

IMGS-261 CORRECTED Homework #3 Due 2/19/2015 (Th)

1. Show that the complex conjugate of the sum of two complex numbers is the sum of the complex conjugates:

$$(z_1 + z_2)^* = z_1^* + z_2^*$$

2. In a similar manner to the previous problem

(a) Find the relationship between z_1 , z_2 , and $(z_1 \cdot z_2)^*$

(b) Find the relationship between z_1 , z_2 , and $\left(\frac{z_1}{z_2}\right)^*$

3. Find the set of all complex numbers z_0 that satisfy the condition $z_0^* = z_0^3$.

4. Use the Euler relation to demonstrate the following (HINT: these can be proven together)

(a) $\cos [3\theta] = 4 \cdot \boxed{\cos^3 [\theta]} - 3 \cdot \cos [\theta]$ (*typographical error in original*)

(b) $\sin [3\theta] = 3 \cdot \sin [\theta] - 4 \cdot \sin^3 [\theta]$

5. Use DeMoivre's theorem to calculate all roots of the following equations where $i \equiv \sqrt{-1}$; express them as both real/imaginary parts and as magnitude/phase (this is the most important problem in this set):

(a) $z^2 - 1 = 0$

(b) $z^3 - 1 = 0$

(c) $z^4 = +1$

(d) $z^8 = +1$

(e) $z^2 - 4i = 4$

6. For this function, plot the real part, imaginary part, magnitude, and phase AND the Argand diagram:

$$f_1 [x] = \begin{cases} x \cdot \cos [2\pi x] + i \cdot x \cdot \sin [2\pi x] & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

7. (OPTIONAL BONUS) Prove the following trigonometric identities by expressing the left side in complex form (same HINT as in problem above and you need the formula for the finite geometric series):

(a) $\sum_{n=1}^N \cos [n\theta] = \frac{\cos \left[\frac{N+1}{2} \cdot \theta\right] \cdot \sin \left[\frac{N\theta}{2}\right]}{\sin \left[\frac{\theta}{2}\right]}$

(b) $\sum_{n=1}^N \sin [n\theta] = \frac{\sin \left[\frac{N+1}{2} \cdot \theta\right] \cdot \sin \left[\frac{N\theta}{2}\right]}{\sin \left[\frac{\theta}{2}\right]}$