

1. In any language you choose and are comfortable with write a computer program to evaluate the discrete Fourier transform (DFT) of an array $f[n]$ with N data points. You can even do this in Excel – there is an article posted on programming the DFT using a spreadsheet at http://www.cis.rit.edu/class/sing320/Fourier_Analysis_spreadsheet.pdf. The N points of the function should be indexed over the interval $0 \leq n \leq N - 1$, so that the formula for the DFT is:

$$\begin{aligned}
 F[k] &= \sum_{n=0}^{N-1} f[n] \cdot \exp \left[-2\pi i \frac{n \cdot k}{N} \right] \\
 &= \left(\sum_{n=0}^{N-1} f[n] \cos \left[+2\pi \frac{n \cdot k}{N} \right] \right) - i \cdot \left(\sum_{n=0}^{N-1} f[n] \sin \left[+2\pi \frac{n \cdot k}{N} \right] \right)
 \end{aligned}$$

where the limits on k are:

$$0 \leq k \leq N - 1$$

Evaluate the real and imaginary parts separately

$$\begin{aligned}
 \text{Re} \{F[k]\} &= \left(\sum_{n=0}^{N-1} f[n] \cdot \cos \left[+2\pi \frac{n \cdot k}{N} \right] \right) \\
 \text{Im} \{F[k]\} &= \left(\sum_{n=0}^{N-1} f[n] \cdot \left(-\sin \left[+2\pi \frac{n \cdot k}{N} \right] \right) \right)
 \end{aligned}$$

2. Graph the following 8-point arrays and evaluate their DFTs. Graph the real part and the imaginary part of the DFTs; for each array you may submit one graph of the DFT with both the real and imaginary parts but you must distinguish the two lines either by color or by “type” (e.g., solid line and dashed line)

a.

n	$f[n]$
0	$+\frac{1}{3}$
1	$+\frac{1}{3}$
2	0
3	0
4	0
5	0
6	0
7	$+\frac{1}{3}$

b.

n	$f[n]$
0	$+\frac{1}{3}$
1	$+\frac{1}{3}$
2	$+\frac{1}{3}$
3	0
4	0
5	0
6	0
7	0

c.

n	$f[n]$
0	+1
1	$\frac{1}{\sqrt{2}}$
2	0
3	$-\frac{1}{\sqrt{2}}$
4	-1
5	$-\frac{1}{\sqrt{2}}$
6	0
7	$+\frac{1}{\sqrt{2}}$

d.

n	$f[n]$
0	+1
1	-1
2	+1
3	-1
4	+1
5	-1
6	+1
7	-1

e.

n	$f[n]$
0	+1
1	+1
2	+1
3	+1
4	-1
5	-1
6	-1
7	-1

MORE →→→→→

3. Evaluate and graph the real and imaginary parts of the DFTs of the following 64-point arrays that are filled with samples of a cosine function with the following periods:
- (a) 64 samples (one cycle of the cosine in the array), i.e. $f[n] = \cos\left[2\pi\frac{n}{64}\right]$
 - (b) 32 samples (two cycles in the array), $f[n] = \cos\left[2\pi\frac{n}{32}\right]$
 - (c) a cosine with period $\frac{64}{3}$ (so that there are three cycles of the cosine in the array)
 - (d) A cosine with period 16 samples (4 cycles in the array)
 - (e) Compare the graphs of the results of the four cases to find the pattern of the results.
 - (f) Also do the same for one cycle of a sine in the array.
4. (OPTIONAL BONUS) Evaluate and graph the magnitude and phase of each DFT array separately