(1) Consider a cosine voltage signal of time that has an amplitude of 10volts and frequency 50Hertz. What is its time period?

\[ \nu_o = \frac{1}{T} \]  where \( T \) is the period and \( \nu_o \) is the frequency: \( T = \frac{1}{\nu_o} = \frac{1}{50} = 0.02 \text{ sec} \).

This signal goes through a half wave rectifier. By definition, a half wave rectifier converts all the negative voltage values to zero but does not change the positive voltage values in the signal. Sketch the waveform at the output. Is this signal periodic? Is it even or odd or neither? What is the period?

As shown above the signal is periodic. This is an even function with a period of 0.02sec.

Sketch the spectrum of \( x(t) \). Include all the numerical values in your sketch and plot frequencies up to 500Hertz.

\[ x(t) = \frac{10}{2} (1 + \frac{2}{3} \cos(\omega_o t) + \frac{3}{5} \cos(2\omega_o t) - \frac{3}{15} \cos(4\omega_o t) + \frac{2}{35} \cos(6\omega_o t) - \frac{2}{63} \cos(8\omega_o t) + \frac{2}{99} \cos(8\omega_o t)) \]
Figure 3: Spectrum of $x(t)$

\[ x(t) = \frac{10}{\pi} + 5 \cos(\omega_0 t) + \frac{20}{3\pi} \cos(2\omega_0 t) - \frac{20}{15\pi} \cos(4\omega_0 t) + \frac{20}{35\pi} \cos(6\omega_0 t) - \frac{20}{63\pi} \cos(8\omega_0 t) + \frac{20}{99\pi} \cos(8\omega_0 t) \]

\[ F\{x(t)\} = \frac{10}{\pi} \delta(\xi) + \frac{2}{\pi}(\delta(\xi + 50) + \delta(\xi - 50)) + \frac{10}{15\pi}(\delta(\xi + 100) + \delta(\xi - 100)) - \frac{10}{15\pi}(\delta(\xi + 200) + \delta(\xi - 200)) + \frac{10}{35\pi}(\delta(\xi + 300) + \delta(\xi - 300)) - \frac{10}{63\pi}(\delta(\xi + 400) + \delta(\xi - 400)) + \frac{10}{99\pi}(\delta(\xi + 500) + \delta(\xi - 500)) \]

As the frequency increases we observe a decreasing trend for the absolute value of the amplitude component.

Suppose we remove all frequencies above 80Hertz from the signal $x(t)$. Write down the expression and draw a rough sketch of what remains.

\[ x(t) = \frac{10}{\pi}(1 + \frac{\pi}{2} \cos(\omega_0 t)) \]
\[ x(t) = \frac{10}{\pi} + 5 \cos(\omega_0 t) \]

\[ F\{x(t)\} = \frac{10}{\pi} \delta(\xi) + \frac{2}{\pi}(\delta(\xi + 50) + \delta(\xi - 50)) \]

(2) Sketch the spectrum of the following on dimensional signal. Distance $x$ is in mm.

\[ f(x) = 3+12 \cos(2\pi x) + 2 \sin(2\pi 15x) + 3 \cos(\pi 18x) + 8 \cos(2\pi 4x) + 10 \sin(2\pi x) + 6 \cos(\pi 12x) + 5 \sin(2\pi 4x) \]

\[ F\{f(x)\} = 3\delta(\xi) + \frac{12}{\pi} (\delta(\xi - 2) + \delta(\xi + 2)) + \frac{2}{\pi} (\delta(\xi - 15) - \delta(\xi + 15)) + \frac{8}{\pi} (\delta(\xi - 9) + \delta(\xi + 9)) + \frac{10}{\pi} (\delta(\xi - 4) + \delta(\xi + 4)) + \frac{6}{\pi} (\delta(\xi - 6) + \delta(\xi + 6)) + \frac{5}{\pi} (\delta(\xi - 4) - \delta(\xi + 4)) \]
Figure 4: Spectrum of $x(t)$ with freq above 80Hz removed

Figure 5: Real part of $f(x)$
As shown in the figures above, the real part of this function is even while the imaginary part is odd.

To synthesize the odd component in the function \( f(x) \) the real part has to be removed. (i.e. get rid of the cosines)

(3) Sketch the two dimensional spectrum of \( f(x, y) \). Provide numerical values for locations and heights of the delta functions.

\[
\begin{align*}
f(x, y) &= 5 + \sum_{y=1}^{3} \left( \frac{1}{y^2} \cos \left[ 2\pi \left( \frac{y}{10} \right) \right] \right) + \sum_{y=1}^{3} \left( \frac{1}{y^2} \cos \left[ 2\pi \left( \frac{y+3}{10} \right) \right] \right) \\
&= 5 + \cos \left( \frac{2\pi y}{10} \right) + \frac{1}{2} \cos \left( \frac{2\pi y}{10} \right) + \frac{1}{2} \cos \left( \frac{2\pi y}{10} \right) + \frac{1}{2} \cos \left( \frac{2\pi y}{10} \right) + \frac{1}{2} \cos \left( \frac{2\pi y}{10} \right)
\end{align*}
\]

\[
\begin{align*}
F \{ f(x, y) \} &= 5\delta(x) + \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right] + \\
&= \frac{1}{2}\left[ \delta(x - \frac{1}{10}) + \delta(x + \frac{1}{10}) \right]\left[ \delta(y - \frac{1}{10}) + \delta(y + \frac{1}{10}) \right]
\end{align*}
\]
Figure 7: Spectrum of $f(x, y)$
The magnitude of the highest spatial frequency present is $\frac{1}{81}$.

(4) a. Would you consider this a band limited signal?

in the case of $x(t)$, this is not a band limited signal. The function $x(t)$ extends from positive to negative infinity.

in the case of $f(x)$, this is a band limited signal. The highest frequency the signal consists is 9 cycles/mm.

in the case of $f(x, y)$, this is also a band limited signal.

b. When you are ready to digitize the signal, what sampling interval will you choose as to avoid aliasing?

in the case of $x(t)$, $\xi_{\text{max}} = 500Hz$ so, $2\Delta x \leq \frac{1}{500}$ and the sampling interval is $\Delta x \leq \frac{1}{1000}$

in the case of $f(x)$, $\xi_{\text{max}} = 15mm$ so, $2\Delta x \leq \frac{1}{15}$ and the sampling interval is $\Delta x \leq \frac{1}{30}$

in the case of $f(x, y)$, $\xi_{\text{max}} = \sqrt{(\frac{\lambda}{\sin \theta})^2 + (\frac{\lambda}{\sin \theta})^2} = \sqrt{\frac{18}{\sin \theta}} = .042$ so, $2\Delta x \leq \frac{1}{1002}$ and the sampling interval is $\Delta x \leq \frac{1}{1004}$.