1. Find the Fourier transforms of the following functions and sketch them as BOTH real and imaginary parts AND as magnitude and phase:

(a) \( f[x] = \text{RECT} \left[ \frac{x}{2} \right] + \text{RECT} \left[ \frac{x}{4} \right] \)

(b) \( h[x] = \frac{1}{2} \text{RECT} \left[ \frac{x - 1}{2} \right] + i \cdot \text{RECT} \left[ \frac{x + 1}{2} \right] \)

(c) \( p[x] = \cos \left[ \frac{x^2}{4} \right] \)

(d) \( r[x] = \sin \left[ \frac{x^2}{4} \right] \)

(e) \( u[x] = p[x] + i \cdot r[x] \)

2. Find expressions for (AND SKETCH) the inverse Fourier transforms of the following functions:

(a) \( R[\xi] = \left( \frac{1}{10} \right) \cdot \left( \delta \left[ \frac{\xi}{10} + 1 \right] + \delta \left[ \frac{\xi}{10} - 1 \right] \right) \)

(b) \( G[\xi] = \text{SINC}^2 \left[ \frac{\xi - 1}{2} \right] \)

(c) \( S[\xi] = \text{TRI} [\xi + 1] + \text{TRI} [\xi - 1] \)

3. Evaluate the Fourier transforms of the outputs of the following operations and sketch them as real-and-imaginary parts and as magnitude-phase:

(a) \( \text{RECT}[x] \ast \text{RECT}[x] \)

(b) \( \text{RECT}[x-1] \ast \text{RECT}[x] \)

(c) \( \text{RECT}[x-1] \ast \text{RECT}[x+1] \)

(d) \( \text{RECT}[x-1] \ast \text{RECT}[x+1] \)

4. Find the Fourier transforms of the following functions and sketch both representations:

(a) \( f[x] = \text{COMB}[x] \cdot \text{RECT} \left[ \frac{x}{4} \right] \)

(b) \( g[x] = \left( \text{COMB}[x] \cdot \text{RECT} \left[ \frac{x}{4} \right] \right) \ast \text{RECT}[2x] \)

(c) \( r[x] = \text{COMB}[x] \cdot \text{SINC} \left[ \frac{x}{4} \right] \)

(d) \( s[x] = \text{COMB}[x] \cdot \text{SINC} \left[ \frac{x}{2} \right] \)

(e) \( t[x] = \text{COMB}[x] \cdot \text{SINC}[x] \)

(f) \( u[x] = \text{COMB}[x] \cdot \text{SINC}[2x] \)

5. In HW#4-4, you solved (or tried to) the convolutions of two scaled Gaussian functions and of two SINC functions:

(a) \( \text{GAUS} \left[ \frac{x}{3} \right] \ast \text{GAUS} \left[ \frac{x}{4} \right] = \int_{-\infty}^{\infty} \exp \left[ -\pi \left( \frac{x}{3} \right)^2 \right] \exp \left[ -\pi \left( \frac{x}{4} \right)^2 \right] \, dx \)

(b) \( \text{SINC} [3x] \ast \text{SINC} [2x] = \int_{-\infty}^{\infty} \left( \frac{\sin [3\pi x]}{3\pi x} \right) \left( \frac{\sin [2\pi (x - \alpha)]}{2\pi (x - \alpha)} \right) \, d\alpha \)

Use the filter theorem and known transforms to evaluate these convolutions, which is MUCH easier than direct integration.