b) Explain whether it is possible to select width parameters and B_0 such that:

$$\text{SINC}\left[\frac{x}{b_0}\right] * \text{SINC}^2\left[\frac{x}{d_0}\right] = g[x] = B_0 \cdot \text{SINC}\left[\frac{x}{b_0}\right]$$