Although similar approaches may be used to investigate the higher order derivatives of the delta function, we shall not do so because of the difficulties involved.

The following additional properties may be "derived" from Eq. (3.38)

\[ \delta^{(k)} \left( \frac{x-x_0}{b} \right) = b^k \delta^{(k)}(x-x_0). \]  

(3.48)

\[ \delta^{(k)}(-x) = (-1)^k \delta^{(k)}(x). \]  

(3.49)

Thus we see that \( \delta^{(k)}(x) \) possesses even symmetry if \( k \) is even and odd symmetry if \( k \) is odd.

In addition we have

\[ \frac{(-1)^k(x-x_0)^k}{k!} \delta^{(k)}(x-x_0) = \delta(x-x_0). \]  

(3.50)

\[ -x \delta^{(1)}(x) = \delta(x), \]  

(3.51)

but care must be exercised in using these results. For example, although Eq. (3.51) is valid, we cannot rearrange it to read \( \delta^{(1)}(x) = -\delta(x)/x \) because the latter expression is not defined in the one-dimensional case.

One more property of interest is that the total area under any of the derivatives is identically zero, i.e.,

\[ \int_{-\infty}^{\infty} \delta^{(k)}(x) dx = 0. \]  

(3.52)

As will be seen later, the delta function derivatives can be used to advantage in evaluating the Fourier transforms of the derivatives of a function.

### 3-4 TWO-DIMENSIONAL FUNCTIONS

In this section we develop the notation for several two-dimensional functions. Such notation will be quite useful in our studies of optical problems.

#### Rectangular Coordinates

Since we will be dealing primarily with separable functions, which can be written as the product of one-dimensional functions, we draw heavily on the notation already established for one-dimensional functions. We specify that \( x_0 \) and \( y_0 \) are real constants and that \( b \) and \( d \) are real, non-zero

constants. In the figures, we shall assume all four of these constants to be positive unless otherwise noted.

#### Rectangle Function

We define the two-dimensional rectangle function to be the product of one-dimensional rectangle functions, i.e.,

\[ \text{rect} \left( \frac{x-x_0}{b}, \frac{y-y_0}{d} \right) = \text{rect} \left( \frac{x-x_0}{b} \right) \text{rect} \left( \frac{y-y_0}{d} \right). \]  

(3.53)

This function, which is illustrated in Fig. 3-24, is often used to describe the transmittance function of a rectangular aperture. It has a "volume" of \( bd \), as may easily be seen from the figure.

#### The Triangle Function

The two-dimensional triangle function is given by

\[ \text{tri} \left( \frac{x-x_0}{b}, \frac{y-y_0}{d} \right) = \text{tri} \left( \frac{x-x_0}{b} \right) \text{tri} \left( \frac{y-y_0}{d} \right). \]  

(3.54)

The graph of this function is shown in Fig. 3-25, where we have chosen \( x_0 = y_0 = 0 \) to simplify the drawing. At first it might seem that the triangle function should be shaped like a pyramid, but as may be seen in the figure, this is not the case. The profile in a direction perpendicular to either axis is always triangular, but the profile along a diagonal is made up of two