Read Chapters 16-18 in the notes

1. Find the DFTs of the following 4-sample sequences for both sets of array indices: 
   \([0 \leq n \leq +3]\) and \([-2 \leq n \leq +1]\):
   (a) \(f_a[n] = [0, 0, 1, 0]\)
   (b) \(f_b[n] = [0, 0, 0, 1]\)
   (c) \(f_c[n] = [i, 0, 0, 0]\)
   (d) \(f_d[n] = \left[ e^{+i\pi \frac{n}{2}} \right]\)
   (e) \(f_e[n] = \left[ e^{+i\pi \frac{n}{3}} \right]\)
   (f) \(f_f[n] = \left[ e^{+i\pi \frac{n}{4}} \right]\)

2. Sketch the following 1-D discrete functions, evaluate and graph their discrete Fourier transforms over an arbitrary number of samples \(N\). The data arrays are centered at \(n = 0\), and \([-\frac{N}{2} \leq n \leq +\frac{N}{2} - 1]\) where \(N\) may be any number. Explicitly note the normalization you are using.
   (a) \(f_a[n] = 1[n]\)
   (b) \(f_b[n] = RECT \left[ \frac{n}{2} \right]\)
   (c) \(f_c[n] = RECT \left[ \frac{n}{3} \right]\)
   (d) \(f_d[n] = RECT \left[ \frac{n}{4} \right]\)
   (e) \(f_e[n] = RECT \left[ n - 1 \right]\)

3. Write a computer program for a 1-D DFT of a function \(f[n]\) composed of \(N = 256\) complex-valued samples (i.e., \(f = a + ib\)). The indices of data are in the range \(0 \leq n \leq N - 1\), but the function is assumed to be centered about \(n = \frac{N}{2} + 1\). The index of spatial frequency runs over the same range but again the spatial frequency \(\xi = 0\) is located at \(k = \frac{N}{2} - 1\). Submit code for this program in any language (C, BASIC, IDL, Excel, . . . ), but you cannot use “canned” FFT programs (except perhaps to check your result). Use the program to compute the DFTs of the following functions in arrays of size 256 samples. Submit graphical output of the real and imaginary parts.
   (a) Centered rectangle functions of “width” 8 and 16 samples
   (b) Rectangle functions of “width” 8 and 16 samples but centered 4 and 8 samples from the center of the array.
   (c) cosine of period 32
   (d) cosine of period 64
   (e) cosine of period 60
   (f) cosine of period 60 multiplied by a Hanning window