Homework Assignment - 4

(1) Consider a digitizer that has performed two dimensional spatial sampling on a 12 by 12 grid. The ideal sampling values at the center of each grid is represented by the matrix of numbers shown in Fig. 1. These sample values have to undergo a quantization process with a 5 bit NON-LINEAR quantizer. The tone transfer curve (TTC) of the quantizer is a "square" function, i.e. it takes the input sampled values and squares it before sending it to the quantization step.

The quantizer performs the following operation on each sample value $f$ that has gone through the TTC:

$$f_q = Q\left(\frac{(f - f_{min})(2^m - 1)}{(f_{max} - f_{min})}\right)$$

where $m$ is the number of bits and $Q$ stands for ROUNDING operation. Assume that some one has set the dynamic range by setting $f_{max} = 60$ and $f_{min} = 0$ and that you are unable to change it.

(a) compute the 12 by 12 matrix values that represent the quantized values. If you wish you may write a small program to do this.
12 by 12 matrix of quantized values:

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(b) plot or sketch the histogram of the quantized image. Use at least 15 bin values for your histogram.
(c) Is there evidence for "clipping" here?

Yes, there is evidence of clipping. After the image is quantized all the values above 60 will get cut off.

(d) Plot or sketch the cumulative histogram.
(e) devise and suggest a procedure that could be applied to the quantized image to correct for non-linearity of the TTC.

To correct for the non-linearity of the TTC, one can increase the dynamic range of the quantizer to the maximum value of the image.

(2) Fig. 2 is a gray scale digitized image and Fig. 3 gives the corresponding digital values in each pixel. Note that "dark" represents a values of 15 and "white" represents 0.
(a) How many number of bits are used by the quantizer?
There are 4 bits used by the quantizer.
(b) Sketch the image histogram $H(f_k)$. 

image for problem 2

data set for problems 2 and 3
(c) Confirm that number of pixels = $\sum H(f_k)$. 

(d) Consider the following gray value transformation:
outputgraylevel\(g = 0\) when \(0 \leq f_k \leq 9\)
outputgraylevel\(g = 15\) when \(10 \leq f_k \leq 15\)
Sketch the transformation curve and write down the look-up-table (LUT)
(e) Apply the transformation and draw the transformed image as a matrix of numbers.

(3) Repeat part (d) and (e) of problem 2 for the following transformation:

\[
g = 0 \quad \text{when} \quad 0 \leq f_k < 7
\]
\[
g = \frac{(f_k - 7)15}{15 - f_k} \quad \text{when} \quad 7 \leq f_k < 15
\]
(4) Write down the transformation equation for the following:
\[ g = 0.59f \text{ where } 0 \leq f < 85 \]
\[ g = 1.76f - 99.6 \text{ where } 85 \leq f < 170 \]
\[ g = 0.65f + 89.5 \text{ where } 170 \leq f < 255 \]

(5) Consider the following transformation. Explain the major problem in implementing it.

The major problem in implementing this transformation is the values of \( f \) that are between 50 and 110 will get multiple values in \( g \).