

0. Read for meaning (i.e., do not just “skim”) Chapters 3 (“Vectors with Real-Valued Components”) and 4.1 - 4.5 in the book (“Complex Numbers” up to, but not including §4.6)

1. For the two unit vectors $\underline{\mathbf{x}}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\underline{\mathbf{x}}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(a) Sketch the vectors

(b) Sketch the vectors after rotation by $-\frac{\pi}{3}$ radians.

(c) Find expressions for the two rotated vectors.

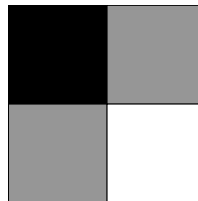
(d) Evaluate the scalare product of the two rotated vectors to demonstrate that they are orthogonal.

2. Consider an arbitrary 2-D real-valued vector $\underline{\mathbf{a}}$ that has the representation $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ in the “normal” basis, i.e.,

$$\underline{\mathbf{a}} = a_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Derive the representation for this same vector in the basis that has been rotated from the “normal” basis by an angle of $-\frac{\pi}{3}$ radians.

3. A 2×2 sampled “object” with real-valued gray values has the form $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$. For example, an image with gray values $a = 0$ (black), $b = c = \frac{1}{2}$ (mid gray), and $d = 1$ (white) would look like:



This object also may be represented as a 4-D vector $\underline{\mathbf{x}}$ by “stacking” the columns:

$$\underline{\mathbf{x}} \equiv \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

(this is the “lexicographically ordered” 1-D vector that represents the 2-D image). We can apply a matrix operator to the this vector that has four columns. Find the matrix operators $\underline{\mathbf{A}}_n$ that produce the following output vectors $\underline{\mathbf{b}}_n$ (“output images”) when applied to $\underline{\mathbf{x}}$:

(a) $\underline{\mathbf{b}}_1$ is vector for the original image after exchanging the rows and columns, so that the 2×2 output image is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

(b) $\underline{\mathbf{b}}_2$ is vector corresponding to the original image rotated by $+90^\circ$

(c) The elements of the output image are the sums of the rows and of the columns of

$$\underline{\mathbf{x}} \text{ so that } \underline{\mathbf{b}}_4 = \begin{bmatrix} a + c \\ b + d \\ a + b \\ c + d \end{bmatrix}.$$

(d) (OPTIONAL BONUS) Find the inverses $\underline{\mathbf{A}}_n^{-1}$ of the matrix operators *if they exist*. If $\underline{\mathbf{A}}_n^{-1}$ does not exist, then explain why.

4. For the distinct complex numbers $z_1 = 1 - 2 \cdot i$ and $z_2 = 3 + i$,

(a) locate the numbers on one Argand diagram of the complex plane (i.e., plot both points on the same diagram)

(b) find expressions as magnitude (“modulus”) and azimuth angle for z_1 and z_2

(c) locate the following points on the same Argand diagram as those in part (a)

1. z_1^*

2. z_2^*

3. z_1^{-1}

4. z_2^{-1}

5. $z_1 + z_2$

6. $z_1 - z_2$

7. $z_1 \cdot z_1^*$

8. $z_1 \cdot z_2^*$

9. $\frac{z_2}{z_1}$