Image Registration
Utilizing the LoG Filter & FWT

Industrial Associates

Karl C. Walli
12 May 2003
Major Themes

- Laplacian of Gaussian (LoG) Filter
  - Develop GCPs

- Registering Multisensor Images
  - Develop Transform

- Fast Wavelet Transform
  - Applications
Laplacian Of Gaussian

- What is it?
- The Filter Design
- Use in Image Registration
LoG Filter Unveiled

Knife Edge Input

Gaussian Filter
“Low Pass-Noise”

1st Derivative

2nd Derivative
“High Pass-Edges”

LoG Accentuates Edges like the Human Eye!

LoG Filter
“Mexican Hat”

0  0  -1  0  0
0 -1  -2 -1  0
-1 -2 16 -2 -1
0 -1  -2 -1  0
0  0  -1  0  0

5x5 Log Filter
Laplacian of GAUS (LoG) Technique

“Cross Roads”  
GAUS Smoothing “Low Pass”  
Laplacian Edge Detect “High Pass”

- Utilize Thresholding to determine:
  - Midpoint of “Cross-Roads”
  - End Points of Building/Roads
  - Circular Tanks and Trees
  - ID Areas w/Similar Rate of Variation
Laplacian of GAUS (LoG) Technique

Note blurring of image due to convolution with GAUS (x,y)

Note relative amplification at the ends of buildings (Horns)
Laplacian of GAUS (LoG) Technique

- Determine Threshold
- Correlate Points
- Transform Image

- Threshold Points become Ground Control Points
- Match Related Points in Each Image
- Use these to determine Polynomial Transform
Image Transformation

- Polynomial Transforms
- Affine Geometric Transforms
- Code Implementation
How is it Done?

• Polynomial Coordinate Transform

\[
x = a_0 + a_1 x' + a_2 y' + a_3 x'y' + a_4 x'^2 + a_5 y'^2 \ldots + \varepsilon_x
\]

\[
y = b_0 + b_1 y' + b_2 x' + b_3 y'x' + b_4 y'^2 + b_5 x'^2 \ldots + \varepsilon_x
\]

Affine Transforms

1st Order Poly

2nd Order Poly

Schott (8.4)
3 Affine Geometric Transforms

Shift: \[ x = a_0 + x' \]
\[ y = b_0 + y' \]

Scale: \[ x = a_1 x' \]
\[ y = b_2 y' \]

Rotation
\[ x = \cos(\theta) x' + \sin(\theta) y' \]
\[ y = -\sin(\theta) x' + \cos(\theta) y' \]

\[ f_i = s \begin{bmatrix} \cos(\theta) + \sin(\theta) \\ -\sin(\theta) + \cos(\theta) \end{bmatrix} f'_i + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \]

So, 3 unknowns: We need at least 3 GCP pairs to Register!
Solving the Equation

Psuedo-Inverse Solution to the Linear Least-Squares Problem.

\[
\begin{bmatrix}
x'_1 & y'_1 & 1 \\
x'_2 & y'_2 & 1 \\
x'_3 & y'_3 & 1 \\
x'_4 & y'_4 & 1
\end{bmatrix} = \begin{bmatrix}
x_1 & y_1 & 1 \\
x_2 & y_2 & 1 \\
x_3 & y_3 & 1 \\
x_4 & y_4 & 1
\end{bmatrix} \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\]

\[
A = U^{-1}X
\]

\[
A = (U^TU)^{-1}U^TX
\]

Over-determined problems can give subpixel Registration Results!!!
The Products of Affine transformations is also Affine!
So, several affines can be combined into one single Transform.
Utility of Composite Transform

- Reduces Sampling Degradations
- Concise Mathematical Notation for Changes to Original Image
- Good Approximation for most Transformations (95%)
- Predicting Transforms - Enables the use of Lower Freq Subbands for Registration of Large Images
Predicting Transforms

One Affine Model can give many potential transforms!

Affines demonstrate great Agility in MultiScale Transformation

(Can Register Pan for Accuracy or use MS Band* for Speed and Efficiency ...)
Relating the Images with Point Matching Techniques

- Key to Image Registration!!!
- Shift and Rotation
- Scale
Pixel Distance Match

\[ \begin{bmatrix} 0 & \sqrt{2} & 3 \\ \sqrt{2} & 0 & \sqrt{5} \\ 3 & \sqrt{5} & 0 \end{bmatrix} \]
\[ \begin{bmatrix} 0 & \sqrt{2} & \sqrt{5} \\ \sqrt{2} & 0 & 3 \\ \sqrt{5} & 3 & 0 \end{bmatrix} \]

Distance: 
\[
d = \sqrt{[(x_1 - x_2)^2 + (y_1 - y_2)^2]}
\]

\[
d = \sqrt{[(2 - 1)^2 + (2 - 4)^2]} = \sqrt{5}
\]
Scale Match

Distance:
$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Compare Ratios:
$$\frac{dist_{ref(1 \to 2)}}{dist_{ref(1 \to 3)}} = \left(\frac{\sqrt{2}}{3}\right) \cong \frac{dist_{warp(2 \to 1)}}{dist_{warp(2 \to 3)}} = \left(\frac{2\sqrt{2}}{6}\right)$$

Ratios of distances from like points is equal!

Compare Distance Ratios!
Angle Matching Criteria

\[ \theta[i,j,k] = \cos \left( \frac{a^2 - b^2 - c^2}{-2 \cdot b \cdot c} \right) \]
Additional Matching Criteria

• LoG Maxima Similarity
  - Normalized Extrema (+/- 10%)

• Match Distance Similarity
  - Post Match Check (+/- 1STD)
  - Only non-Rotated Images
RMSDE Matching Criteria

• Use other matches to predict Warp GCP

• Compute RMS Distance Error from actual

• Compute Overall RMSDE to determine Accuracy of Registration (Subpixel)

• Allows Statistical Analysis of Match Set
  – Iteratively strip off 1STD from Mean until Subpixel Accuracy is achieved
  – Works in concert w/ENVI Match Point Analysis
Preliminary Results

- Sample Registration - LANDSAT
- Large Image Manipulation 8k x 8k
- Sharpening using the FWT
Decimated to 1/4 size using FWT
Fast Automation Feature 50pts @ 50%
ROI Tool for Supervised Focus
Affine Transform of Decimated Image
Reference Image
Warp Image  Registered  Predicted
Reference Image
Warp Image  Registered  Predicted
Image “Sharpening” using the Fast Wavelet Transform
Mallat’s FWT Pyramid Representation

<table>
<thead>
<tr>
<th>FWT2 LP/LP</th>
<th>FWT2 HP/LP (Rows/Cols)</th>
<th>FWT1 HP/LP (Rows/Cols)</th>
<th>FWT1 LP/HP (Rows/Cols)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Detail</td>
<td>HP/HP</td>
<td>Diagonal Detail</td>
<td>Horizontal Detail</td>
</tr>
</tbody>
</table>

1 FWT Iterations

Scale in “x” & “y”

Detail in “x” & “y”

Detail in “y”

2 FWT Iterations

Full FWT Decomposition

Original Image
Test Case

• Preprocess Sharpening Image (CITIPIX)
  • Scale to Dyadic Size (640pix to 512pix)

• Decompose (FWT) Sharpening Image to Ref Image Scale
  • 2 FWT iterations (128pix)

• Register Images

• Transfer Detail Planes to Reference Image

• Sharpen with FWT⁻¹
Preprocess Sharpening Image

CITIPIX

- Scale to 512x512
- Histogram Match
- Rotation/Translate

Test case is 4x Sharper than HYMAP (128x128), original is ~20x Sharper.
Decimate Hi-Resolution Image

Two Iterations of the FWT
Switch Image Scale Planes

CITIPIX Scale Plane  HYMAP Image

128x128  128x128
HYMAP w/CITIPIX Detail Planes

Two Iterations of the FWT⁻¹
Original HYMAP Image
Sharpened (2x) HYMAP Image

Grayscale Values ABS( )
Sharpened (4x) HYMAP Image

ABS(Grayscale Values)
Summary

• LoG Technique Promising for Auto GCP Selection
  • Can detect similar locations in Multi-Sensor Images

• Point Matching Scheme useful for Poly Transform
  • Translation, Scale, and Rotation

• FWT shows utility for registration & sharpening
  • Gracefully Decompose Hi-Res Image
  • Preserve Overall Radiometry (on average)