

## SIMG-232 LABORATORY #2

Week of 3/15/2004, Writeup Due 3/30/2004 (T)

# 1 Snell's Law, Dispersion, and the Prism

## 1.1 References:

*Physical Optics*, Robert W. Wood, Optical Society, 1988 (reprint of book originally published in 1911)

*Waves*, Frank Crawford, Berkeley Physics Series, McGraw-Hill, 1968.

*The Nature of Light*, V. Ronchi, Harvard University Press, 1970.

*Rainbows, Haloes, and Glories*, Robert Greenler, Cambridge University Press, 1980.

*Sunsets, Twilights, and Evening Skies*, Aden and Marjorie Meinel, Cambridge University Press, 1983.

*Light and Color in the Outdoors*, M.G.J. Minnaert, Springer-Verlag, 1993.

## 1.2 Rationale:

Many optical imaging systems used for scientific purposes (e.g., in remote sensing or astronomy) generate images from light collected in relatively narrow bands of wavelengths. Three common choices exist for "splitting" the light into its constituent wavelengths. The simplest is by inserting "bandpass" filters into the light path that pass light in only narrow selected bands (you will use such filters in part of this lab). The second method uses an optical element formed from a periodic pattern of transparent and opaque regions. This "grating" forces different wavelengths to travel along paths at angles proportional to the wavelength. The third method, which is the subject of this lab, also "disperses" the light, but uses the physical mechanism of differential refraction due to the inherent property that different wavelengths travel at different speeds in glass.

## 1.3 Theory:

From the study of refraction in geometrical optics, we know that light rays incident on an interface between two transparent materials are refracted at an angle determined by the incident angle and by the phase velocities of light in the two materials. The ratio of the phase velocity of light in vacuum to that in a material is the refractive index  $n$ :

$$n = \frac{c}{v}$$

where  $c = 3 \cdot 10^8 \frac{m}{s}$ . The "phase velocity"  $v$  in the medium is the ratio of the angular frequency  $\omega$  and the "wavenumber"  $k$ , which is equal to the product of the wavelength  $\lambda$  and the temporal frequency  $\nu$ :

$$v = \frac{\omega}{k} = \lambda \cdot \nu$$

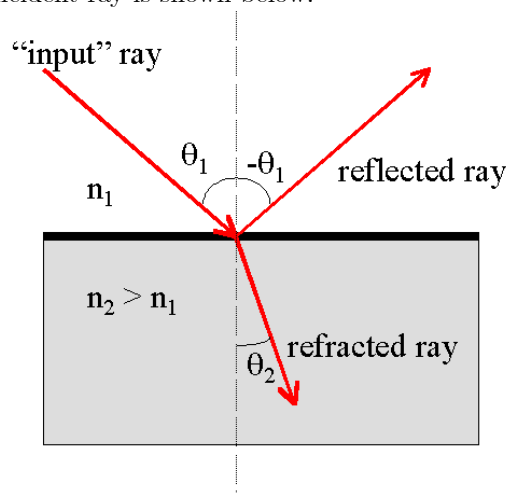
By combining the two equations, we obtain an expression for the refractive index in terms of the vacuum velocity  $c$ , the angular frequency  $\omega$ , and the wavenumber  $k$ .

$$n = c \cdot \left( \frac{k}{\omega} \right)$$

Snell's law relates the indices of refraction to the angles of light rays in two media. If the angles of the incident and refracted rays are  $\theta_1$  and  $\theta_2$ , respectively (measured from the vector normal to the surface), then Snell's law is:

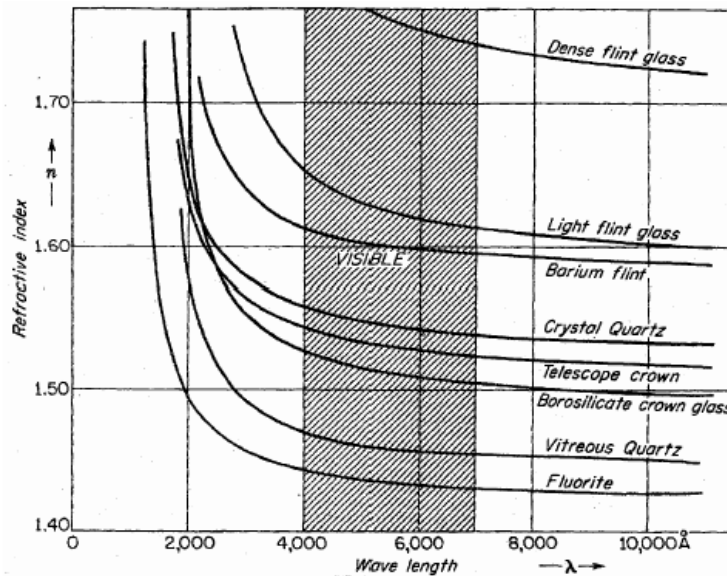
$$n_1 \sin [\theta_1] = n_2 \sin [\theta_2]$$

A plot of the effect on an incident ray is shown below:



*Illustration of Snell's Law.*

Elementary discussions of refraction often assume that the indices of refraction do not vary with the temporal frequency  $\nu$  of the light. For real materials, the situation is more complicated. Obviously, the phase velocity of light waves is slower in a material than in vacuum, because  $n \geq 1$  (except for a few very special cases, such as refraction of x-rays in some materials). But waves with different  $\lambda$  travel at different velocities in media; long waves travel faster than short waves, except over very narrow ranges centered about very specific wavelengths (the resonances), which are beyond the scope of this laboratory. Therefore,  $n$  is smaller for longer waves than for shorter. A plot of the refractive indices of several media vs. wavelength, i.e., a plot of  $n[\lambda]$  for several media, is shown:



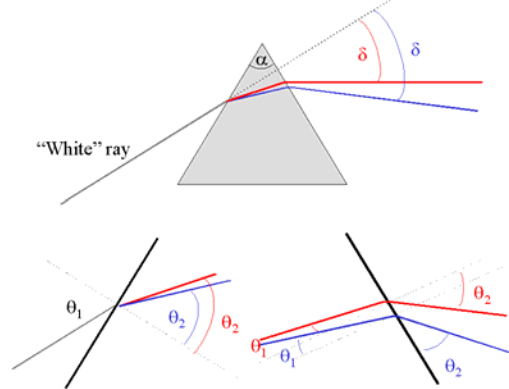
*Refractive Index  $n$  vs. Wavelength  $\lambda$  for several media. Note that  $n$  decreases with increasing  $\lambda$ , which means that the velocity of light in the medium *increases* with increasing  $\lambda$ .*

White light can be dispersed into its constituent frequencies (or equivalently, its wavelength spectrum) by utilizing this differential velocity of light in materials combined with Snell's law. The intensity of the dispersed light can be measured and plotted as a spectrum, which is a valuable tool for determining the chemical constituents of the light source or an intervening medium. In fact, spectral imaging (forming images of objects in narrow wavebands of light) has become a hot topic in imaging because of advances in dispersing materials, computer technologies, and processing

algorithms.

The best-known method for dispersing light is based on differential refraction within a block of glass with flat sides fabricated at an angle (usually  $60^\circ$  but other angles are common). Of course, this was the scheme used by Newton to perform the first spectral “analysis” of white light (spreading it into its constituent colors) in 1671. Note that Newton also “synthesized” white light by recombining the colored components. Newton was the first to investigate the spectral components of white and colored light, and in this way was arguably the first to perform a Fourier analysis. Newton did make a famous error in this study by assuming that the effect of the dispersion was proportional to the refractive index of the glass. In other words, he assumed that the dispersion is a property of the index and does not differ with material.

Consider a prism with apex angle  $\alpha$  shown below. A light ray is incident at angle  $\theta_1$ .

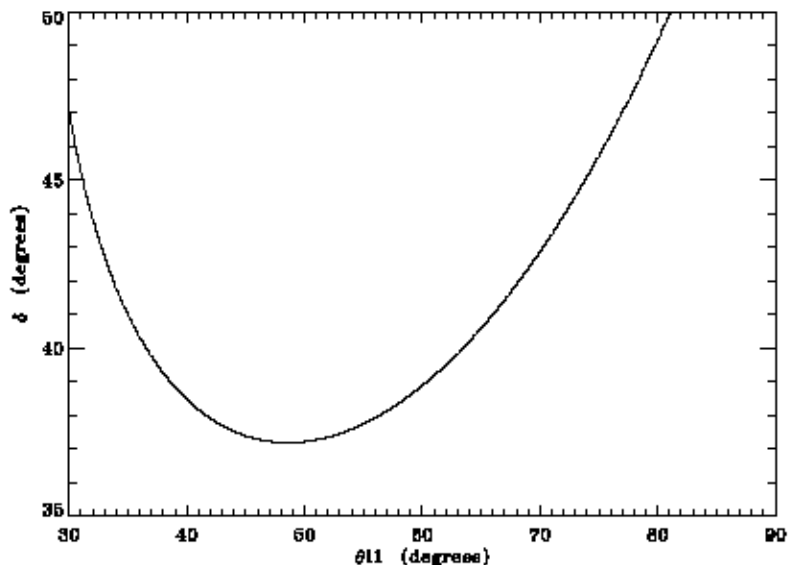


*Schematic of refraction by a prism with apex angle  $\alpha$ . The “deviation” of the incident ray is  $\delta$ , which is shown for both red and blue light. The two diagrams at the bottom are magnified views of the refraction of the rays at the two interfaces.*

We also saw that when one applies Snell’s law to refraction by a prism, geometrical arguments can be used to derive the “total deviation”  $\delta$  of a ray from its original path. The formula is a trifle complicated!

$$\delta = \theta_1 - \alpha + \sin^{-1} \left[ \left( \sqrt{n^2 - \sin^2 [\theta_1]} \right) \cdot \sin [\alpha] - \cos [\alpha] \cdot \sin [\theta_1] \right]$$

where the angle of incidence is  $\theta_1$ , the apex angle of the prism is  $\alpha$ , and the index of refraction is  $n$  for the specific wavelength. This is presented only for its impressive complexity, not because we need to use it. A graph of  $\delta$  vs. incident angle  $\theta_1$  for a fixed index  $n$  and some apex angle  $\alpha$  demonstrates that an angle exists where the deviation of the prism is a minimum, which can be called the “minimum deviation angle” and signified by  $\delta_{\min}$ . The light deviated by this angle passes symmetrically through the prism, i.e., the angle of incidence to the prism and the angle of “departure” are equal. Note that the angle of minimum deviation depends on the refractive index  $n$ , and thus the minimum deviation angle for different wavelengths will be different.



Graph of the deviation angle  $\delta$  vs. angle of incidence  $\theta_1$  for  $n = 1.5$  and apex angle  $\alpha = 60^\circ$ . Note that  $\delta$  exhibits a minimum at some angle  $\theta_{\min} \simeq 48^\circ$ .

For a given prism with apex angle  $\alpha$  and minimum deviation angle  $\delta_{\min}$ , the equation may be rewritten to specify the index of refraction  $n$  in terms of  $\delta_{\min}$  and  $\alpha$ :

$$n = \frac{\sin \left[ \frac{\delta_{\min} + \alpha}{2} \right]}{\sin \left[ \frac{\alpha}{2} \right]}$$

By measuring the minimum deviation angle  $\delta_{\min}$  for different wavelengths  $\lambda$ , you can evaluate  $n[\lambda]$ , *i.e.*, the index of refraction as a function of wavelength. This determines the *dispersion* of the glass used in the prism.

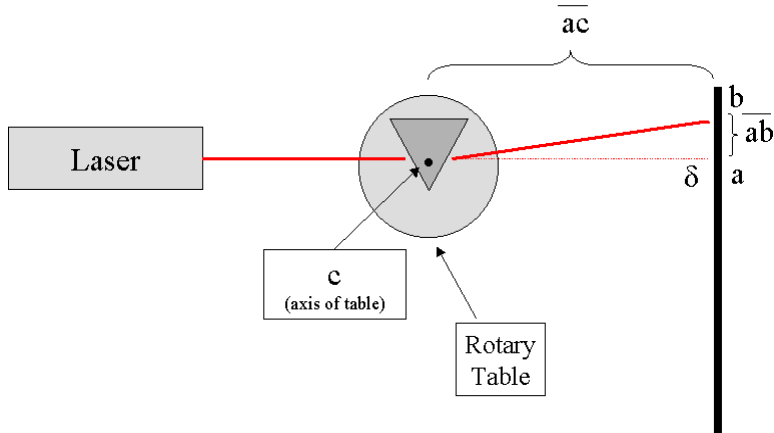
White light also may be dispersed by a different physical means known as *diffraction*. Though the mathematical description of diffraction will be considered later, its properties are introduced in this laboratory.

## 1.4 Procedure:

To do the experiments below, you will need:

1. Glass equiangular prism ( $\alpha = 60^\circ$ )
2. Equiangular (or approximately so) prism made of glass slides that can be filled with water or mineral oil (Use small pieces of the essential tool for all laboratories – duct tape, which is officially sanctioned by the Department of Homeland Security!).
3. Rotary table or angle sheet on which to place the prisms to change the orientation easily
4. He:Ne laser
5. White-light source with optical fiber output
6. Colored filters made of glass or gelatin (“gels”)
7. Aluminum foil (to make slits to reduce the spreading of the light passed into the prism).
8. Diffraction Grating(s)

Set the prism on the rotary table and arrange the laser so you can point the laser beam through the prism easily, as in the figure. DO NOT LOOK DIRECTLY AT THE LASER BEAM. The prism should be able to rotate through a large range in the angle of incidence on the input face. Use a book with a white piece of paper taped to it or a piece of white paper taped to the wall as a screen. Make sure before taking any data that the laser spot remains on your screen as you rotate the prism.



*Apparatus for measuring the minimum deviation of a prism.*

1. Measure the distance from the center of the rotary table to the screen by taking a number of independent measurements (3 to 5). Each member of the group should make independent measurements of the length and the uncertainty without revealing the results to the partners. The measurements should be averaged. What is the standard deviation  $\sigma$  of the result? Use  $\frac{\sigma}{\sqrt{N}}$  as an estimate of the uncertainty in your distance measure (note that the uncertainty is expected to decrease as the number  $N$  of measurements is increased).
2. Mark the position of the laser spot on the screen in the case where no prism is present. This will give you the position of the undeviated beam (position “a” in the drawing).
3. Measure the apex angle of the glass prism supplied. You can draw lines parallel to the faces of the prism, extend these lines with a ruler, and then use a protractor to make the measurement.
4. Place the prism in the center of the rotary table or angle sheet. Find the position where the reflected beam travels directly back to the laser source; this is the location where the face of the prism is perpendicular to the laser beam. Rotate the prism until you see a spot of laser light that has been refracted by the prism. Mark the position of the laser spot for different incident angles for as large a range of angles and of spot positions as you can measure. Take at least 8 data points, recording the value of the incident angle (as judged by the position of the rotary table) and the distance from the undeviated spot position.
5. Convert the distances from the previous step to deviation angles  $\delta [\theta_1]$ . This can be done by noting that:

$$\frac{\overline{ab}}{\overline{ac}} = \tan [\delta]$$

where  $\overline{ab}$  is the distance from  $a$  to  $b$  in figure and  $\overline{ac}$  is the distance from  $a$  to  $c$ , both of which you have measured.

6. Now remove your glass prism and fill the hollow prism made from the microscope slides with the distilled water provided. Repeat steps 1 through 5. Obviously, you cannot mount this prism vertically, so this is a more subjective experiment. But try to estimate the difference in the deviation between the water and glass prisms.

7. Repeat steps 1-5 after filling the microscope-slide prism with a liquid, such as water or mineral oil (or both).
8. Go back to the glass prism and use an optical fiber light source and a slit instead of the laser. Put the fiber source at least a couple of feet away from the prism. Make a slit that is a few millimeters wide and about 1 cm long. Measure the angle of incidence as best you can, and then measure the deviation  $\delta$  for three different colors: red, green (or yellow), and blue. If you have trouble, you may want to consider holding filters from the optics kit in front of the slit to cut out the unwanted colors. In any case, you will probably have to turn the lights off and shield your screen from the ambient light to be able to see the slit on your screen.
9. Now, replace the prism with a diffraction grating and describe the spectrum of the white light. Note particularly the differences compared to the spectrum generated by the prism. Sketch approximate paths of light emerging from the grating, and carefully note the differences in path for red and blue light. If available, compare the spectrum obtained from diffraction gratings with different rulings (spacings).

## 1.5 Analysis

In your lab write-up, be sure to complete the following:

1. For each of the three prisms, graph the deviation angle  $\delta$  as a function of the incident angle on the input face of the prism,  $\theta_1$ . Find the minimum deviation angle,  $\delta_{\min}$  and estimate the uncertainty in this quantity.
2. Using the equation, calculate the index of refraction for the glass, water, and mineral oil used, based on your value for  $\delta_{\min}$  and  $\alpha$  in each case.
3. From your measurements, derive the index of refraction for red ( $\lambda \simeq 700 \text{ nm}$ ), green ( $\lambda \simeq 550 \text{ nm}$ ) and blue ( $\lambda \simeq 450 \text{ nm}$ ) light. Do your data show an decrease in  $n$  with increasing wavelength? The laser is also red ( $\lambda \simeq 632.8 \text{ nm}$ ). How well does your value of  $n$  agree between the laser measurements and the red filter measurement?

## 1.6 Questions:

1. Imagine that a source emits a pulse of light  $1 \mu\text{s}$  long which is composed of equal amplitudes of all wavelengths. The light is incident on a block of glass of thickness 20 km and whose refractive index as shown above.
  - (a) What is the physical length of the pulse [m] when the light enters the glass?
  - (b) What is the time required for red light and for blue light to traverse the glass?
  - (c) What is the physical length of the pulse emerging from the glass.
  - (d) Describe the "color" of the emerging pulse.
2. If the dispersion of the glass is as shown in the plot of refractive index, and if the light is incident on the equiangular prism at at the angle shown, determine and sketch approximate paths followed by red light ( $\lambda = 600 \text{ nm}$ ) and by blue light ( $\lambda = 400 \text{ nm}$ ) and find the approximate difference in emerging angle (the "dispersion angle", often denoted by  $\delta$ ). Sketch the path of the light from incidence through the glass to the emergence. Note carefully which color is deviated more.
3. Sketch the path followed by the light if it is incident from within a block of the same type of glass onto a "prism-shaped" hole.
4. Describe the relative values of the velocity of the average wave and the modulation wave for if the material has the dispersion curve shown. What if the dispersion curve is reversed (*i.e.*, if  $n_{\text{red}} > n_{\text{blue}}$ )?

5. Now that you've finished this lab on dispersion, reconsider the question of the formation process of rainbows. Some answers given at the beginning of the quarter speculated that rainbows were caused by diffracted or by scattered sunlight. Recall (either from experience or research) and sketch the relative locations of the rainbow, the viewer, and the sun. On your sketch, show the path followed by the different colors from the sun to the eye. In other words, explain the sequence of colors in the rainbow.

**BONUS QUESTION:**

6. Find a description of and an explanation for the atmospheric phenomenon known as the "*green flash*", which is occasionally seen at the final instant of sunset from locations with an unobstructed horizon. Explain why the effect is not called a "*blue flash*".