IMGS-616-20141  Homework Assignment #6  Due 10/30/2014 (Th)
(NEXT EXAM will be held Thursday, 11/6)

0. In Chapter 7 (2-D special functions), read §7.2, §7.3.1-§7.3.3, §7.3.5, §7.4 (you may skip the sections
on rotating functions; also read Chapter 10 (2-D transforms), and Chapter 11 (transforms of circular
functions)

1. Evaluate the volumes of and graph 1-D axial profiles (i.e., \( f[x_0] \) and \( f[0, y] \)) of the following functions as “top views”. Also find expressions for and sketch the even and odd parts.
   
   (a) \( f[x, y] = SINC \left[ \frac{x}{2}, y \right] \)
   
   (b) \( g[x, y] = RECT \left[ \frac{x}{2}, \frac{y}{4} \right] - RECT \left[ x, \frac{y}{2} \right] \)
   
   (c) \( p[x, y] = CYL \left( \frac{\sqrt{x^2 + y^2}}{2} \right) - CYL \left( \frac{\sqrt{x^2 + y^2}}{2} \right) \)
   
   (d) \( q[x, y] = [CYL \left( \frac{r}{2} \right) - CYL (r)] \cdot STEP \left[ y \right] \)

2. Find the Fourier transforms of the following 2-D separable functions and sketch them as profiles or as “images”:
   
   (a) \( COR \left[ \frac{x}{2}, 2y \right] \), where “COR” is the “corral” function defined in §7.3.6
   
   (b) \( RECT [x, y] * (\delta [x] \cdot 1 [y]) \)
   
   (c) \( RECT [x, y] * (\delta [x - 1] \cdot 1 [y - 1]) \)
   
   (d) \( RECT [x, y] * CROSS [x, y] \), where \( CROSS [x, y] = \delta [x] \cdot 1 [y] + 1 [x] \cdot \delta [y] \)
   
   (e) \( COR [x, y] * COR [x, y] \)

3. Use the Fourier transforms of \( \exp [\pm i \pi x^2] \) to derive the 2-D transform
   
   \( \mathcal{F}_2 \{ \exp [\pm i \pi (x^2)] \cdot \exp [\pm i \pi (y^2)] \} = \mathcal{F}_2 \{ \exp [\pm i \pi (x^2 + y^2)] \} = \mathcal{F}_2 \{ \exp [\pm i \pi r^2] \} \)

4. Find the results of the convolution and sketch it:
   
   (a) \( CYL (r) * (\delta [x, y + 2] + \delta [x, y - 2]) \)
   
   (b) \( GAUS (r) * \delta [x - 1, y] \)

5. Evaluate the Fourier transforms of the following functions and sketch them:
   
   (a) \( CYL (r) * (\delta [x] \cdot (\delta [y + 2] + \delta [y - 2])) \)
   
   (b) \( GAUS (r) * \delta [x - 1, y] \)
   
   (c) \( J_0 (2\pi r) + J_0 (4\pi r) \)
   
   (d) \( \exp \left[ -i \pi \frac{r^2}{4} \right] + \exp \left[ +i \pi \frac{r^2}{4} \right] \)

6. Find the transfer function of the imaging systems with the following impulse responses:
   
   (a) \( h_a (r) = J_0 (2\pi r) + J_0 (\pi r) \)
   
   (b) \( h_b (r) = SOMB \left( \frac{r}{10} \right) \)
   
   (c) \( h_c (r) = -r^2 GAUS (r) \)

7. Evaluate AND SKETCH the results of the following 2-D operations, where the symbols “∗” and “★” denote 2-D convolution and correlation, respectively:
   
   (a) \( CYL (r) * (\delta [x] \cdot 1 [y]) \)
   
   (b) \( \cos \left[ x\pi \right] \cdot SINC [\eta] + SINC [\xi] \cdot \cos \left[ \eta \pi \right] \) ★ \( \left( \cos \left[ 2\pi \xi \right] \cdot SINC \left( \frac{\eta}{2} \right) + SINC \left( \frac{\xi}{2} \right) \cdot \cos \left[ 2\pi \eta \right] \right) \)
   
   (c) \( J_0 (2\pi \rho_0 r) * J_0 (2\pi \rho_1 r) \), where \( \rho_0 \neq \rho_1 \)