IMGS-261 Homework #10 Due 05/14/2015 (Tu)

In the following expressions, L_0 and Δx are real-valued parameters and the *COMB* function is defined:

$$COMB\left[x\right] \equiv \sum_{n=-\infty}^{+\infty} \delta\left[x-n\right]$$

A COMB function scaled by the width b_0 may be evaluated as:

$$COMB\left[\frac{x}{b_0}\right] = \sum_{n=-\infty}^{+\infty} \delta\left[\frac{x}{b_0}-n\right] = \sum_{n=-\infty}^{+\infty} \delta\left[\frac{x-n\cdot b_0}{b_0}\right] = |b_0|\cdot \sum_{n=-\infty}^{+\infty} \delta\left[x-n\cdot b_0\right]$$

The spectrum of the *COMB* function is:

$$\mathcal{F}_1\left\{COMB\left[x\right]\right\} = COMB\left[\xi\right]$$

Use these definitions to sketch these functions and find expressions for and sketch their Fourier transforms:

1. $$g\left[x\right] = \frac{1}{L_0} COMB\left[\frac{x}{L_0}\right] * RECT\left[\frac{x}{L_0/2}\right]$$
2. $$g\left[x\right] = \frac{1}{L_0} COMB\left[\frac{x}{L_0}\right] * RECT\left[\frac{x}{L_0}\right]$$
3. $$g\left[x\right] = \frac{1}{L_0} COMB\left[\frac{x}{L_0}\right] * TRI\left[\frac{x}{L_0/2}\right]$$
4. $$g\left[x\right] = \frac{1}{L_0} COMB\left[\frac{x}{L_0}\right] * TRI\left[\frac{x}{L_0}\right]$$
5. $$g\left[x\right] = \left(\frac{1}{\Delta x}\cdot COMB\left[\frac{x}{\Delta x}\right]\right)\cdot \cos\left[2\pi\left(\frac{x}{8\cdot\Delta x}\right)\right]$$
6. $$g\left[x\right] = \left(\frac{1}{\Delta x}\cdot COMB\left[\frac{x}{\Delta x}\right]\right)\cdot \cos\left[2\pi\left(\frac{x}{4\cdot\Delta x}\right)\right]$$
7. $$g\left[x\right] = \left(\frac{1}{\Delta x}\cdot COMB\left[\frac{x}{\Delta x}\right]\right)\cdot \cos\left[2\pi\left(\frac{x}{2\cdot\Delta x}\right)\right]$$
8. $$g\left[x\right] = \left(\frac{1}{\Delta x}\cdot COMB\left[\frac{x}{\Delta x}\right]\right)\cdot \cos\left[2\pi\left(\frac{x}{1\cdot\Delta x}\right)\right]$$
9. For numbers 5-8, determine if the cosine function may be recovered by convolution with a SINC-function interpolator $h\left[x\right] = A_0\cdot SINC\left[\frac{x}{b_0}\right]$ and find the allowed ranges of the parameters A_0 and b_0.