IMGS-261 Homework #5 Due 3/12/2015 (Th)

Finish reading Chapter 5 of the book; we will eventually be using material from SOME of Chapter 6, so you might want to start reading about the rectangle, triangle, and other functions up through §6.1.9.

1. Extend the properties of the 2-D and 4-D cases to

   (a) Derive (or just "write down") the normalized eigenvectors of the 16×16 circulant matrices as real-and-imaginary parts and as magnitude+phase angle.

   (b) On separate graphs for each eigenvector, sketch the numerical values of the samples of each eigenvector as as real-and-imaginary parts and as magnitude+phase.

2. For each matrix, find all eigenvalues and the corresponding eigenvectors, the diagonal form for the matrix with the eigenvalues on the diagonal, and the inverse of the diagonal matrix

   (a) $\underline{\mathbf{A}}_1 = \begin{bmatrix} -1 & 0 \\ 0 & +2 \end{bmatrix}$ (not circulant)

   (b) $\underline{\mathbf{A}}_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & +3 \end{bmatrix}$ (not circulant)

   (c) $\underline{\mathbf{A}}_3 = \begin{bmatrix} 0 & +1 \\ +1 & 0 \end{bmatrix}$ (circulant)

   (d) $\underline{\mathbf{A}}_4 = \begin{bmatrix} +\frac{1}{2} & +\frac{1}{2} \\ +\frac{1}{2} & +\frac{1}{2} \end{bmatrix}$ (circulant)

   (e) $\underline{\mathbf{A}}_5 = \begin{bmatrix} +\frac{1}{3} & +\frac{1}{3} & +\frac{1}{3} \\ +\frac{1}{3} & +\frac{1}{3} & +\frac{1}{3} \\ +\frac{1}{3} & +\frac{1}{3} & +\frac{1}{3} \end{bmatrix}$ (circulant)

3. The following definitions of the action of systems designated by the 8×8 matrix $\underline{\mathbf{A}}_n$ on 8-element input vectors $\underline{\mathbf{x}}_n$, which may be interpreted as samples $f[n]$ of the input function $f[x]$ to produce output vectors $\underline{\mathbf{b}}_n$, which are, in turn, samples $g[n]$ of the output function $g[x]$. represent the outputs of a system for input vectors $\underline{\mathbf{x}}_n$, which may be interpreted as samples of the input function $f[n]$. In each case, the functions f are assumed to be periodic over 8 samples, so that the values that "disappear off the edge" on one side will "reappear" on the other side. These all respresent common operations in image processing.

   (1) $\underline{\mathbf{A}}_1$ is the 8×8 matrix that calculates the numerical difference of pairs of pixels separated by one pixel:

$$g_1[n] = (-f[n-1] + f[n+1])$$

(2) $\underline{\mathbf{A}}_2$ calculates the difference of adjacent pixels from the result of $\underline{\mathbf{A}}_1$

(3) $\underline{\mathbf{A}}_3$ is the 8×8 matrix that calculates the numerical difference of the pixel and the average of its neighbors:

$$g_3[n] = f[n] - \frac{1}{2} \cdot (f[n+1] - f[n-1])$$

(a) Write down the 8 normalized eigenvectors for all of the $\underline{\mathbf{A}}_n$ (since all are circulant, there is only one set of 8; the length of each vector is unity).

(b) Write down the single diagonalizing matrix $\underline{\mathbf{D}}$ for all the $\underline{\mathbf{A}}_n$ – this is the matrix that generates a diagonal matrix $\underline{\mathbf{\Lambda}}$ via:

$$\underline{\mathbf{D}}^{-1}\underline{\mathbf{A}}\underline{\mathbf{D}} = \underline{\mathbf{\Lambda}}$$

(c) For each of the three cases, write down the corresponding 8×8 matrix $\underline{\mathbf{A}}_n$.

(d) For each case, evaluate the diagonal form $\underline{\mathbf{\Lambda}}_n$ and thus the eight eigenvalues λ_n; you may do this by hand using the methods from class and then just insert them in the appropriate locations in the diagonal matrix, but it is MUCH preferred to use a computer program, e.g., *Matlab*^{TM}**,** *Mathematica*^{TM}, etc., which are available from the RIT computing infrastructure.

(e) For each of the matrices, make graphs of the magnitude and of the phase of the eigenvalues on a plot where the indices k of the eight eigenvalues λ_k are on the horizontal axis and the complex-valued amplitude is plotted along the vertical axis. In other words, the x-axis coordinates are labeled by $k = 1$, $k = 2$, ..., $k = 8$. You may want to make two graphs for each case so that the magnitude and phase are plotted separately.

(f) Determine if each matrix is invertible.