

IMGS-261      Homework #7      Due 04/09/2015 (Th)  
**Second Test on Thursday 16 April**

In the book, read the sections in Chapter 6 about the functions we have discussed and read Chapter 8

Do the following problems:

1. Sketch the following functions and evaluate their continuous Fourier transforms

$$(a) f_1[x] \equiv \begin{cases} 0 & \text{if } |x| > 2 \\ \frac{1}{2} & \text{if } |x| = 2 \\ 1 & \text{if } |x| < 2 \end{cases}$$

$$(b) f_2[x] \equiv \begin{cases} 0 & \text{if } x > 4 \\ \frac{1}{2} & \text{if } x = 4 \\ 1 & \text{if } 0 < x < 4 \\ \frac{1}{2} & \text{if } x = 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$(c) f_3[x] = \exp[-x] \cdot STEP[x]$$

$$(d) f_4[x] = \exp\left[-\frac{x}{2}\right] \cdot STEP\left[\frac{x}{2}\right]$$

2. The operation of “convolution” of two functions  $f[x]$  and  $h[x]$  is defined:

$$f[x] * h[x] = \int_{-\infty}^{+\infty} f[\alpha] \cdot h[x - \alpha] dx$$

Evaluate the convolution of  $f[x] = \cos\left[2\pi\frac{x}{2} + \frac{\pi}{4}\right]$  with the following functions for  $h[x]$  and sketch (or plot) the results:

$$(a) h_1[x] = RECT[x]$$

$$(b) h_2[x] = \frac{1}{2}RECT\left[\frac{x}{2}\right]$$

3. Evaluate the convolutions of the following functions by “direct integration” and sketch (or plot) the results:

$$(a) g_1[x] = RECT[x] * RECT[x]$$

$$(b) g_2[x] = RECT\left[\frac{x}{2}\right] * RECT\left[\frac{x}{2}\right]$$

$$(c) g_3[x] = RECT[x] * RECT[x] * RECT[x]$$

4. Evaluate and sketch (or plot) the following convolutions:

$$(a) (\exp[-x] \cdot STEP[x]) * RECT[x]$$

$$(b) (\exp[-x] \cdot STEP[x]) * (\exp[-x] \cdot STEP[x])$$