Direct measurement of the transverse velocity of dark spatial solitons

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We describe the direct experimental measurement of the transverse propagation velocities of dark spatial solitons. Good agreement is obtained from a comparison of the velocities measured experimentally and the velocities predicted by the two-dimensional theory of Zakharov and Shabat [Sov. Phys. JETP **37**, 823 (1973)].

Dark-soliton solutions to the two-dimensional nonlinear Schrödinger equation with self-defocusing nonlinearity have been known since the work of Zakharov and Shabat[1] (Z&S). These solutions are characterized by nonzero boundary conditions as the transverse coordinate (i.e., beam size for spatial or pulse width for temporal solitons) goes to $\pm\infty$ and by a minimum in the field amplitude at the center of the dark excitation. Experimental investigation of such solutions has been hampered by the apparent need for extremely large beam powers in order to form the proper constant background on which the dark solitons may propagate. Theoretically, infinite power would be required for the boundary condition to be satisfied. However, recently two independent sets of experiments on dark temporal solitons[2,3] in silica-glass fibers, and some numerical work,[4] demonstrate that it is possible to have (temporal) two-dimensional dark pulses propagate on a finite-width bright background. Although these dark pulses are not solitons in the strict definition of the word (owing to slow changes of the dark-pulse width and amplitude as the bright pulse propagates[4]), the propagation is clearly soliton-like, i.e., the solitary waves have the characteristic field profile of solitons, they are stable over long propagation distances and under collisions, and they obey certain conservation laws. Thus previous authors have labeled these pulses dark solitons, and, to maintain consistency, we adopt that nomenclature in this Letter.

Until recently, work on spatial dark solitons had mostly been overlooked, presumably because three-dimensional (two transverse and one longitudinal) systems were not expected to produce stable solitons. For example, it is well known that bright excitations are not stable in three-dimensional Kerr media.[5] However, our earlier investigations[6,7] demonstrated that dark solitons can exist as stripes in the cross section of a cylindrically symmetric laser beam. What is more, a grid of dark solitons was also observed, suggesting that neither the higher dimensionality of the system nor the beam broadening destabilizes the solitons. These experiments were conducted by placing either a single wire or a wire mesh at the input face of a nonlinear medium with $n_2 < 0$ (e.g., sodium vapor at $\lambda_0 \geq 589.0$ nm and various liquids exhibiting large thermal nonlinearities). The solitons were photographically recorded at the output face of the nonlinear medium and also in the far field, where spectacular transformations of the intensity profiles were observed.

It is the purpose of this Letter to report further results for the dark-spatial-soliton system discussed above.[6,7] Specifically, we report good agreement between the transverse velocities of dark solitons, generated experimentally by partially occluding a cw optical beam with a single wire of varying thickness before the beam enters a defocusing medium, and the theoretical predictions of those transverse velocities.[1]

The experimental arrangement consisted of a 0.6328-μm laser beam of approximately 3.5-mW cw power incident upon a cell containing a 200-mm path length of gasoline dyed slightly green in order to enhance its defocusing thermal nonlinear refractive-index properties in the red region of the optical spectrum. This type of nonlinear-optical material has been studied for several years.[8] The beam was nominally collimated at the entrance face of the cell with a spot size of 1 mm. Metallic wires ranging from a minimum diameter of 25 μm to a maximum diameter of 757 μm were positioned at the center of the beam near the entrance face of the sample cell in order to create an appropriate initial condition for the generation of dark spatial solitons. The near-field intensity profile of the beam at the output face of the cell was then imaged onto a screen.

A simplified schematic diagram of the experiment is shown in Fig. 1(a). The wire occludes a region of the

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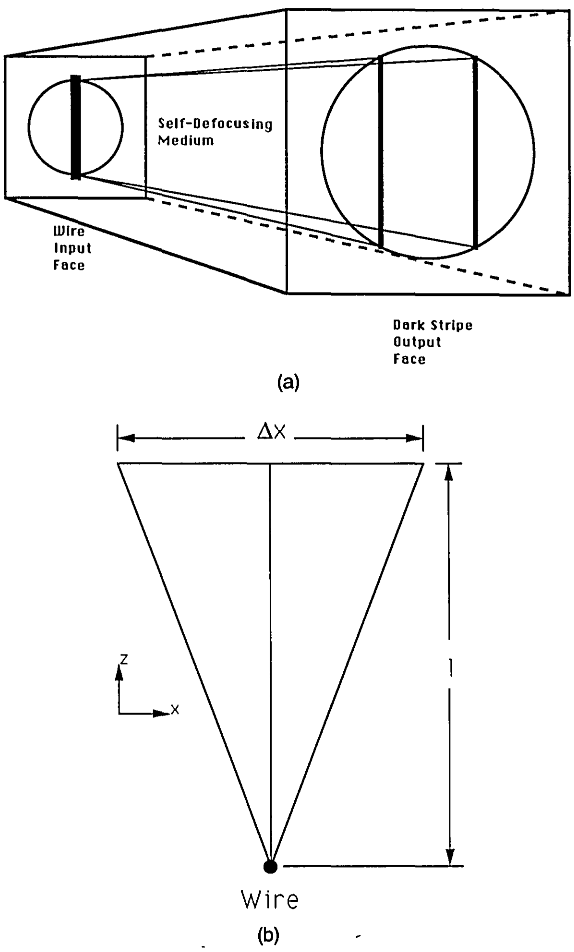

Fig. 1. Schematic of the experimental arrangement. The incident optical beam is partially occluded by a wire and propagates through a nonlinear material of length 200 mm. The dark, even initial condition generated by the wire breaks up into one or more transversely counterpropagating dark soliton pairs. (a) Illustrates this process, and (b) indicates the definition of certain experimentally measured variables.

incident beam, resulting in the dark, even initial condition first investigated by Z&S. The transverse velocity of the nth soliton pair generated from the dark, even initial condition is obtained from the spacing (Δx_n) between the solitons that compose the nth pair, as is shown in Fig. 1(b). The spacing between the two solitons is related to the soliton eigenvalue (λ_n) (Ref. 1) as follows:

$$\lambda_n = \frac{\Delta x_n}{2l}\left(\frac{2n_0}{|n_2|E_0{}^2}\right)^{1/2}, \tag{1}$$

where for our experimental arrangement $n_2E_0{}^2/2 = -2.84 \times 10^{-5}$ is taken as the nonlinear refractive-index change at the center of the Gaussian beam (in good agreement with the value measured by counting the far-field interference fringes resulting from the defocusing of the unoccluded beam), Δx_n is the spacing between the two solitons that compose the nth pair, and l = 200 mm is the optical path length in the sample. In order to compare the experimental results with the available theory, we also normalize the wire width to

$$a = \left(\frac{|n_2|E_0{}^2}{2n_0}\right)^{1/2} n_0k_0\frac{x_w}{2}, \tag{2}$$

where x_w is the unnormalized wire diameter and $k_0 = 2\pi/0.6328\ \mu$m is the vacuum wave number. These two parameters can then be readily compared with the analytical results predicted for the two-dimensional infinite-beam case[1]:

$$\lambda_n = \cos(2\lambda_n a). \tag{3}$$

Experimental results are shown in Fig. 2. Figure 2(a) shows a photographic image of a typical experimental result. In this case, a wire of width 324 μm occluded the beam, resulting in the transmission of four pairs of dark solitons. It is interesting to note here that the three-dimensional perturbation associated with the Gaussian tail of the background bright beam in the transverse direction parallel to the dark stripes manifests itself by a broadening of the dark soliton width near the tail. This broadening is a result of the local conservation of the soliton constant $E_B{}^2\kappa_s{}^2$, where $E_B{}^2$ is the local background intensity and $\kappa_s{}^2$ is the square of the soliton half-width. In Fig. 2(b) the theoretical[1] dispersion curves for the three innermost

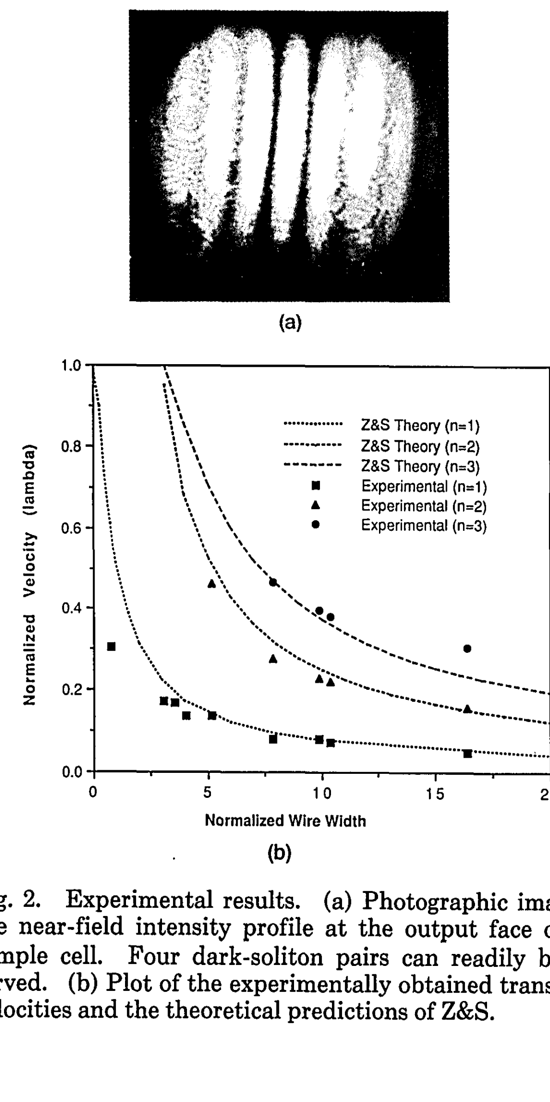

Fig. 2. Experimental results. (a) Photographic image of the near-field intensity profile at the output face of the sample cell. Four dark-soliton pairs can readily be observed. (b) Plot of the experimentally obtained transverse velocities and the theoretical predictions of Z&S.

soliton pairs are plotted along with the experimental data. As can be seen, the agreement between experiment and theory is good, especially above $\sim a = 2.5$. Similar agreement was obtained for higher-order pairs but is not shown in the interest of clarity of presentation.

The experimental results plotted in Fig. 2(b) indicate that Eq. (3) is a good model for the transverse propagation velocity of dark spatial solitons in liquids exhibiting strong thermal nonlinearities. However, this model does not incorporate effects related to the spreading of the bright background due to defocusing or diffraction. Such spreading will result in an interaction between the dark solitons and the edge of the bright background beam. In order to investigate this interaction of the dark solitons with the edge of the finite-width bright background optical beam, we may define a soliton-propagation condition as follows:

$$\Theta_{\text{soliton}} < \frac{1}{n_0 k_0 \omega_0} + \left(\frac{|n_2|E_0{}^2}{2n_0}\right)^{1/2}, \qquad (4)$$

where $\Theta_{\text{soliton}} \cong \Delta x_n/2l = \lambda_n(|n_2|E_0{}^2/2n_0)^{1/2}$ is the angular spread between the optical axis of the bright background beam and the direction of the soliton propagation and ω_0 is the beam waist. This condition implies that, in order for soliton propagation to occur, the propagation angle Θ_{soliton} must be smaller than the sum of the linear diffraction angle $1/n_0k_0\omega_0$ plus the nonlinear defocusing angle $(|n_2|E_0{}^2/2n_0)^{1/2}$. A condition is thus obtained for λ_n:

$$\lambda_n < 1 + \left(\frac{2n_0}{|n_2|E_0{}^2}\right)^{1/2} \frac{1}{n_0 k_0 \omega_0}. \qquad (5)$$

This condition is always satisfied by virtue of Eq. (3).

The result given in inequality (5) implies that dark solitons generated by a dark band, such as is the case for the current experiment, will not have propagation angles larger than the sum of the linear diffraction and nonlinear defocusing angles of the bright background beam. Thus the interaction between a dark soliton and the bright-beam edge should be reduced such that the soliton will not walk off the edge of the bright beam. However, a rather strong interaction may still take place, depending on the geometry of the problem.

Several additional observations may be made concerning the experiment. It is possible for an observer positioned to the side of the experiment in the direction parallel to the orientation of the wire to observe the breakup of a single, dark initial condition into one or more dark soliton pairs. This is a rather striking effect, although it is difficult to photograph or otherwise to record. A second observation concerns the role played by the nonlocality inherent in thermal nonlinearities such as the one used in this experiment. For the results discussed above, sufficiently low power levels were used that manifestations of nonlocality such as severe distortion of the far-field pattern owing to convection currents were not observed. However, following the collection of those data, the input power was increased until the nonlocality began to play a prominent role. Despite these conditions, the dark solitary waves were still manifest, leading us to conclude that such waves are apparently robust even in the presence of strongly nonlocal effects. Finally, although the nonlinear medium was not strongly absorbing, the long path length used (200 mm) was a reasonable fraction of an absorption length. The beam attenuation did not appear to affect the stability of the dark solitons. Similar observations have been made regarding bright solitons in optical fibers.

The results presented in this Letter suggest several applications for dark spatial solitons. For example, the collision properties of dark solitons are such that optical logic gates may be constructed by using appropriately placed detectors to observe the spatial shifts undergone by the dark solitons during the collision interaction. Further, dark spatial solitons may be used in certain types of nonlinear waveguiding application.

In conclusion, we have reported the direct measurement of the transverse velocities of dark, spatial solitary waves existing upon a finite, initially Gaussian bright background. Good agreement was obtained between these experimental results and a two-dimensional theory developed by Zakharov and Shabat. Applications of this phenomenon are numerous and include optical communications and optical switching.

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