

IMGS-261      Homework #4   Due 2/26/2015 (Th)

1. For  $z_1 = 4 \cdot \exp \left[ +i \cdot \left( -\frac{5\pi}{6} \right) \right]$

- (a) Evaluate the real and imaginary parts of  $z_1$
- (b) Show the location of  $z_1$  on an Argand diagram.
- (c) Evaluate the real and imaginary parts of  $(z_1)^2$  and show the location on the same Argand diagram
- (d) Evaluate the real and imaginary parts of the two distinct values of  $(z_1)^{\frac{1}{2}}$  and show their locations on the same Argand diagram.

2. For each of the following vectors, find an expression for an orthogonal vector of unit length:

(a)  $\begin{bmatrix} +1 + i \\ +1 + i \end{bmatrix}$

(b)  $\begin{bmatrix} -1 + i \\ -2 - i \end{bmatrix}$

(c)  $\begin{bmatrix} -i \\ i \end{bmatrix}$

(d)  $\begin{bmatrix} \exp \left[ -i\frac{\pi}{3} \right] \\ \exp \left[ +i\frac{\pi}{4} \right] \end{bmatrix}$

3. In each case, calculate the “projection” of the vector  $\underline{\mathbf{x}}$  onto the direction of the vector  $\underline{\mathbf{a}}$ , i.e., the length of the “input” vector  $\underline{\mathbf{x}}$  in the direction of the “reference” vector  $\underline{\mathbf{a}}$ .

(a)  $\underline{\mathbf{x}} = \begin{bmatrix} +1 \\ +1 \\ 0 \end{bmatrix}, \underline{\mathbf{a}} = \begin{bmatrix} 0 \\ +1 \\ +1 \end{bmatrix}$

(b)  $\underline{\mathbf{x}} = \begin{bmatrix} +i \\ +i \\ 0 \end{bmatrix}, \underline{\mathbf{a}} = \begin{bmatrix} 0 \\ +i \\ +i \end{bmatrix}$

(c)  $\underline{\mathbf{x}} = \begin{bmatrix} +i \\ +i \\ 0 \end{bmatrix}, \underline{\mathbf{a}} = \begin{bmatrix} 0 \\ -i \\ -i \end{bmatrix}$

(d)  $\underline{\mathbf{x}} = \begin{bmatrix} 1 + i \\ 1 \\ 1 - i \end{bmatrix}, \underline{\mathbf{a}} = \begin{bmatrix} 1 \\ 1 + i \\ 1 - i \end{bmatrix}$

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4. The following  $2 \times 2$  matrix operators represent “imaging systems”

$$\underline{\mathbf{A}}_1 = \begin{bmatrix} -2 & 0 \\ 0 & +\frac{1}{2} \end{bmatrix}$$

$$\underline{\mathbf{A}}_2 = \begin{bmatrix} +1 & +1 \\ -1 & +1 \end{bmatrix}$$

$$\underline{\mathbf{A}}_3 = \begin{bmatrix} +1 & +i \\ -i & +1 \end{bmatrix}$$

$$\underline{\mathbf{A}}_4 = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

For each, do the following:

- (a) Describe in words the action of this system on an arbitrary 2-D input object vector  $\underline{\mathbf{x}}$ .
- (b) Determine whether the system is invertible.
- (c) If the answer to (b) is “yes”, then find the inverse matrix.

5. For the 4-D input vector:

$$\underline{\mathbf{x}} = \begin{bmatrix} \exp \left[ +i \cdot \frac{\pi}{2} \right] \\ 2 \cdot \exp \left[ +i \cdot \frac{3\pi}{4} \right] \\ \sqrt{2} \cdot \exp \left[ +i \cdot \pi \right] \\ \exp \left[ -i \cdot \frac{\pi}{2} \right] \end{bmatrix}$$

- (a) evaluate the projections of this vector onto each of the vectors that oscillate with frequency  $\xi = 0, \frac{1}{4}, \frac{1}{2},$  and  $\frac{3}{4}$  cycles per sample as real and imaginary parts
- (b) convert the projections to magnitude and phase angle.