Peering into darkness with a vortex spatial filter

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Received August 23, 2000

I propose to use as a window the dark core of an optical vortex to examine a weak background signal hidden in the glare of a bright coherent source. Applications such as the detection of an astronomical object, forward-scattered radiation, and incoherent light are described whereby signal enhancements of at least 7 orders of magnitude may be achieved.

OCIS codes: 100.5090, 050.1970, 120.1880, 350.1260, 290.0290.

It is sometimes desirable to attenuate the intense glare of a bright coherent beam of light to enhance the detection of an incoherent background signal or a weak nearly collinear source. For example, forward-scattered radiation occurs in many optical systems, but its detection is often difficult. Sources adjacent to bright bodies are often obscured in dazzling starlight. An optical vortex phase mask provides a means with which one can discriminate between two sources by forming a dark core within bright beams. In effect this core opens a window, allowing one to examine signals from other sources of light. By using this scheme one is able to enhance the discrimination between two signals by orders of magnitude.

Vortices, or phase defects, often occur in coherent radiation, e.g., in cylindrically guided waves.[1] Laguerre–Gaussian laser beams,[2] scattered light,[3,4] optical caustics,[5,6] and optical vortex solitons.[7,8] In cylindrical coordinates a field propagating in the z direction and containing a single vortex has a simple form:

$$E(r, \phi, z; t) = A(r, z)\exp(im\phi)\exp[i(\omega t - kz)], \quad (1)$$

where A is a circularly symmetric amplitude function, ω and $k = 2\pi/\lambda$ are, respectively, the frequency and the wave number of the monochromatic field of wavelength λ, and m is the topological charge (a signed nonzero integer). Total destructive interference occurs at $r = 0$, and thus $A(r = 0, z) = 0$ for all values of z.[9]

In the spatial filtering scheme proposed in this Letter, a transmission phase mask is used to produce an optical vortex. One means of creating the mask is to etch[10] a substrate material with refractive index n_0 to depth $\Delta d = [(m\lambda\phi)/2\pi(n_0 - 1)]$. The mask will resemble a single revolution about a spiral staircase, and the transmission of a planar wave front will experience an azimuthally varying retardation with a maximum phase difference that corresponds to an integer number of wavelengths. Although the transmitted wave front will have a step discontinuity at $\phi = 0$, the optical field will be everywhere harmonic and continuous, except at the origin, where the intensity must vanish.

An analytical expression for amplitude A in Eq. (1) may be found from the wave equation and boundary conditions. First we consider the case when a single light source is many focal lengths from a lens and the vortex mask is placed near a lens of focal length f and diameter D (see Fig. 1). Owing to the cylindrical symmetry of the optical system, the intensity in the focal plane (fp) of the lens may be described by use of Bessel functions.[11,12] Analytic solutions to such problems are difficult to interpret, and it is more convenient to compute the beam profiles numerically.[13] Radial plots of the amplitude profiles in the focal plane are shown in Fig. 2 for $|m| = 0-4$. We can see that both the beam and the core sizes increase with increasing values of $|m|$. Moreover, the region in the vicinity of the core (where $|A|$ varies as $r^{|m|}$) becomes darker as $|m|$ increases. The case $m = 0$ (the Airy disk) with a zero at $r = R_{\text{diff}} = 1.22\lambda f/D$ is also graphed in Fig. 2 to demonstrate the relative shifts of the amplitude peaks: $R_{|m|=1} = 0.64R_{\text{diff}}$, $R_{|m|=2} = 1.03R_{\text{diff}}$, $R_{|m|=3} = 1.37R_{\text{diff}}$, and $R_{|m|=4} = 1.71R_{\text{diff}}$.

Let us now address what is seen within the vortex core. If there are no other light sources we simply see the darkened core of the original beam. However, if other light sources exist, they may be detected within the core. In this fashion the vortex opens a window, allowing us to see a background field without the glare of the original field. Interferometric nulling is another technique for detecting a weak off-axis source by canceling the field.[14,15] The effective aperture of this window depends on both the topological charge of the mask and the relative intensity of the two fields. From a detection point of view, good discrimination of the signals through the aperture will be achieved if the transmitted power of the original source is less than of the second source.

First we determine the amount of power from the original beam that is transmitted through a circular aperture in the focal plane. The fraction of transmitted power is shown in Fig. 3 for $m = 0-4$. As expected, the transmission through an aperture of any given size is reduced as the value of m increases. For example, when the aperture diameter is one quarter of the diameter of the Airy disk, $R_{\text{ap}}/R_{\text{diff}} = 0.25$, the throughput power is reduced by a factor of

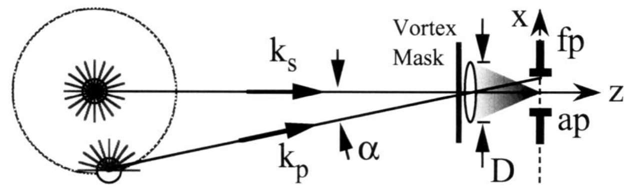

Fig. 1. Direct rays from a star and reflected rays from a planet make an angle α at the objective of a telescope with focal length f and diameter D.

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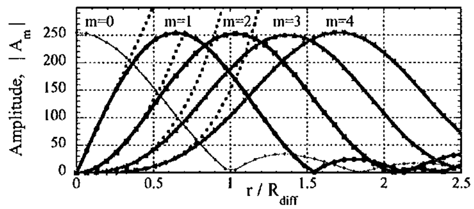

Fig. 2. Line plots of the field amplitude through the center of vortex cores for values of topological charge, $m = 1-4$. Solid curves are drawn to aid the eye. $m = 0$ corresponds to the Airy disk, which has a first minimum at $r = R_{\text{diff}}$. Dotted curves, least-squares fits to a power-law curve proportional to $r^{|m|}$.

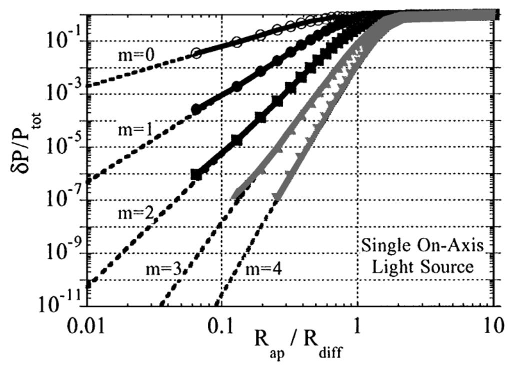

Fig. 3. Fraction of the total beam power transmitted through an aperture of radial size R_{ap} located in the focal plane and centered on the vortex core. Dashed curves, predictions from a least-squares fit analysis for several values of the topological charge, m.

$\delta P/P_{\text{tot}} = 0.02$ for $m = 1$ and of $\delta P/P_{\text{tot}} = 10^{-7}$ when $m = 4$. These are small values, even in comparison to the amount of light (~26%) through the aperture when no vortex is present ($m = 0$). In theory, one may achieve an arbitrary degree of rejection of the original beam by reducing the aperture size or increasing the topological charge of the phase mask. A least-squares fit suggests that near the core, where $R_{\text{ap}} \ll R_{\text{diff}}$, the fraction of power decreases as $(R_{\text{ap}}/R_{\text{diff}})^{1.44+1.82m}$. Thus, if the throughput power ratio is known for some aperture size, it may be estimated for other sizes:

$$(\delta P/P_{\text{tot}})_2 \approx (\delta P/P_{\text{tot}})_1 (R_{\text{ap},2}/R_{\text{ap},1})^{1.44+1.82m}. \quad (2)$$

Next we consider how the second source field is affected by the vortex mask. If, on the one hand, this field is incoherent, the mask will have little effect on the focused beam profile, and the net power through the aperture will be the sum of the individual powers. In this case the signal detected from the background field is simply the integrated intensity across the aperture. Assuming that the aperture is centered on the optical axis, as depicted in Fig. 1, the background signal will be a maximum when the source also lies along the optical axis ($\alpha = 0$). Using an $|m| = 4$ vortex mask and a pinhole that is one fourth of the diameter of the Airy disk may reduce the glare of the primary source by a factor of 10^7, as indicated in Fig. 3.

However, if the background light is transversely coherent, the mask will also produce a vortex in the corresponding focal spot of the background source. In addition, if the background is mutually coherent with the source, the two fields may interfere. For the sake of discussion we shall refer to the original beam as starlight and to the background beam as light reflected from an orbiting planet. We assume that the two fields radiate from distance monochromatic point sources and that they subtend an angle $\alpha \ll 1$ in the xz plane at a lens of focal length f and diameter D, as shown in Fig. 1. The field at the entrance pupil may be written as

$$E = [A_s + A_p \exp(ik_{x,p}x)\exp(i\Phi)]\exp[i(\omega t - kz)], \quad (3)$$

where A_s is the amplitude of the starlight, A_p and Φ are the amplitude and the phase, respectively, of the planetary light, $k = |k_p| = |k_s| = 2\pi/\lambda$ is the wave number, λ is the wavelength, and $k_{x,p} \approx k\alpha$ is the transverse wave number of the planetary light. We are interested in determining time-averaged intensities and therefore ignore the rapidly oscillating factor $\exp[i(\omega t - kz)]$. When $\alpha f \sim R_m$, planetary light will shine within the core of the stellar vortex. Placing a pinhole at the location of the stellar vortex will permit the detection of planetary light without the glare of starlight. To demonstrate this, let us examine the net field in the focal plane of the lens, which is related to the Fourier transform of the field at the entrance pupil. Assuming a lossless vortex mask with a transmittance function $G(r, \phi) = \exp(im\phi)$ yields a field in the focal plane that is proportional to

$$\text{FT}\{EG\} = A_s \text{FT}\{\exp(im\phi)\} + A_p \text{FT}\{\exp(-ik\alpha x) \times \exp(im\phi)\exp(i\Phi)\}, \quad (4)$$

where FT is a two-dimensional Fourier integral over the area of the lens. If the phase, Φ, is independent of the transverse coordinates, we can see that the terms on the right-hand side of Eq. (4) are related by means of the sifting property of the Fourier transform, and

$$\text{FT}\{EG\} = A_s \xi(x, y) + A_p \exp(i\Phi)\}\xi(x - \alpha f, y), \quad (5)$$

where $\xi(x, y) = \text{FT}\{\exp(im\phi)\}$. The Fourier-transform operation leaves the vortex intact, such that $\xi(0, 0) = 0$.[11,12] Thus the core of the starlight vortex is located at the origin, (0,0), and that of the planetary light is displaced at point $(f\alpha, 0)$.

The optimum planetary signal is expected to occur when $\alpha f = R_m$. For example, Fig. 4 shows the net intensity profiles for $m = 1-4$ that correspond to two distant point sources of equal luminosity subtending an angle $\alpha = \theta_{\text{diff}} = 1.22\lambda/D$. When the two fields are mutually coherent [Fig. 4(a)], interference between the fields produces an elliptical focal spot that has an m number of composite vortices that are aligned along the bisector of the two beams. When the fields are mutually incoherent [Fig. 4(b)], the added intensities also produce an elliptical spot; however, composite vortices do not fully develop. The cross in each profile in Fig. 4 marks the location of the vortex core of the primary beam. When a pinhole is placed at this position,

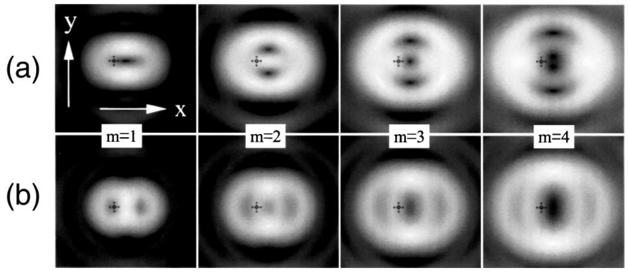

Fig. 4. Combined vortex beams in the focal plane for two equally luminous sources subtending an angle $\alpha = \theta_{\rm diff}$. Topological charge m of both beams is identical. Each beam is transversely coherent, and the two beams are mutually (a) coherent and (b) incoherent. Crosses, location of the optical axis (which coincides with the vortex core of the primary beam).

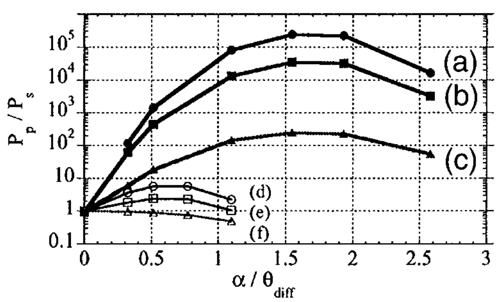

Fig. 5. Enhancement of the weak secondary signal over the strong primary signal for several values of angular aperture size $\theta_{\rm ap}$, angular distance between sources α, and topological charge m. Data points are shown for $m = 4$ for $\theta_{\rm ap}/\theta_{\rm diff}$ values of (a) 0.19, (b) 0.32, and (c) 0.65 and for $m = 1$ for $\theta_{\rm ap}/\theta_{\rm diff}$ values of (d) 0.19, (e) 0.32, and (f) 0.65. Solid curves were drawn to aid the eye.

it is evident that a large signal (almost entirely from a secondary source) will be detected.

We determine the amount of discrimination afforded by the vortex mask by calculating the power transmitted through the focal-plane aperture for both sources. For example, consider the mutually incoherent case shown in Fig. 4(b). The ratio of the transmitted beam powers, P_p/P_s, is plotted in Fig. 5 as a function of angular separation α for several values of the aperture size. As expected from Fig. 3, a greater enhancement is found for larger values of topological charge m and for smaller values of the angular aperture size, $\theta_{\rm ap} = R_{\rm ap}/f$. Note that $\theta_{\rm ap}/\theta_{\rm diff} = R_{\rm ap}/R_{\rm diff}$. Furthermore, the integrations also confirm that the optimum performance occurs when $\alpha = R_m/f$.

The significance of Fig. 5 may be understood by an example, such as this case: $m = 4$ and $\theta_{\rm ap}/\theta_{\rm diff} = 0.19$ at $\alpha/\theta_{\rm diff} \approx 1.7$. We obtain this curve by assuming equally luminous sources. Under this condition, the transmitted power of the secondary beam is roughly 2×10^5 times larger than that of the primary beam. However, if the secondary source is 2×10^5 times less luminous than the primary source, the transmitted power of the two beams will be equal. In the latter case, the vortex detection scheme provides a means to see the faint signal that would otherwise have been obscured by the glare of the bright primary source. For example, the relative luminosity of a planet the size of Jupiter with an orbital radius of 1 astronomical unit about a distant star varies periodically with a maximum value that is roughly 10^{-7} times that of the star. We estimate[16] from Fig. 3 that this signal could be detected with an $|m| = 4$ vortex mask and a pinhole that is one tenth of the diffraction-limited spot size. In conclusion, weak sources of on-axis incoherent radiation or off-axis coherent radiation can be detected when an optical vortex phase mask is used to eliminate the glare of a bright superimposed beam.

This study was supported by grants from the Research Corporation (Cottrell Scholar Program) and the National Science Foundation. I thank James H. Burge (University of Arizona, Tucson, Ariz.) for discussions of astronomical applications. My e-mail address is gswartzlander@optics.arizona.edu.

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