1. Light of a single wavelength $\lambda_0$ illuminates two small apertures in an opaque screen (the apertures can be considered to be points) located at $[x, y] = [\pm \frac{d}{2}, 0]$. The light travels down the $z$-axis a distance $L$ where it encounters a screen. The pattern of irradiance on the screen is sinusoidal fringes that vary along the $x$-axis. Determine the period of the fringes (from maximum to maximum) as a function of $L, \lambda_0$, and $d$. From the notes,

$$D_{\text{mod}} = \frac{\lambda}{\sin [\theta]}$$

**but** $\sin [\theta] \approx \frac{d}{L} \implies D_{\text{mod}} \approx \frac{\lambda}{\frac{d}{L}}$

$$\implies d \cdot D_{\text{mod}} \approx L \cdot \lambda$$
2. A particular optical system has an aperture shape that is a rectangle with sides of length $d_1$ and $d_2$, given by:

$$A[x, y] = \begin{cases} 
1 & \text{if } -\frac{d_1}{2} \leq x \leq \frac{d_1}{2} \text{ and } -\frac{d_2}{2} \leq y \leq \frac{d_2}{2} \\
0 & \text{otherwise}
\end{cases}$$

Calculate the irradiance pattern formed by the system imaging a point source at infinity in the Fraunhofer diffraction limit. (Don’t worry about the overall scale or normalization, but please do calculate the Fourier integral explicitly.)

The aperture clearly can be written in the form:

$$A[x, y] = \text{RECT}\left[\frac{x}{d_1}, \frac{y}{d_2}\right]$$

The wavefronts from a point source at infinity are planar, so the amplitude at the aperture is:

$$E[x, y, z = 0] = E_0 \cdot A[x, y] = E_0 \cdot \text{RECT}\left[\frac{x}{d_1}, \frac{y}{d_2}\right]$$

The irradiance in the image plane is proportional to the squared magnitude of the scaled Fourier transform of the aperture function. The steps in the process:

$$\mathcal{F}_2\{E_0 \cdot A[x, y]\} = E_0 \cdot \mathcal{F}_2\left\{\text{RECT}\left[\frac{x}{d_1}, \frac{y}{d_2}\right]\right\} = E_0 \cdot d_1 d_2 \cdot \text{SINC}[d_1 \xi, d_2 \eta]$$

The frequency coordinate $\xi$ is scaled back to the space coordinate $x$ via the proportionality constant:

$$\xi = \frac{x}{\lambda f}$$

where $\lambda$ is the wavelength and $f$ is the focal length of the imaging system, which was not specified. Thus the observed irradiance is:

$$I[x, y] \propto |E_0 \cdot d_1 d_2 \cdot \text{SINC}\left[\left(\frac{d_1}{\lambda f}\right) x, \left(\frac{d_2}{\lambda f}\right) y\right]|^2$$

$$I[x, y] \propto E_0^2 d_1^2 d_2^2 \cdot \text{SINC}^2\left[\left(\frac{d_1}{\lambda f}\right) x, \left(\frac{d_2}{\lambda f}\right) y\right]$$
3. The diameter of a telescope objective is 120 mm and the focal length is 1500 mm. Light with a mean wavelength of $\lambda = 550$ nm from a distant star enters the telescope as a (nearly) collimated beam. Compute the radius of the central disk of light in the image of the star on the focal plane of the lens. Assume no aberrations from the lens or the atmosphere.

(zzzzz) The aperture function is:

$$a(r) = CYL\left(\frac{r}{120 \text{ mm}}\right) \implies A(\rho) = \pi \frac{(120 \text{ mm})^2}{4} \cdot SOMB(120 \text{ mm} \cdot \rho)$$

The first zero of the sombrero function is at the first zero of $J_1(120 \text{ mm} \cdot \rho)$, which occurs at:

$$120 \text{ mm} \cdot \rho_0 \approx 1.22 \implies \rho_0 \approx \frac{1.22}{120 \text{ mm}} = 0.0102 \text{ cycles per mm}$$

In the Fraunhofer diffraction region, the spatial frequency is scaled by the product of the wavelength and focal length:

$$r_0 \approx \rho_0 \cdot \lambda f$$

$$\approx 0.0102 \text{ mm}^{-1} \cdot 550 \text{ nm} \cdot 1500 \text{ mm}$$

$$r_0 \approx 8.39 \mu m = 8.39 \times 10^{-3} \text{ mm}$$
4. Assuming that the eye can resolve the image of an object that subtends one arcminute, determine the distance at which a normal eye can see a black circle of diameter 150 mm on a white background.

(more zzzz) One arcminute can should be scaled to radians:

\[
1' = \frac{1^\circ}{60} = \frac{\pi \text{ radians}}{180^\circ \cdot 60} = 2.91 \times 10^{-4} \text{ radians} = 29.1 \text{ milliradians}
\]

The angle \( \theta \) clearly is small so it is easy to determine the distance at which 150 mm subtends 29.1 mrad.

\[
\frac{150 \text{ mm}}{L} \approx 2.91 \times 10^{-4} \implies L \approx \frac{150 \text{ mm}}{2.91 \times 10^{-4}} \approx 5.15 \times 10^5 \text{ mm} = 515 \text{ m} = L
\]