1. Let $U$ be a discrete random variable and let $V = aU + b$ where $a$ and $b$ are constants. Show that $E[V] = aE[U] + b$.

2. Let $U$ be a discrete random variable. Show that $|E[U]| \leq E[|U|]$ and specify the conditions that must be true for equality to hold. [Hint: Use the triangle inequality $|\sum_k r_k| \leq \sum_k |r_k|$.

3. Let $V = a_1U_1 + a_2U_2 + \cdots + a_nU_n$. Carry out an analysis similar to that of Example 3.2.2 to find $E[V]$ in terms of the expectations of the $U_k$.

4. Suppose that $U_k$ is a binomial random variable that takes on the value 1 with probability $p$ and the value 0 with probability $(1 - p)$. Let $V = U_1 + U_2 + \cdots + U_n$ be the sum of $n$ such binomial random variables. Show that $E[V] = np$.

5. Let $U_1$ and $U_2$ be statistically independent random variables, and let $V = U_1U_2$. Show that $E[V] = E[U_1]E[U_2]$. Make specific use of the assumption of statistical independence.

6. Let $X$ be a discrete random variable with the Poisson probability distribution

$$P[X = k] = \frac{\mu^k e^{-\mu}}{k!} \quad k = 0, 1, 2, \ldots$$

(a) Show that $\sum_{k=0}^{\infty} P[X = k] = 1$

(b) Show that $E[X] = \mu$

(c) Show that $\text{var}(U) = \mu$. That is, the expected value and the variance of a Poisson distribution are equal.

7. Let $X$ be a normal random variable with the probability density function

$$f_X(x) = \frac{1}{\sqrt{2\pi}s}\exp\left[-\frac{(x-a)^2}{2s^2}\right]$$

(a) Show that $\int_{-\infty}^{\infty} f_X(x)dx = 1$. Hint: Consider a function $f_{XY}(x,y) = f_X(x)f_Y(y)$ with $X$ and $Y$ identically distributed and statistically independent. It turns out to be easier to do the integral $\int \int f_{XY}(x,y)dxdy$ than it is to do $\int f_X(x)dx$ because you can make a change of variable that converts the 2D integral from rectangular to polar coordinates. The integration in polar coordinates is very easy. If you can do this you will find $\int \int f_{XY}(x,y)dxdy = 1$, from which you reach the desired conclusion.

(b) Show that $E[X] = a$, so that writing $\mu$ in the position occupied by $a$ is a sensible thing to do.
(c) Show that \( \text{var}(X) = s^2 \) so that writing \( \sigma \) in the position occupied by \( s \) is a sensible thing to do.

8. Verify the results for the mean and standard deviation of the Rayleigh distribution that are given in Example 3.4.1. Plot the Rayleigh distribution for \( b = 1, 4, 9, 25 \) and observe the changes in the graph.

9. Calculate the characteristic function \( M_X(j\omega) = E[e^{j\omega X}] \) for a random variable \( X \) that has a Poisson distribution

\[
P[X = k] = \frac{\mu^k e^{-\mu}}{k!} \quad k = 0, 1, 2, \ldots
\]

Use the characteristic function to compute the moments \( E[X] \), \( E[X^2] \) and \( E[X^3] \).