

Broadband nulling of a vortex phase mask

Grover A. Swartzlander, Jr.

College of Optical Sciences, University of Arizona, Tucson, Arizona 85721

Received May 24, 2005; revised manuscript received June 15, 2005; accepted July 18, 2005

A pulse transmitted through a helical vortex phase mask undergoes a temporal Hilbert transform. The fluence transmitted into the unfavorable plane wave mode is found to increase as the square of the bandwidth and, to first order, is independent of the topological charge. © 2005 Optical Society of America

OCIS codes: 100.5090, 070.6610, 260.2030, 350.1260.

Optical vortices enjoy a wide range of applications for quasi-monochromatic light, including optical spatial filtering,¹ optical tweezers,² quantum cryptography³ and communication,⁴ high-resolution spectroscopy,⁵ and semiconductor patterning.⁶ This broad range of uses is afforded by robust attributes: vortices exist as natural modes in many systems, they exhibit a distinct point of destructive interference in coherent light, and they may be readily produced in the laboratory by various techniques. Additional applications may be possible once the broadband properties of vortices are fully explored. Colored vortices and other optical singularities have been investigated.⁷ Spectral anomalies have been reported,^{8–10} and temporal coherence properties under broadband illumination have been measured.¹¹ Here we explore the broadband transmission properties of a vortex phase mask, giving special attention to the amount of light transmitted into the nonvortex mode. This latter mode limits the use of a vortex phase mask for spatial filtering applications such as the search for extrasolar planets.¹² The topological dispersion¹¹ of the vortex phase mask provides a means to both null the peak of a pulse via a temporal Hilbert transform and significantly attenuate the on-axis fluence.

Optical vortices are modes that include a broad family of separable cylindrical wave functions.¹³ The complex amplitude of the scalar field of such modes may be expressed as a separable function in cylindrical coordinates (r, θ, z) :

$$F_m(r, \theta, z) = A_m(r, z) \exp(im\theta), \quad (1)$$

where m is a mode number called the topological charge. A lossless phase mask is frequently used to transform an initial distribution, $F_{m'}$, into $F_m = t(\theta)F_{m'}$, where the azimuthally dependent transmission function is given by $t(\theta) = \exp[i(m - m')\theta]$. Typically the mask is used to create a vortex of charge m on a beam having $m' = 0$. For example, the initial distribution may represent a uniform plane wave or a Gaussian beam. We thus assume $m' = 0$ and write

$$t(\theta) = \exp(im\theta). \quad (2)$$

A single mask that satisfies Eq. (2) across a broad spectrum does not yet exist. On the other hand, it is simple to design such a mask for a single wavelength or over a small band of wavelengths. For example, a vortex mask may be made by azimuthally varying the thickness of a substrate by an amount

$$d(\theta) = d_{\text{base}} + \Delta d(1 - \theta/2\pi), \quad (3)$$

where $\Delta d = m_0\lambda_0/[n_s(\lambda_0) - n_0(\lambda_0)]$, d_{base} is the minimum thickness, m_0 is the topological charge produced at the design wavelength λ_0 , and the refractive indices of the substrate and the surrounding media at the design wavelength are $n_s(\lambda_0)$ and $n_0(\lambda_0)$, respectively. The effective topological charge generated at any frequency ω is given by

$$m(\omega) = m_0 \frac{\omega}{\omega_0} \left[\frac{n_s(\omega) - n_0(\omega)}{n_s(\omega_0) - n_0(\omega_0)} \right], \quad (4)$$

where $\omega_0 = 2\pi c/\lambda_0$ is the design frequency and ω is the angular frequency associated with the light source. We note that $m(\omega)$ is not necessarily an integer. For most materials the factor in brackets in Eq. (4) differs much less than the ratio ω/ω_0 . For mathematical convenience we therefore assume $n_s(\omega) - n_0(\omega) \approx n_s(\omega_0) - n_0(\omega_0)$, and we write

$$m(\omega) = m_0\omega/\omega_0. \quad (5)$$

If m_0 is an integer, then $m = m_0p$ is also an integer whenever $\omega = p\omega_0$, where $p = 1/m_0, 2/m_0, \dots, 1, 2, 3, \dots$. The frequencies satisfying $\omega = p\omega_0$ will be called principal frequencies.

Although the topological charge varies continuously across the spectrum, it is sometimes advantageous^{11,14} to rewrite the field as a Fourier series of modes having an integer topological charge. For this purpose we write

$$t(\theta, \omega) = \sum_{l=-\infty}^{\infty} C_l(\omega) \exp(il\theta), \quad (6a)$$

$$C_l(\omega) = (2\pi)^{-1} \int_{-\pi}^{\pi} t(\theta, \omega) \exp(-il\theta) d\theta, \quad (6b)$$

where $C_l(\omega)$ shall be called the l th order vortex spectrum. Inserting Eq. (5) into Eq. (2), one may readily show that

$$C_l(\omega) = \text{sinc}(m_0\pi\omega/\omega_0 - l\pi). \quad (7)$$

The vortex spectra, $C_0(\omega)$ and $C_{\pm 1}(\omega)$, are plotted in Fig. 1 for the case $m_0 = 1$. As expected, at $\omega = \omega_0$ only $C_{\pm 1}$ has a nonzero value. We also see that $C_0 = 0$ at the principal frequencies, varying linearly in the vicinity

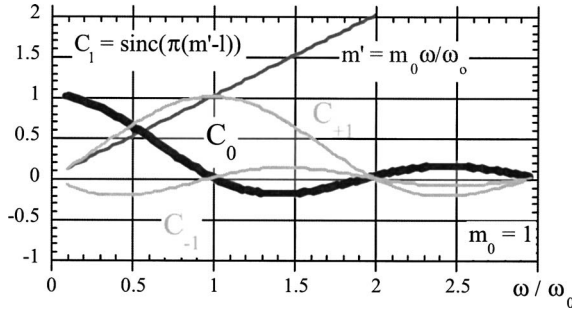


Fig. 1. Vortex spectra for the orders $l=0, +1, -1$ for the case $m_0=1$.

of $\omega-l\omega_0$. The discussion below makes use of this linear variation to null the peak of the transmitted pulse along the optical axis.

To describe the spatiotemporal effects of the vortex mask, we write the scalar input field

$$E_{\text{in}}(r, \theta, z; t) = G(t)F(r, z)\exp(i\Omega t)\exp(-ik_z z), \quad (8)$$

where $G(t)$ describes the temporal profile of the beam, Ω is the carrier frequency, and k_z is the projection of the carrier wave vector along the optical axis. Applying the paraxial approximation, we set $k_z = n\Omega/c$, where n is the refractive index. The temporal pulse and its spectrum are related by the Fourier transform pair:

$$g(\omega) = \int_{-\infty}^{\infty} G(t)\exp(i\Omega t)\exp(-i\omega t)dt, \quad (9a)$$

$$G(t)\exp(i\Omega t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(\omega)\exp(i\omega t)d\omega. \quad (9b)$$

If the maximum thickness of the mask, $d_{\text{base}} + \Delta d$, is much less than the characteristic diffraction length, the field at the output plane of the mask may be written as

$$E_{\text{out}}(r, \theta, z=0; t) = \sum_{l=-\infty}^{\infty} E_l(r, \theta, z=0; t), \quad (10)$$

where we have assigned the plane $z=0$ to the output plane of the mask. The transmitted vortex modes are given by

$$E_l(r, \theta, z=0; t) = G_l(t)F(r, z=0)\exp(il\theta)\exp(i\Omega t), \quad (11)$$

where

$$G_l(t) = (2\pi)^{-1} \exp(-i\Omega t) \int_{-\infty}^{\infty} g(\omega)C_l(\omega)\exp(i\omega t)d\omega, \quad (12a)$$

$$g_l(\omega) = C_l(\omega)g(\omega). \quad (12b)$$

Material dispersion in the thin layer has been ignored in Eq. (12). When the source spectrum is band limited and $C_l(\omega)$ is an odd function over the spectral band, Eq. (12a) may be interpreted as a temporal Hilbert transform. In this case the peak of the pulse becomes zero valued.

The zeroth-order vortex spectrum, $C_0(\omega)$, is of special interest because it is the only mode whose intensity does not vanish along the optical axis. From a spatial filtering point of view, this limits the nulling ability of a vortex mask. Thus we see that the Fourier series decomposition in Eq. (6) provides a means of quantifying the nulling efficiency of a vortex mask without the need to determine the net field. The net field is complicated and may contain vortices in different locations.^{14,15} Below we determine the relative fluence, η , transmitted into this mode. For convenience we assume that the input field is planar, $F(r, z=0)=1$:

$$\begin{aligned} \eta &= \frac{\int_{-\infty}^{\infty} |E_{l=0}(r, \theta, z=0; t)|^2 dt}{\int_{-\infty}^{\infty} |E_{\text{in}}(r, \theta, z=0; t)|^2 dt} = \frac{\int_{-\infty}^{\infty} |G_{l=0}(t)|^2 dt}{\int_{-\infty}^{\infty} |G(t)|^2 dt} \\ &= \frac{\int_{-\infty}^{\infty} |g_{l=0}(\omega)|^2 d\omega}{\int_{-\infty}^{\infty} |g(\omega)|^2 d\omega}. \end{aligned} \quad (13)$$

Parseval's theorem has been invoked to obtain the right-hand side of Eq. (13).

The following analytical and numerical calculations establish important characteristics of Eq. (13). Analytical results are made possible by assuming a uniform band-limited spectrum. Let $g(\omega) = (2\Delta\omega)^{-1/2}$ for $\Omega - \Delta\omega \leq \omega \leq \Omega + \Delta\omega$ and $g(\omega) = 0$ otherwise, such that $\int_{-\infty}^{\infty} |g(\omega)|^2 d\omega = 1$. An evaluation of η is considerably simplified by assuming that $\Omega = \omega_0$, integrating over the variable $\omega'' = \omega/\Omega - 1$, and expanding the integrand to $O(\omega''^3)$, assuming that $\omega'' \ll 1$ and m_0 is an integer:

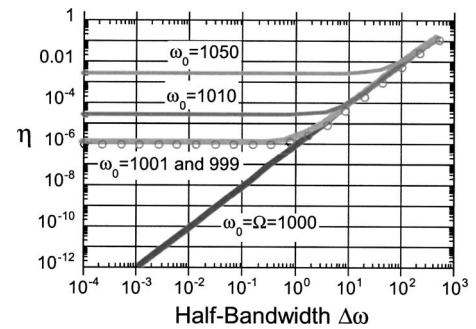


Fig. 2. Transmitted fluence (joules per square centimeter) of the plane-wave mode through a vortex mask (design parameters, $m_0=1$ and ω_0), normalized to the net fluence of the input pulse having a uniform band-limited frequency distribution of half-width $\Delta\omega$ and center Ω .

$$\eta = \frac{1}{2\Delta\omega} \int_{\Omega-\Delta\omega}^{\Omega+\Delta\omega} \text{sinc}^2(m_0\pi\omega/\Omega) d\omega$$

$$= \left(\frac{1}{2(m_0\pi)^2\Delta\omega} \right) \int_{-\Delta\omega/\Omega}^{\Delta\omega/\Omega} \frac{\sin^2(m_0\pi\omega'')}{(\omega''+1)^2} d\omega'' \approx \frac{1}{3} \left(\frac{\Delta\omega}{\Omega} \right)^2. \quad (14)$$

Numerical integration of Eq. (13), whose values are plotted in Fig. 2, reveals that the right-hand side of relation (14) is an excellent approximation. Furthermore this result is seen in Fig. 2 to be valid in the broad bandwidth regime when $\Omega \neq \omega_0$. On the other hand, when the bandwidth is small, direct integration shows that $\eta_{\Delta\omega/\Omega \rightarrow 0} = C_0^2(\omega_0) = \text{sinc}^2(m_0\pi)$.

An ideal vortex-nulling filter requires that $\eta \ll 1$ over a broad bandwidth. In the small-bandwidth regime design parameter errors (ω_0 or m_0) significantly affect the nulling efficiency. An extinction of more than 8 orders of magnitude is shown in Fig. 2 when $m_0=1$, $\Delta\omega=0.1$, and ω_0 is selected to coincide with the center frequency of the source, Ω . Errors in ω_0/Ω as small as 0.1% limit the extinction to, at best, 6 orders of magnitude. The limited performance of the mask is attributed to the linear frequency dependence of the effective topological charge shown in Eq. (4). In principle an ideal frequency-independent vortex mask may be produced by using a high-dispersion material.

The author is grateful to Greg Foo (University of Arizona) and David Palacios (Jet Propulsion Laboratory) for their comments. This work was supported by the U.S. Army Research Office and the State of Arizona. G. A. Swartzlander's e-mail address is grovers@optics.arizona.edu.

References

1. S. N. Khonina, V. V. Kotlyar, M. V. Shinkaryev, V. A. Soifer, and G. V. Uspleniev, *J. Mod. Opt.* **39**, 1147 (1992).
2. A. Ashkin, *Biophys. J.* **61**, 569 (1992).
3. H. Sasada and M. Okamoto, *Phys. Rev. A* **68**, 012323 (2003).
4. J. Leach, J. Courtial, K. Skeldon, S. M. Barnett, S. Franke-Arnold, and M. J. Padgett, *Phys. Rev. Lett.* **92**, 013601 (2004).
5. V. Westphal and S. W. Hell, *Phys. Rev. Lett.* **94**, 143903 (2005).
6. M. D. Levenson, T. Ebihara, G. Dai, Y. Morikawa, N. Hayashi, and S. M. Tan, *J. Microlithogr., Microfabr., Microsyst.* **3**, 293 (2004).
7. M. V. Berry and S. Klein, *Proc. Natl. Acad. Sci. U.S.A.* **93**, 2614 (1996).
8. G. Gbur, T. D. Visser, and E. Wolf, *Phys. Rev. Lett.* **88**, 013901 (2002).
9. G. Popescu and A. Dogariu, *Phys. Rev. Lett.* **88**, 183902 (2002).
10. J. T. Foley and E. Wolf, *J. Opt. Soc. Am. A* **19**, 2510 (2002).
11. G. A. Swartzlander, Jr., and J. Schmit, *Phys. Rev. Lett.* **93**, 093901 (2004).
12. G. A. Swartzlander, Jr., *Opt. Lett.* **26**, 497 (2001).
13. For a review of singular optics, see M. Vasnetsov and K. Staliunas, eds., *Optical Vortices*, Vol. 228 of *Horizons in World Physics* (Nova Science, 1999), and G. A. Swartzlander, Jr., "Singular optics/optical vortex references," <http://www.u.arizona.edu/~grovers/SO/so.html>.
14. M. V. Berry, *J. Opt. A, Pure Appl. Opt.* **6**, 259 (2004).
15. D. Palacios, "An optical vortex coherence filter," PhD. dissertation (Worcester Polytechnic Institute, June 2004).