Spatial coherence singularities and incoherent vortex solitons

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We study spatially localized optical vortices created by self-trapping of partially incoherent light with a phase dislocation in a biased photorefractive crystal. In contrast to a decay of coherent self-trapped vortex beams due to the azimuthal modulational instability, the incoherent vortices are stabilized for large values of the spatial incoherence; this was confirmed by experiment. We analyze the spatial coherence properties of the incoherent optical vortices and reveal the existence of ringlike singularities in the spatial coherence function of a vortex field that can characterize the stable propagation of vortices through nonlinear media. © 2005 Optical Society of America

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1. INTRODUCTION

Vortices are fundamental objects in physics, and they can be found in different types of linear and nonlinear coherent systems. A typical scalar vortex has the amplitude vanishing at its center and a well-defined phase associated with the circulation of momentum around the helix axis.[1] In optics, vortices are associated with phase dislocations (or phase singularities) carried by optical beams.[2] The past decade has seen a resurgence of interest in the study of optical vortices,[3] owing in part to readily available, computer-generated holographic techniques for creating phase singularities in laser beams.

In a nonlinear medium the singular optical beam undergoes self-focusing, and it becomes self-trapped, creating a stationary ringlike structure with zero intensity at the center and a phase singularity.[4] However, such an optical vortex soliton is known to be highly unstable in self-focusing nonlinear media[5]; it decays by splitting into several fundamental (no nodes) solitons flying off the soliton ring.[6–8] This effect has been observed experimentally in different nonlinear systems, including saturable Kerr-like nonlinear media,[9,10] biased photorefractive crystals,[11] and quadratic nonlinear media[12] operating in the self-focusing regime. This effect is also expected to occur in other physical systems including the attractive Bose–Einstein condensates.[13]

A number of recent theoretical studies,[14–19] including rigorous analysis of the linear stability of a self-trapped vortex beam,[20,21] suggest that the stable propagation of spatial and spatiotemporal vortexlike stationary structures may become possible in models with competing nonlinearities in the presence of a large higher-order defocusing nonlinearity; however such materials are not yet known and no stable coherent vortex solitons have so far been observed in experiment.

Recently, stable propagation of spatially localized optical vortices in a self-focusing, biased, nonlinear, photorefractive crystal has been observed experimentally in the case when the vortices are created by partially incoherent light carrying a phase dislocation.[22] In particular, it was shown, both experimentally and theoretically, that single- and double-charge optical vortices can be stabilized in self-focusing nonlinear media when the spatial incoherence of the light exceeds a certain threshold. This effect can be compared with the somewhat similar effect of soliton-stripe stabilization by partially coherent light[23,24] following the use of the incoherence to eliminate the modulation instability.[25,26]

The successful experimental observation of stable, self-trapped vortex beams created by partially incoherent light calls for additional studies of the specific properties of partially coherent light carrying phase singularities and propagating in a nonlinear medium. Indeed, if a vortex-carrying beam is partially incoherent, the phase-front topology is not well defined, and statistics are required to quantify the vortex phase. In the incoherent limit neither the helical phase nor the characteristic zero intensity at the vortex center can be observed.

However, several recent studies have shed light on the question how phase singularities can be unveiled in incoherent light fields propagating in linear media.[27,28] In particular, Palacios *et al.*[28] used both experimental and numerical techniques to explore how a beam transmitted through a vortex phase mask changes as the transverse coherence length at the input of the mask varies. Assuming a quasi-monochromatic, statistically stationary light source and ignoring temporal coherence effects, they demonstrated that robust attributes of the vortex remain in the beam, most prominently in the form of a ring dislocation in the cross-correlation function.

The purpose of this paper is twofold. First we study numerically the effect of vortex stabilization through analysis of the spatial coherence function of a vortex beam propagating in a self-focusing nonlinear medium. We reveal the specific features of the coherence function and demonstrate its importance for the study of singular beams in nonlinear media. Second by applying the modal theory approach we provide a deeper physical insight into the effect of the vortex stabilization by partially coherent light observed in experiment.

The paper is organized as follows. In Section 2 we introduce our numerical model that is based on the coherent density approach and that describes the propagation of partially incoherent light in a slow-response nonlinear medium such as a biased photorefractive crystal. Section 3 describes some examples of stable, partially incoherent vortex solitons, including the experimental results. In Section 4 we introduce the spatial coherence function and analyze its properties, while Section 5 is devoted to a simplified approach based on truncated modal expansion. Section 6 concludes the paper.

2. MODEL AND NUMERICAL APPROACH

To study numerically the propagation of partially incoherent optical vortices in a biased photorefractive nonlinear medium, we employ the coherent density approach.[29] This approach is based on the decomposition of an incoherent light source into a superposition of (infinitely) many coherent components E_j that are mutually incoherent, having slightly different propagation directions:

$$E(\mathbf{r},t) = \sum_j E_j(\mathbf{r})\exp\{i\mathbf{k}_{\perp j}\mathbf{r}\}\exp[i\gamma_j(t)], \tag{1}$$

where $\mathbf{k}_{\perp j}=k(\alpha_j\mathbf{e}_x+\beta_j\mathbf{e}_y)$ is the transverse wave vector of the jth component having direction cosines α_j and β_j, $\mathbf{r}=x\mathbf{e}_x+y\mathbf{e}_y$, $\gamma_j(t)$ is a random variable that changes on the time scale of the coherence time of the light source, and $k=2\pi/\lambda$ is the wave number. Throughout this paper we use a number $N=1681$ of mutually incoherent components to account for the beam incoherence. The spatial grid used for the numerical simulations consists of 256×256 grid points. The vortex is introduced by means of a phase mask at the input face ($z=0$) of the medium. To avoid complexities that may arise from incoherent light sources having abrupt boundaries, we assume the source has a Gaussian profile

$$E_j(\mathbf{r}) = \left\{ \frac{1}{\sqrt{\pi}\theta_0} \exp[-(\alpha_j^2+\beta_j^2)/\theta_0^2] \right\}^{1/2} A(\mathbf{r}), \tag{2}$$

where

$$A(\mathbf{r}) = (r/w_0)^2 \exp(im\phi)\exp(-r^2/\sigma^2) \tag{3}$$

is the complex vortex profile, ϕ is the angular variable, and θ_0 is a parameter that controls the coherence properties, i.e., larger value of θ_0 means less coherence.

If we scale the lengths in the transverse directions to $x_0=1\ \mu\text{m}$ and the length in the propagation direction to $z_0=2kx_0^2$, where we choose $k=2\pi/(230\ \text{nm})$, the propagating field $E_j(\mathbf{r},z)$ can be described by the nonlinear Schrödinger equation

$$i\frac{\partial E_j(\mathbf{r},z)}{\partial z} + \nabla_\perp^{\ 2} E_j(\mathbf{r},z) + \eta(\mathbf{r},z)E_j(\mathbf{r},z) = 0, \tag{4}$$

where $\eta(\mathbf{r},z)$ accounts for the nonlinear refractive-index change in the material. This equation was numerically solved by use of a split-step technique, i.e., one propagation step consists of linearly propagating the beam for half a step-width, calculating the nonlinear refractive index, applying the nonlinear phase shift to the beam, and finally propagating the beams linearly for another half step-width. The step width was chosen to be 0.005 in dimensionless units.

We assume a photorefractive medium with a saturable nonlinearity having a response time much larger than the coherence time of the light source. In this case η depends on the average intensity $I=\Sigma_j|E_j|^2$ and we write

$$\eta(\mathbf{r},z) = \frac{I(\mathbf{r},z)}{1+sI(\mathbf{r},z)}, \tag{5}$$

where s is the saturation parameter. Whereas numerical solutions of Eq. (4) may be readily computed by use of the coherent density approach, we also adopt below the equivalent multimode theory[30] to provide a physical insight into our findings.

3. PARTIALLY INCOHERENT VORTEX SOLITONS

Experimental results, similar to those reported earlier in Ref. 22, were obtained for a vortex beam generated in a self-focusing, biased photorefractive strontium barium niobate crystal. The rotating diffuser was used to introduce randomly varying phase and amplitude of the input light beam on time scales much shorter than the response time of the crystal. By adjusting the position of the diffuser to near (away from) the focal point of the lens in front of the diffuser, the degree of light coherence was increased (decreased). After the rotating diffuser the light was sent through a computer-generated hologram to imprint a vortex phase on the light beam. Such a partially coherent vortex beam was then sent into the photorefractive crystal. All other details of the experimental setup and measurements can be found in Ref. 22.

The experimental results are summarized in Fig. 1 (lower row), and they are compared with the corresponding numerical results [see Fig. 1 (upper row)] obtained in the framework of the theoretical model introduced in Section 2. First both numerics and experiment reproduce the well-known result that the coherent single-charge ($m=1$) vortex beam cannot propagate stably in a self-focusing nonlinear medium (left-hand plots). Indeed, when the diffuser is removed from the experimental setup and a 2.5 -kV biasing voltage is applied on the photorefractive crystal to create a Kerr-type, self-focusing nonlinear medium, the vortex beam breaks up into two pieces. This vortex breakup observed in a self-focusing medium is due to the azimuthal instability, and it has been observed previously.

When the rotating diffuser is used, the degree of coherence of the vortex beam varies, and we observe clearly that the vortex beam can be stabilized by the reduction of the degree of coherence, as is summarized in Fig. 1. Above a certain value of the coherence parameter θ_0, the generated stable, partially incoherent vortex soliton is observed at the output face of the crystal.

4. SPATIAL COHERENCE FUNCTION

To quantify the second-order coherence properties of the singular beam propagating in a nonlinear medium, we calculate the mutual coherence function

$$\Gamma(\mathbf{r}_1,\mathbf{r}_2;z)=\langle E^*(\mathbf{r}_2,z,t)E(\mathbf{r}_1,z,t)\rangle, \tag{6}$$

where the angle brackets stand for averaging over the net field $E(\mathbf{r},z,t)=\sum_{j=1}^{N}E_j(\mathbf{r},z)\exp[i\gamma_j(t)]$. Again we assume that for photorefractive nonlinearities the random phase factors $\gamma_j(t)$ vary on a time scale much faster than the response time of the medium. For the linear propagation Palacios *et al.*[28] demonstrated that the phase singularities occur in the cross correlation $\Gamma(-\mathbf{r},\mathbf{r})$ of an incoherent vortex beam, where the origin of the coordinate system is chosen to coincide with the vortex center.

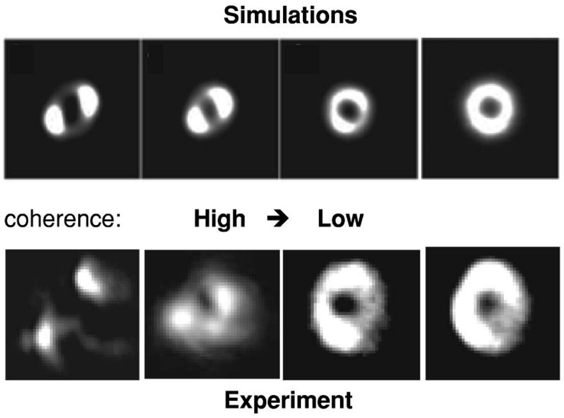

Fig. 1. Comparison between numerical (upper row) and experimental (lower row) results for the vortex stabilization effect. Numerical results are shown for the vortex after 9 mm of propagation for (from left to right) the coherent case and for three partially incoherent cases at $\theta_0=0.14°$, $\theta_0=0.29°$, $\theta_0=0.38°$, respectively.

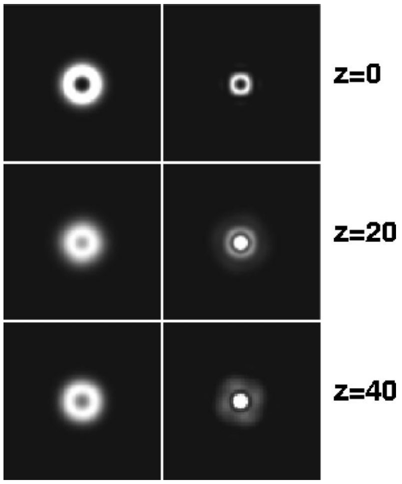

Fig. 2. Images of the intensity (left column) and the modulus of the cross correlation (right column) of an incoherent vortex with $\theta_0=0.64°$ (strong incoherence). Contrary to the case of the linear propagation, there is a local intensity minimum in the beam's center. The cross correlation, however, shows the same ring of phase singularities as predicted in the linear theory.

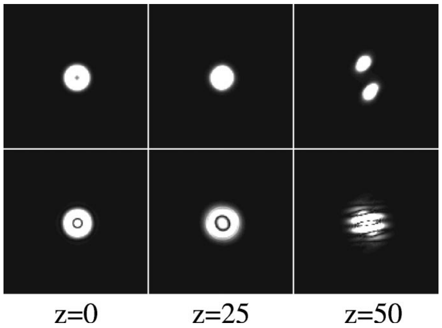

Fig. 3. Images of the intensity (upper row) and the modulus of the cross correlation (lower row) for the breakup of an incoherent vortex at $\theta_0=0.37°$ (weak incoherence), when the ring is not preserved.

In Figs. 2 and 3 we show the numerical results for the stable and unstable nonlinear evolution of an incoherent vortex and the corresponding evolution of the vortex cross-correlation function. In these examples, we solve numerically the model of Eqs. (1)–(5) with the parameters $w_0=1.8$, $\sigma=1.5$ and $s=0.5$. The size of the numerical solution range corresponds to the domain 35×35 μm.

First we notice that in the nonlinear case the beam intensity has a local minimum in the center of the vortex, even after propagating many diffraction lengths. This is contrary to the case of linear propagation in which a beam with the same degree of coherence θ_0 has maximum intensity in the center of the vortex after only a few diffraction lengths. Also if we had chosen to propagate an incoherent ring of light without topological charge instead of an incoherent vortex, we would also observe a maximum at the beam center, similar to the coherent case reported in Ref. 31. Thus we can state that the coherence function of the vortex manifests itself in the intensity distribution of the light beam after propagating through a nonlinear medium. In fact, the intensity profile remains reminiscent of a vortex, even if the intensity does not drop quite to zero in the center of the beam.

Analyzing the structure of the beam cross-correlation function, we clearly observe, similar to the case of linear propagation,[28] a ring of phase singularities in the cross-correlation function $\Gamma(-\mathbf{r},\mathbf{r})$ that is preserved when the vortex is stabilized (see Fig. 2) or disintegrates and decays when the vortex breaks up (see Fig. 3).

Thus as the first result of our numerical studies we state that the phase singularities in cross correlation predicted for the incoherent vortices propagating in linear media also survive the propagation through a nonlinear medium. This is not self-evident, considering that in the nonlinear case the separate components that form an incoherent light beam do interact, contrary to the linear case. A physically intuitive explanation of how this ring of phase singularities develops under linear propagation is given in Ref. 28. However, this issue becomes more complicated for propagation in a nonlinear medium.

In addition in Fig. 4 we show the situation in the far field. All parameters are identical to those used in Fig. 2. In the far field we observe again a ringlike structure of the cross-correlation function $\Gamma(-\mathbf{f},\mathbf{f})$, where $\mathbf{f}$ stands now for the spatial coordinates in the far field. The intensity distribution in the far field can also show a local minimum in the center of the beam, contrary to what one would obtain if the vortex were propagating through a linear medium,[28] and also in contrast to the result we would obtain if we were propagating a light beam without topological charge. This emphasizes the importance of the interaction between the beam coherence function and the nonlinearity.

5. MODAL THEORY APPROACH

Although the coherent density approach can be used to simulate the propagation of partially incoherent light with an arbitrary accuracy, it is of little use when it comes to an explanation for the results obtained from the numerical simulations such as those presented above. A deeper physical insight can be obtained by use of the modal theory of incoherent solitons.[32] According to the modal theory the incoherent solitons can be regarded as an incoherent superposition of guided modes of the waveguide induced by the total light intensity. Since we are considering incoherent vortices that induce circularly symmetric waveguides, the guided modes are also circularly symmetric. To explain our numerical findings we construct numerically, by a standard relaxation technique,[33] a partially incoherent vortex soliton that consists of the circularly symmetric modes with the topological charges $m=0$, 1, and 2:

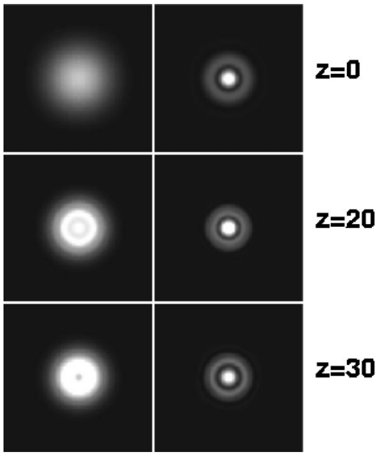

Fig. 4. The intensity (left column) and the cross correlation (right column) of the far field. The effects of the nonlinearity on the intensity distribution can be clearly seen, whereas the cross correlation maintains more or less the structure one would expect in the case of linear propagation.

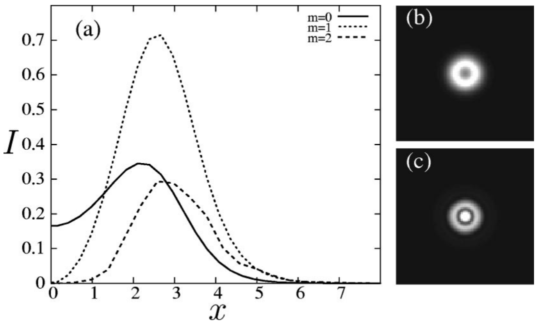

Fig. 5. A composite soliton calculated by using the three modes with the topological charges $m=0,1,2$. (a) Profiles of the three components, (b) total intensity of the vortex soliton, (c) vortex cross correlation $\Gamma(-\mathbf{r},\mathbf{r})$.

$$E(\mathbf{r},t)=\sum_{m=0}^{2}E_m(\mathbf{r})\exp(im\varphi)\exp[i\gamma_m(t)].$$

A more precise modeling of incoherent vortices would require more modes. Here we restrict ourselves to three modes only, assuming that for a partially incoherent vortex the $m=1$ component should be dominant and that the next strongest components should be those with topological charge $m'=m\pm1$, i.e., $m'=0,2$. Indeed we find that the main features of incoherent vortex solitons can be explained qualitatively by use of these three modes only.

For this three-mode composite vortex soliton, the relative intensity of the $m=0$ and $m=2$ modes controls the overall beam coherence, as compared with the $m=1$ main vortex mode. However, to assure that the total topological charge of the beam,

$$m_{\text{tot}}=\text{Im}\left\{\left\langle\int E^{*}(\mathbf{r}\times\nabla E)\mathrm{d}\mathbf{r}\right\rangle\right\}\mathbf{e}_z\bigg/\int I\mathrm{d}\mathbf{r}, \qquad (7)$$

is equal to one, we have to choose the $m=0$ and $m=2$ components of equal intensity. To check whether this simple approach yields the results that agree at least qualitatively with the full numerical model of an incoherent vortex soliton, we calculate the resulting shape of the vortex components, the total intensity, and the cross-correlation $\Gamma(-\mathbf{r},\mathbf{r})$ shown in Fig. 5. Comparing Fig. 2 and Fig. 5, we note the presence of two similar features: (i) the local minimum of the intensity in the center of the beam and (ii) the ringlike structure of the cross correlation. Hence these two phenomena can be explained by considering a simple modal representation of the incoherent vortex consisting of only three modes with the topological charges $m=0$, 1, and 2.

First the local minimum in the center of the beam can be explained by the fact that the waveguide induced by the $m=1$ and $m=2$ components affects the $m=0$ mode in such a way that it also develops a local intensity minimum in its center, a fact well known from vortex-mode vector solitons.[5] Second the ringlike structure of the cross correlation comes from the different radial extent of the single components. As is known from the physics of vortex-mode vector solitons,[5] the $m=0$ component has the smallest radial extent, whereas the $m=1$ and $m=2$ components have larger radii. Hence the cross correlation given by

$$\Gamma(-\mathbf{r},\mathbf{r}) = \sum_{m,m'=0}^{2} \langle E^{*}_{m'}(-\mathbf{r})E_{m}(\mathbf{r})\rangle = \sum_{m=0}^{2} E^{*}_{m}(-\mathbf{r})E_{m}(\mathbf{r})$$

is dominated for small $\mathbf{r}$ by the autocorrelated $m=0$ component, whereas the $m=1$ component dominates for larger $\mathbf{r}$. For even larger $\mathbf{r}$ the $m=2$ component can also come into play, which can eventually result in a second ring of cross correlation.

6. CONCLUSIONS

We have demonstrated stable propagation of optical vortices in a self-focusing nonlinear medium when the vortices are created by self-trapped partially incoherent light with a phase singularity propagating in a slow-response nonlinear medium such as a photorefractive crystal. The vortex azimuthal instability is found to be suppressed for light incoherence above a critical value. To get a deeper physical insight into the effect observed in both numerics and experiment, we have studied the phase singularities in the spatial coherence function employed earlier in the linear optics and demonstrated that they survive propagation through nonlinear media when the singular beam creates an incoherent vortex soliton. Our results emphasize the importance of the spatial coherence function in studies of the propagation of incoherent singular beams. Not only the phase structure, but also the intensity distribution strongly depends on the initial form of the coherence function of the light beam as it enters a nonlinear medium.

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