IMGS-261-20142 Homework #1 Due 2/5/2015 (Th)

0. Skim Chapter 1, Read Chapter 2 of the book AND read the article on the Fourier transform from Scientific American, June 1989, pp. 86-95 that is posted online at http://www.cis.rit.edu/class/simg320/The_Fourier_Transform_Bracewell_SciAm.pdf

1. An arbitrary function $f[x]$ may be decomposed into the unique even and odd parts $f_e[x]$ and $f_0[x]$, so that $f[x] = f_{\text{even}}[x] + f_{\text{odd}}[x]$. The criteria that are satisfied by the even and odd parts are respectively:

$$f_{\text{even}}[x] = f_{\text{even}}[-x]$$
$$f_{\text{odd}}[x] = -f_{\text{odd}}[-x]$$

where the substitution of $-x$ for x in the argument "reverses" the function (left to right). Use these definitions to derive the expressions for the even and odd parts of the arbitrary function $f[x]$.

2. Derive expressions for the even and odd parts of the sinusoid:

$$f[x] = A_0 + A_1 \cos\left[2\pi\frac{x}{X_0} + \phi_0\right]$$

where A_0 is the constant "bias" of the function, A_1 is the amplitude of the sinusoid, X_0 is the "period" of the sinusoid (the "length per cycle"), and ϕ_0 is the initial phase (measured in radians). Note that we will generally substitute the "spatial frequency" $\xi_0 \equiv (X_0)^{-1}$, which is measured with reciprocal units ("cycles per unit length", e.g., "cycles per mm"), so that the more common expression will be:

$$f[x] = A_0 + A_1 \cos[2\pi\xi_0 x + \phi_0]$$

Sketch/plot the following functions AND evaluate and sketch (or plot) their even and odd parts.

   (a) $f_1[x] = 2 \cdot \cos\left[2\pi\frac{x}{2} + \frac{\pi}{2}\right]$

   (b) $f_2[x] = 2 \cdot \cos\left[2\pi\frac{x}{2} - \frac{\pi}{4}\right]$

3. For a sinusoidal functions whose phase is a power of the coordinate:

$$f[x] = \cos\left[\pi\left(\frac{x}{\alpha_0}\right)^n + \phi_0\right]$$

   (a) Graph the function for $\alpha_0 = 1$, $\phi_0 = 0$, and $n = 1$ and 2. (you may do this by hand or with computer software, but you will need to know how to sketch functions without use of computers)

   (b) What are the "dimensions" ("units") of the parameter α_0?

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4. Plot (or sketch) the following cosine functions:

   (a) $f_0[x] = \cos[2\pi x]$

   (b) $f_1[x] = -\frac{1}{3} \cdot \cos\left[2\pi \cdot \frac{x}{\left(\frac{1}{3}\right)}\right] = -\frac{1}{3} \cdot \cos[2\pi \cdot 3 \cdot x]$

   (c) $f_2[x] = +\frac{1}{5} \cdot \cos\left[2\pi \cdot \frac{x}{\left(\frac{1}{5}\right)}\right] = +\frac{1}{5} \cdot \cos[2\pi \cdot 5 \cdot x]$

   (d) $g[x] = f_0[x] + f_1[x] + f_2[x]$ (this is easiest to do with computer help, but may be done by hand)

   (e) If the sequence of functions is continued, i.e., the n^{th} function in the sequence is:

$$f_n[x] = (-1)^n \frac{1}{2n+1} \cos[2\pi \cdot (2n+1) \cdot x]$$

   what do you expect to be the limiting result of the sum of N such functions where $N \to \infty$?

5. For the function $f[x]$ with the definition:

$$f[x] \equiv \begin{cases} 1 & \text{if} \quad |x| < \frac{1}{2} \\ \frac{1}{2} & \text{if} \quad |x| = \frac{1}{2} \\ 0 & \text{if} \quad |x| > \frac{1}{2} \end{cases}$$

   Sketch the following functions based on this definition along with the even part and odd part:

   (a) $f[x]$ (which shows why this function is called a "rectangle").

   (b) $f[x-1]$

   (c) $f[2x-2] = f\left[\frac{x-1}{\left(\frac{1}{2}\right)}\right]$