Ph.D. Candidacy Exam Solutions

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Abstract

The following are solutions to the candidacy exam given to Emmett J. Ientilucci for the Ph.D. thesis entitled, “Hyperspectral Sub-Pixel Target Detection Using Hybrid Algorithms and Physics Based Modeling”. The first set of questions by Dr. Bajorski explores issues regarding the principal component and maximum noise fraction transforms. Due to time constraints, a data set with low noise, to show similarities between the PCA and MNF transforms, was not evaluated. The second set of questions from Dr. Kerekes deals with issues regarding calibration errors and their effect on algorithm performance. The third set of questions from Dr. Rhody looks at the least-squares problem from a fundamental linear algebra standpoint. The last set of questions by Dr. Schott addresses the issue of how to describe, characterize, and use a physics-based target space in the proposed algorithm. Due to time constraints, the integration of the target space into the stochastic mixture tuned matched filter was not documented.

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1 Questions from Dr. Bajorski

Assume we measure the variability in multivariate data using the trace of the variance-covariance matrix of the data.

**Question 1** Is the data variability in the original space the same as the variability after the MNF transform? Why?

**Question 1 Answer**

The overall variability after the MNF transform will not be the same as in the original space. In short, it will have been rotated, whitened, and rotated again based on a signal-to-noise ratio (SNR). We can show this by following the covariance through the MNF process.

Assume we have data matrix $A$ with covariance $\Sigma_A$. We assume that the signal and noise components in the data are uncorrelated. In doing so, we can write the overall covariance as signal plus additive noise. That is,

$$\Sigma_A = \Sigma_S + \Sigma_N.$$  \hspace{1cm} (1)

We first perform a standard principal components (PC) transform on the data based on the covariance of the noise, $\Sigma_N$. If we rotate with eigenvector matrix $E$ the resulting covariance matrix will be

$$\Sigma_{P_1} = E^T \Sigma_S E + E^T \Sigma_N E$$

$$= \Sigma_{P_1 S} + \Lambda_{P_1 N}. \hspace{1cm} (2)$$

This rotation has decorrelated the noise component of the original covariance but not the signal component. Now that we have pushed all the variation due to noise onto the diagonal, we use the inverses-square root of it to whiten the data. This further alters the covariance as

$$\Sigma_{P_2} = (\Lambda_{P_1 N}^{-1/2})^T \Sigma_{P_1 S} (\Lambda_{P_1 N}^{-1/2}) + (\Lambda_{P_1 N}^{-1/2})^T \Lambda_{P_1 N} (\Lambda_{P_1 N}^{-1/2})$$

$$= \Sigma_{P_2 S} + I_N \hspace{1cm} (3)$$

where we have now turned the noise component into an identity matrix.
Finally, we apply a standard PC transform to the noise-whitened data with eigenvector matrix $W$ which is computed from the covariance $\Sigma_{P_2}$. The resulting covariance in MNF space is then

$$\Sigma_{P_3} = W^T \Sigma_{P_2} S W + W^T I_N W$$

$$= \Lambda_{P_3} S + I_N$$

$$= \Lambda_{P_3} + I_N$$

(4)

since the orthonormal vectors $W$ have the property $W^T W = I$. This last transformation produces a covariance that is diagonal where the noise component is isotropic with unit variance. This matrix is clearly different from our original matrix $\Sigma_A$.

Question 2 Assume that we transformed data using MNF, then eliminated $k$-least desirable bands, and finally back-transformed the data into the original space. Give formulas for the total variability and noise variability in the final back-transformed data using only mathematical notation.

Question 2 Answer

In the previous question we derived what the covariance would look in MNF space. If we wanted to know what the total variability was in that space we would simply compute the trace() of the covariance matrix $\Lambda_{P_3}$. Assuming we have $p$-spectral bands ($p \times p$ matrix) the total variability would be

$$\sigma^2_{T,MNF} = \sum_{i=1}^{p} \sigma^2_{i,i}$$

(5)

where $\sigma^2_{i,i}$ are diagonal elements from the matrix $\Lambda_{P_3}$. We can also compute the total variability due to noise since it is assumed to be separable. This is performed in a similar manner by computing the trace() of the noise covariance $I_N$. Since this is the identity, the total variability due to noise is simply $p$.

In a standard PC space, after ordering the variances (or eigenvalues), the latter bands usually have variation in them due to noise, since the larger variations (assumed to be due to information) are pushed to the earlier bands. A procedure often practiced is to truncate the $k$-least desirable bands in the PC space then back-transform the data to the original space in hopes of reducing the noise contribution in the imagery. However, if the variation is not due to noise, then one runs the risk of throwing away valid image information.
This same procedure is performed using the MNF transform. Here, the information content is not so much related to pure variance as it is image quality. This is due to the fact that the third step in the algorithm computes a PC transform on a signal-to-noise quantity.

In the MNF space we can eliminate $k$-least desirable bands from a total of $p$-bands and back-transform to the original space, keeping tract of the covariance. Then we can compute the total variance and total noise variance in this space, due to the truncated matrices in MNF space.

If the MNF covariance is $\Sigma_{P_3[p \times p]}$, then keeping $r = p - k$ bands has the form $\Sigma_{P_{st}[r \times r]}$, where the subscript $t$ denotes the truncation. Similarly, the truncated noise matrix will be $\mathbf{I}_{Nt[r \times r]}$. In practice, these latter bands are simply zeroed out while the array dimension is maintained for multiplication purposes. In order to get covariance $\Sigma_{P_2t}$, we have to invert $\mathbf{W}$. That is

$$
\Sigma_{P_2t} = (\mathbf{W}^{-1})^T \mathbf{A}_{P_3St} \mathbf{W}^{-1} + (\mathbf{W}^{-1})^T \mathbf{I}_{Nt} \mathbf{W}^{-1}
$$

$$
= \mathbf{W} \mathbf{A}_{P_3St} \mathbf{W}^T + \mathbf{W} \mathbf{I}_{Nt} \mathbf{W}^T
$$

$$
= \Sigma_{P_2St} + \mathbf{I}_{Nt}
$$

(6)

since $\mathbf{W}$ is orthonormal with properties $\mathbf{W}^{-1} = \mathbf{W}^T$ and $\mathbf{W}^T \mathbf{W} = \mathbf{I}$. We then need un-whiten the data through a similar inversion with the inverse-square root matrix to get back to the covariance $\Sigma_{P_{1t}}$. That is

$$
\Sigma_{P_{1t}} = (\mathbf{A}_{P_{1N}}^{1/2})^T \Sigma_{P_2St}(\mathbf{A}_{P_{1N}}^{1/2}) + (\mathbf{A}_{P_{1N}}^{1/2})^T \mathbf{I}_{Nt}(\mathbf{A}_{P_{1N}}^{1/2})
$$

$$
= \Sigma_{P_{1St}} + \mathbf{A}_{P_{1Nt}}
$$

(7)

Finally, we rotate back to the original space with the inverse of $\mathbf{E}$ to obtain

$$
\Sigma_{At} = (\mathbf{E}^{-1})^T \Sigma_{P_{1St}} \mathbf{E}^{-1} + (\mathbf{E}^{-1})^T \mathbf{A}_{P_{1Nt}} \mathbf{E}^{-1}
$$

$$
= \mathbf{E} \Sigma_{P_{1St}} \mathbf{E}^T + \mathbf{E} \mathbf{A}_{P_{1Nt}} \mathbf{E}^T
$$

$$
= \Sigma_{St} + \Sigma_{Nt}
$$

(8)

At this point we can compute the total variability and noise variability by computing the $\text{trace}()$ of $\Sigma_{At}$ and $\Sigma_{Nt}$. Thus the total variability can be defined as

$$
\sigma_{T,At}^2 = \sum_{i=1}^{p} \sigma_{At[i,i]}^2
$$

(9)
\[ \sigma^2_{\text{At}[i,i]} \] are diagonal elements from the matrix \( \Sigma_{\text{At}} \). Similarly, the noise variability can be expressed as

\[
\sigma^2_{\text{T,Nt}} = \sum_{i=1}^{p} \sigma^2_{\text{Nt}[i,i]} \tag{10}
\]

where \( \sigma^2_{\text{Nt}[i,i]} \) are diagonal elements from the matrix \( \Sigma_{\text{Nt}} \).

**Question 3**

Assume that all bands are uncorrelated. *(i)* Find the noise fractions for all bands. *(ii)* Assume that we perform MNF and PCA transforms (back to the original space) and decide to keep the best (in the sense of each of the two methods) 5 (out of the total of 10) bands in each case. Is it possible that the two transformed images are totally “unrelated” (that is, uncorrelated)? Show an example. In such a case, how would you decide which of the two transformations is better?

**Question 3 Answer**

*(i)* If we assume that all the bands are uncorrelated, then we must have a diagonal covariance matrix where the diagonal terms \( \sigma^2_{i,i} \) for \( i = 1, 2, \ldots, p \), represent the variance for each band and all of the off-diagonal terms are zero, since there is no co-variance (i.e., uncorrelated).

The noise fraction can be defined as the per-band noise variance \( \sigma^2_{\text{Nt}[i]} \) divided by the per-band total variance, \( \sigma^2_{\text{T}[i]} \) for \( i = 1, 2, \ldots, p \)-spectral bands, where \( \sigma^2_{\text{T}[i]} = \sigma^2_{\text{S}[i]} + \sigma^2_{\text{Nt}[i]} \). That is

\[
\rho_i = \frac{\sigma^2_{\text{Nt}[i]}}{\sigma^2_{\text{T}[i]}}, \quad i = 1, 2, \ldots, p. \tag{11}
\]

*(ii)* If we perform a standard PC on image \( A \), keep the “best” 5 bands, then back transform and compare the result to a similar procedure using and MNF transform, we will see that the two methods can produce related imagery if the noise is small in \( A \). The imagery will become more “unrelated” as the noise variance increases. An explanation follows.

**Low Noise**

A standard PC transform will simply try to rotate the data so the new axes are in the direction of maximum image variance, hence the new transformed covariance will be diagonal. If there is low noise and high signal variation, then the larger eigenvalues (first
5, for example) will be due to signal with the smaller ones due to noise. We can keep these 5 bands and invert back to the original space with some adequate noise elimination.

The MNF transform try’s to maximize a signal-to-noise (SNR) variance rather than an image variance. This SNR will be large in the event that we have low noise. Thus, like the PC transform, the larger eigenvalues (first 5, for example) will be due to signal with the smaller ones due to noise. Therefore, one can achieve similar results using either the PC or MNF transforms.

**High Noise**

In the case of dominant variation due to noise, the PC transform will still try the rotation, but the direction of maximum variance will be due to noise and not signal. More importantly, the largest eigenvalues (when ordered) will be due to noise variance not signal. We also could have the case of the noise level in one band being higher than the signal in another band. When we decide to keep the first 5 (after ordering) and invert, we are simply keeping the variance due to noise in the back transform. There is no mechanism for separating out the noise variation.

With the MNF transform, we at least have a heads up on the covariance of the noise before we start the process. If the noise is large, then the SNR will be small and the PC transform will move this information to the latter eigenvalues, not the first five. Now if we select the first 5 and back transform, we will have a different image than the one generated using the PC transform with the same level of (high) noise.

**16 Band Example: High Noise**

The multispectral data set that was used for this analysis came from the Modular Imaging Spectrometer Instrument (MISI). The data set under test was an image of the Lake Ontario shoreline near Russell Station located in Rochester, NY. The data was collected on September 9, 2001. The MISI instrument was flown at an altitude of 5000 feet with a ground speed of 128 knots.

Figure 1 shows each of the sixteen (16) NIR bands (730 nm to 985 nm) from the spectrometer. Below each image is the total variance for that particular band (in digital counts). Each image has been linearly scaled by the same amount. Doing so illustrates the lack of dynamic range or variance in the later bands. These low variability images show up as having lower contrast. This particular set of NIR MISI images exhibits a fair
amount of noise, as can be seen by the large “strip” across the center of the image in most of the bands.

**PCA Transform Results**

Once we have determined the form of the rotational transform, we can then apply it to the original MISI data set. All 16 bands of the ordered transformed data can be seen in Figure 2 along with their corresponding variances.

It is clear that the first principal component contains the majority of the information, as can be seen by the large variance value. This tells us that the data set, from band to band, is very correlated. The first few rotations seem to contain all the variability, however, after bands 5 or 6 one might argue that the variance values are mainly due to noise, not relevant information.

The magnitude of this first principal component can be more clearly seen if we plot the eigenvalues (see Figure 3a). Here we see the variance values being dominated by the first principal component. The plot tells us how many independent factors there are in the data set. When the plot levels off, the remaining principal components do not explain much about the data set. Therefore, only the first $k$ principal components really matter in terms of explaining the variation in the data. This result can also be seen by plotting the total variability explained in the data (see Figure 3b). Here, we keep a running sum of how much each band adds to the total variability. Clearly, we can see that 99.8% of the variability is explained by summing the first 5 bands.

**PCA Noise Reduction**

It is clear by both visual inspection of the principal component images, as well as the eigenvalue plot, that only the first 5 or 6 bands (or less) contain relevant variance information about the image data set. The remaining bands have variances that are dominated by noise. Each of the later bands has a (noise) variance around $\sigma^2 \approx 2$ (digital counts). Since these later bands do not contribute significantly to relevant information, they may be omitted or zeroed out during an inverse transform.

Take for example band 13. This band has a significant amount of instrument noise. In the inverse transform, we can zero out some of the later bands thus only keeping the first few bands. This inverse transform was performed while keeping the first PC band only, the first 3 PC bands, and the first 5 PC bands. The results of this are seen in Figure 4.
Figure 1: MISI NIR 16-band data (730 nm to 985 nm). All images scaled the same.
Figure 2: MISI NIR data in PC transformed or rotated space (all with similar scaling).
Figure 3: (a) Plot of eigenvalues for PCA transform on MISI data and (b) total percent of variability as a function of band number.

It is clear that some noise is introduced back into the original space as we include more of the later PC bands.

**MNF Transform Results**

The noise adjusted principal component algorithm was then applied to the NIR MISI data set. The results of this transformation can be seen in Figure 5 along with the individual band variances. Initially, we see the same trend as we did when we used the standard PC transform (see Figure 2). However, the variance values (eigenvalue) for the MNF are not as large. Furthermore, we see that a significant amount of noise has been pushed to the later bands while the earlier bands seem to be free of the “stripping” seen in Figure 2.

The eigenvalues from the transform can be seen in Figure 6. Figure 6a shows the same trend as that from the standard PC transform. However, Figure 6b tells us that more bands are required in order to explain a larger amount of image information.

**MNF Noise Reduction**

The MNF inverse transform was then implemented on the data similar to that of the PC inverses. The first inverse involved keeping the first PC band only while the second and third involved keeping 3 and 5 bands, respectively. The results of this can be seen in Figure 7. Immediately we see a significant visual improvement over that from the standard
Figure 4: Inverse principal component transform analyzing band 13 keeping (a) 1, (b) 3, and (c) 5 bands. All images scaled with similar gain to that of the original MISI image in Figure 1.

PC inverse transform. There is no “stripping” artifacts evident in any of the images. If we look at the imagery in MNF transformed space, we do not see the stripping in the earlier bands that was evident in the PC transformed space.

We have presented a case in which keeping 5 bands in transformed space and inverting, using the same high-noise data set but different algorithms, has resulted in an (expected) difference between the resultant images.

At this time, however, the investigator has yet to put together a test case on low-noise data that shows when the two presented methods yield similar results.

When to Choose One Over the Other

The choice of method depends on the amount of noise variation, relative to the signal, in the imagery. The MNF transform improves upon the PC method at high noise levels while yielding similar results at low noise levels. One would think to use MNF all the time, and forgo the choice of which one was better, assuming you have access to noise information (i.e., noise covariance).

In order to choose which of the two transformation yielded a better result (given the situation of keeping $k$ bands), one would have to define the term “better”. Visual inspection is always a good qualitative indicator, as seen in the results presented in the
band 1 ($\sigma^2 = 724.4$) band 2 ($\sigma^2 = 36.5$) band 3 ($\sigma^2 = 9.8$) band 4 ($\sigma^2 = 4.3$)

band 5 ($\sigma^2 = 1.5$) band 6 ($\sigma^2 = 1.5$) band 7 ($\sigma^2 = 1.1$) band 8 ($\sigma^2 = 1.0$)

band 9 ($\sigma^2 = 1.0$) band 10 ($\sigma^2 = 1.0$) band 11 ($\sigma^2 = 1.0$) band 12 ($\sigma^2 = 1.0$)

band 13 ($\sigma^2 = 1.0$) band 14 ($\sigma^2 = 0.99$) band 15 ($\sigma^2 = 0.98$) band 16 ($\sigma^2 = 0.97$)

Figure 5: Misi NIR data in NAPC transformed or rotated space (all with similar scaling).
Figure 6: (a) Plot of eigenvalues for MNF transform on MISI data and (b) total percent of variability as a function of band number.

Additionally, if we are looking to reduce noise effects by eliminating some of the latter bands in the forward transform, then perhaps we could look at the variability in homogenous regions throughout the back transformed images. We would find uniform regions in the original, PC, and MNF images and compare them in terms of variability. If the areas are uniform, then the only variation should be due to noise. The image with the lowest variability would prove to be better. This could be performed across all bands.
Figure 7: Inverse MNF transform analyzing band 13 keeping (a) 1, (b) 3, and (c) 5 bands. All images scaled with similar gain to that of the original MISI image in Figure 1.


2 Questions from Dr. Kerekes

The title of your thesis and some of the methods you are investigating include the term “invariant”. As I understand it, the idea behind these approaches is to use physics based modeling to predict the range of expected illumination and atmospheric conditions, in order to capture the essential, consistent features of the target and enable robust detection performance. As such, the models traditionally produce spectra with units of at-sensor spectral radiance with perfect calibration. In the ideal world, these spectra will match identically with spectra measured by a sensor and calibrated to the same physical units.

However, the world is not ideal, and the sensing process rarely leads to perfect, or even highly accurate, calibrated data. Aside from pure radiometric calibration errors, there are many sensor artifacts which are common in data, including the HYDICE data you will use for your investigation.

My question is aimed at encouraging you to consider how typical characteristics (errors) of real sensor data will affect your expected conclusions and the insights into detection algorithms gained from your thesis. To this end, please respond to the following.

NOTE: These questions could be very open-ended with the an entire thesis devoted to their answers. For this exercise, please limit the scope to a handful of known, characterizable, significant error sources and artifacts.

Question 1 Lists common sensor-induced artifacts and sources of error in hyperspectral imagery and discuss qualitatively the impact of each on the primary algorithms used in your thesis work.

Question 1 Answer

The approach taken in this research includes the use of physics-based modeling (PBM) to generate a target space of sensor-reaching spectra, in radiance units. These spectra may be used in place of traditional image-derived target spectra, for example. This target information is then used as input to a detection algorithm, along with background information, to seek out targets of interest.

The clear difference between the PBM approach and the image derived approach, for example, is that the PBM target space has perfect calibration. That is, there are no sources error or variability in the space other than from the original reflectance and propagation model which created it. Certainly a target space of this nature fails to resemble
real world (target) data to some degree. As a result, real calibration issues need to be addressed as well as their impact, or lack-there-of, on detector performance. Below is a list of some of the more important sources of error and how they relate to algorithms used in this research. By no means is this an exhaustive list of errors. The following was based on working with hyperspectral data as well as a literature review on common sources of sensor errors.

**Illumination Angle**

A source of variability the PBM can account for is illumination angle. This variability can be factored into the atmospheric propagation model. However, for the presented work, the time of day (TOD) was known and was not included as a variable in the model. That is not to say we have the illumination angle exactly correct. The generated target space vectors will be for a fixed illumination angle. However, the actual target illumination angle(s) may be slightly different thus potentially impacting the detection process.

However, the magnitude of these errors may not be that significant to the results obtained with the two detectors used in this research, namely the SMTMF and the GMTMF. We can follow the possible sources of error for this parameter at a microscopic level. That is, we have to consider the accuracy in which the user logged the time of day. This may have only been off by a few minutes or less. Then we have to consider the time of day itself. If the TOD was in the morning, then the illumination angle change per minute will be greater than if the TOD was high-noon. We can then bracket the actual illumination angle. This range of angles would then have to be factored into the atmospheric propagation model. Here it is believed that other varied parameters such as aerosol or visibility will dominate over the potentially subtle error in illumination angle. Therefore, its impact on the target space, and ultimately the detection outcome, will be minimal.

**Material Reflectance**

Material reflectance is another source of variability unaccounted for. The target spaces are made using a single measured reflectance spectrum. This can come from the actual material itself or from a spectral library database. In either case, we are assuming that our single reflectance spectrum is representative of the target variability. Clearly this is not the case. Materials have variability at almost all scales. To characterize the material adequately, we would have to obtain a family of spectral reflectance measurements which
would be representative of the material of interest. We would then use our physics-based model to create multiple target spaces, one for each reflectance. This certainly would present a computational logistics issue. Finally, we would merge all our target spaces into one and use it in the detection process.

More often than not, however, we tend to seek out man-made objects which have significantly less variability than say natural objects such as grass, trees, and dirt. This significantly reduces the impact of material variability. The question still remains, how much is this impact reduced? One factor that dictates this answer is the size of the pixels (ground resolved distance) in the imagery and how many of them you have on a target. If the GRD is small and there are many pixels on a target, then a single reflectance spectrum may not be adequate in the target space. However, when the GRD is large and you have very few pixels on a target then we will have some averaging of the material variability and perhaps the single reflectance spectral is enough (assuming it is close in reflectivity to the actual target you are looking for). Also one needs to consider, as we did in the illumination case, the actual range of variations in material reflectance and how that range compares in magnitude and impact to other parameters such as visibility and aerosol content. Overall, it is believed that the impact of material reflectance on detection performance will be minimal.

**Detector Noise**

Another source of variation not included in the physics-based target space are effects due to sensor noise. One such source is detector noise which can come from processes such as photon or shot noise, thermal noise, and multiplexer/readout noise [1]. These noise sources are expected and are due to random fluctuations in the output signal. Shot noise is simply due to the random arrival of the photons themselves. The thermal noise noise can often be reduced by cooling the detector with a dewar to a temperature of 65K. Finally, the readout noise is due to photon-generated electrons at well sites not being fully read out of the detector.

The above mentioned sources of noise will all be in an actual image cube, including an image derived target pixel, for example. However, our physics-based modeled target space will not contain any of these errors. The effect detector noise has is in altering the variability of the spectral target radiance vectors. If it is large enough, it could mask important features that are unique to a particular target thus impacting overall detector
performance. This source of error should be included in the presented algorithms.

The test imagery that will be used throughout this research is the HYDICE forest radiance I data set. Therefore, the next set of noise sources are all specifically related to this sensor.

Heartbeat

The HYDICE sensor, in particular, exhibits what is called heartbeat noise [2]. It is believed that this variation in background signal is caused by the pressure wave at the end of the dewar cold finger interacting with the detector chip assembly [3]. Furthermore, this periodic variation has been seen under both illuminated and dark conditions.

This source of noise has been adequately modeled and can be removed from dark imaged dark frames. However, it is most difficult to eliminate under conditions in which the detector is fully illuminated due to phase variations. An additional study [4] concluded that the heartbeat variation was signal independent with maximum amplitudes being less than 1% of the signal. Therefore, it was concluded that this source of error was not significant. This conclusion would also tell us that this source of error should not affect detection performance.

Absolute Calibration Error

In 1997, a discrepancy was discovered in the absolute radiometric calibration of the HYDICE data. More specifically, there was an error with the sensors in-flight tungsten-halogen calibration lamp reference source. This error is most pronounced in the 1.7 to 2.5 $\mu$m region, as seen in Figure 8. The source of error does not come into play if one uses the radiance data only or is converting the data to apparent reflectance based on within-scene information (e.g., use of in-scene calibration panels or the empirical line method (ELM)). However, when using the physics-based target space with the HYDICE radiance imagery, this source of does become important, and would affect detection performance. Therefore, the bulb correction would have to be implemented on the imagery prior to any detection studies.

FCU Non-Uniformity

The HYDICE sensor also has an error associated with the flight calibration unit (FCU) which calibrates the sensor both radiometrically and spectrally. The error manifests itself
as a non-uniformity across the sensors field of view (FOV). During calibration the FCU is moved into the sensors FOV for imaging. However, the output of the FCU (or integrating sphere) does not fill the entire FOV or the sensor. Because of this, the FOV of the sensor must be illuminated sequentially from left to right in three fixed positions (left, center, and right). The partially illuminated images are then combined to form a full image which was to be used to convert raw data to radiance. Due to the combining of partially illuminated frames, there is a non-uniformity across the focal plane array. However, a calibration method has been developed which showed a reduction in the radiometric uncertainty from 30% to 5% [5, 6].

Certainly, uncertainty in knowledge of the spectral radiance will have an effect on any spectral processing algorithms, especially ones that rely on spectral shape and absolute magnitude. However, the important issue is whether the 5% uncertainty is significant relative to other errors. That is, are other errors dominant over the FCU non-uniformity? If so, then we need not concern ourselves with this source of error. For the application of the algorithms proposed in this research, this source of error would affect results. However, further investigation into the references given might be needed in order to draw final conclusions.

**Spectral Jitter**

As expected with many imaging spectrometer instruments, the HYDICE sensor has an
error associated with spectral shift (or jitter) across the focal plane array. To check this center wavelength shift, a helium-neon (HeNe) LASER source illuminates several columns along the edge of the array during in-flight calibration. This information is logged and used for spectral calibration. The jitter associated with the HYDICE sensor is thought to be caused by mechanical vibrations within the system. Shifts ranging from 0.3 to 4.5 nm have been reported, depending on channel number [2].

Spectral jitter will undoubtedly affect detector performance due to the shifting of channel centers in the data relative to spectral curves found in the physics-based target space. However, is the specific shift in the HYDICE data significant? Earlier reports [2], that used a spectral matched filter (analogous to the algorithms used in this report) showed that there was almost no difference in the detection probabilities when detecting subpixel objects. Based on this previous study, and the type of detection algorithm used, it is assumed that a similar conclusion can be drawn about the affect of spectral jitter on the detection algorithms used in this research.

**Rugate Filter**

In order to suppress the thermal background, a rugate filter is placed at the entrance window of the sensors dewar. This filter has a bandpass that encompasses the sensors spectral coverage (400 - 2500 nm). The transmission within the bandpass is highly structured with many absorption features. As long as the spectral calibration does not change between collections of the FCU, the complex signature can be normalized out. In the event that the spectral calibration does change (possibly due to effects like spectral jitter), the transmission profile will not be completely normalized out and some residual structure will be left behind.

Studies have shown [2] that the residual transmission profile left behind did not impact HYDICE’s signal-to-noise, even when the spectral jitter used was that of a typical channel shift from a typical run. For detection, it is believed that this source of error (other than the spectral shift, which was previously discussed) will not play a large role in detection performance outcome.

**Question 2** Define and write a plan for a simulation experiment that can be conducted as part of your thesis work that will investigate the quantitative impact of sensor error sources and artifacts on your algorithm comparisons.
Figure 9: Test image used for noise analysis. (a) 400 × 400 image with sensor grid over top and (b) image convolved to sensor resolution resulting in a 10 × 10 test image.

Question 2 Answer

In the previous section we listed general sources of error related to the problem at hand (illumination and material variability effects), as well as sensor specific sources or error, some of which were specific to HYDICE (detector noise, heartbeat, absolute calibration error, FCU non-uniformity, spectral jitter, and effects due to the rugate filter transmission). In this section we address some of the more important sources and how we might design experiments to test their impact on detector performance.

Many of the sources of error listed above change the spectral variability of the data, some more than others. We can do an error propagation for each one to develop an overall “variance change” equation for the spectra or we can simply test the overall effects by taking note of the impact random variation has on detection performance.

Noise Sensitivity Study for GMTMF

In order to assess the impact of random noise on the algorithm, we first would have to define a test image. This image would have linear mixed pixels consisting of $p$-bands by $n$-pixels. Actually, the spatial location of the mixed pixels has no impact on detector performance. We conjure one up purely for aesthetic reasons. One such image that has been put together can be seen in Figure 9.

Here we have put together 100 simulated pixels (10 × 10) such that each pixel is
a linear combination of \( \ell \)-materials plus additive Gaussian noise \( \epsilon \). The \( j \)th pixel (for \( j = 1,2,\ldots,n \)) would then be mixed as

\[
\text{pixel}_j = \sum_{i=1}^{\ell} a_i a_i + \epsilon_i
\]

(12)

where \( a_i \) is an end-member for material \( i \) and \( a_i \) is its fraction or weight. After defining each pixel we can generate a set of end-members for the projection matrix \( P_{\perp B} = I - BB^\dagger \). This projection matrix can then be used to compute the \( y \) term in the GMTMF algorithm.

We then need to apply the same noise to the target space vectors as \( t = t + \epsilon \). We then normalize \( t \) per the algorithm with the quantity \( \beta \) to obtain the vector \( y_t = t\beta^{-1} \).

The target and background (normalized) vectors are then used to obtain the abundance variable, \( a \). We obtain our infeasibility metric as

\[
\hat{z} = \frac{\|z\|}{\|r\|}
\]

(13)

where \( z^2 = e\beta^{-1} - a^2 \) and \( r = a\sigma_t + (1 - a)\sigma_z \). The standard deviations come from the variability in the (normalized) target and background spaces, respectively.

We can then generate a plot of the infeasibility scores versus the abundance estimates. In this plot we can “trim” the resultant pixels by selecting only those that have both a high abundance and low infeasibility. Furthermore, we can define the threshold for the infeasibility as \( 1\sigma, 2\sigma, etc. \).

During the mixing process, we can mix a single material (e.g., concrete) throughout the scene to have a range of fractions from 0.01 to 1 (1 to 100%). This would be the “target” pixel used in an ROC curve.

Finally, we can generate ROC curves for the target of interest as a function of additive Gaussian random noise.

**Spectral Shift Study for GMTMF**

Assuming we had a detector noise-free system, we would still want to study the affects of errors due to spectral shift. One simple way to study this effect would be to spectrally shift the target vectors in a predictable manor. Each wavelength value \( \lambda \) in vector \( t \) could be shifted to a new value \( \lambda_n \) as

\[
\lambda_n = \lambda + \varpi s
\]

(14)
where \( \varpi \) is the bandwidth and \( s \) is the fractional amount of shift (relative to the bandwidth). We could test the affects of the shift by incrementing \( s \) from 0 - 1. At a maximum shift of 1, we find that the spectrum is shifted by the amount of the bandwidth. For this study we choose not to shift anymore than this, for in a previous study on shift [2] using the HYDICE sensor, the authors found that there was no impact on target detection performance when the target spectra was shifted by 40% of the channel bandwidth, which was the largest amount observed in the data. However, our model lets us shift up to 100% of the channel bandwidth.

Once the spectra is shifted we could use the same methodology as in the random noise experiment described above. That is, we would use the same simulated data set and obtain the same parameters necessary for the algorithm. Impact on the amount of shift, as a function detection probability, could then be summarized in similar ROC curves perviously discussed.

**Other Comments**

Tests for the effects of illumination change are much more difficult to perform than say random noise testing for it would involve iterative interactions with the atmospheric propagation model. Since we know the TOD (thus the illumination angle), we just have to bracket the error associated with the change in Sun angle. We could compute two target spaces, one for each extreme angle in our bracket estimate, and use this result in our detection algorithms. We could then compare ROC curves for each case.

Material variation would involve altering the reflectance parameter in the physics-based equation, which generates our sensor-reaching radiance spectra. From this we would obtain multiple target spaces to use in the detection algorithm. We could generate ROC curves as a function of material reflectance.

Perhaps a more elegant method would be to assign a variation to the reflectance parameter in the physics-based equation. We would then propagate this variance through the target space so that we have a variance for each radiance spectra (similar to the noise experiment). We could then compute ROC curves as a function of this variability.
3 Questions from Dr. Rhody

These questions are closely related to approximation and projection.
Let \( X \) and \( Y \) be Hilbert spaces, and let

\[
A : X \to Y \\
B : Y \to X
\]

be bounded linear operators with satisfy

\[
\langle Ax, y \rangle = \langle x, By \rangle
\]

for all \( x \in X \) and \( y \in Y \).

**Question 1** Show that the vector \( x \) of minimum norm, that satisfies \( Ax = y \), is given by

\[
x = Bz
\]

where \( z \) is any solution of \( (AB)z = y \).

*Proof.* Let \( n \) be in the null space of \( A \) (\( n \in N(A) \)). If, \( x_r \) is a solution of \( Ax = y \), then so is \( x_r + n \). Since \( N(A) \) is closed (because \( X \) is a Hilbert space), there must be a unique vector \( x \) of minimum norm satisfying \( Ax = y \). This solution must be orthogonal to \( N(A) \) through the orthogonality condition \( x \perp N(A) \). Thus

\[
x \in [N(A)]^\perp = \mathcal{R}(B)
\]

by definition. Hence \( x = Bz \) for some \( z \in Y \). Since \( Ax = y \), we must have \( (AB)z = y \). \( \square \)
Question 2  Show that the matrix equation $A\mathbf{x} = \mathbf{d}$ has a solution if and only if $\mathbf{v}^H \mathbf{d} = 0$
for every vector $\mathbf{v}$ such that $B\mathbf{v} = 0$.

Proof. The proof of this is known as the Fredholm alternative theorem and is related to the existence of the solution to a linear equation. What we set out to show is that the equation $A\mathbf{x} = \mathbf{d}$ has a solution if and only if $\langle \mathbf{d}, \mathbf{v} \rangle = 0$.

Assume that $A\mathbf{x} = \mathbf{d}$, and let $\mathbf{v}$ be in the left nullspace ($\mathbf{v} \in \mathcal{N}(B)$). Then through substitution we have

$$\langle \mathbf{d}, \mathbf{v} \rangle = \langle A\mathbf{x}, \mathbf{v} \rangle = \langle \mathbf{x}, B\mathbf{v} \rangle = \langle \mathbf{x}, \mathbf{0} \rangle = 0.$$ 

\[\square\]

Question 3  Suppose that an equation $A\mathbf{x} = \mathbf{d}$ has no solution. Show that the solution $\hat{\mathbf{x}}$ that minimizes

$$\|A\hat{\mathbf{x}} - \mathbf{d}\|$$

satisfies

$$BA\hat{\mathbf{x}} = B\mathbf{d}.$$ 

The norm $\| \cdot \|$ is the induced norm.

Proof. The stated question (and following proof) provide an alternative view to the pseudoinverse or least-squares operator. If the equation $A\mathbf{x} = \mathbf{d}$ has no solution, then $\mathbf{d}$ is not in the column space (range) of $A$. That is $\mathbf{d} \notin \mathcal{R}(A)$. Minimizing $\|A\mathbf{x} - \mathbf{d}\|$ is equivalent to minimizing $\|\hat{\mathbf{d}} - \mathbf{d}\|$, where $\hat{\mathbf{d}} \in \mathcal{R}(A)$. By the projection theorem, we have

$$\hat{\mathbf{d}} - \mathbf{d} \in \left[\mathcal{R}(A)^\perp\right].$$

Furthermore,

$$\hat{\mathbf{d}} - \mathbf{d} \in \mathcal{N}(B)$$

since the range of $A$ is the orthogonal complement of the left nullspace $\mathcal{N}(B)$. That is

$$\left[\mathcal{R}(A)^\perp\right] = \mathcal{N}(B).$$
We can rewrite this using the operator $B$ as

$$B(\hat{d} - d) = 0$$

which reduces to

$$B\hat{d} = Bd.$$  

Upon substitution we can write

$$BA\hat{x} = Bd$$

since $A\hat{x} = \hat{d}$. This simplifies to

$$BA\hat{x} - Bd = 0$$

We now see that $A\hat{x} - d \in N(B)$. This says that the error will be orthogonal to the range of $A$, $(\mathcal{R}(A))$ and has minimal length by the projection theorem.

**Question 4** Relate the above to the pseudoinverse of $A$.

*Description.* The pseudoinverse operator $A^\dagger$ takes a point from the subspace $Y$ back to a point in the row space such that $\hat{x} \in \mathcal{R}(B)$ with $\hat{x}$ having minimum norm or shortest length. That is, the operator maps $d$ to $\hat{x}$ for each $d \in Y$. 

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4 Questions from Dr. Schott

Your proposal implies the use of physics based modeling (PBM) to predict target behavior. Your algorithms require a statistical description of the target space.

Question 1 How do you propose to use PBM to predict the target space?

Question 1 Answer

Traditionally, target detection algorithms rely on atmospherically compensated imagery. Furthermore, many of the (non-anomaly) algorithms assume the target is deterministic and does not vary. In the event that one wishes to include target variability, it is usually in the form of variation due to image or sensor noise.

An algorithm such as OSP projects a known target against all the pixels in the image. As a result, the algorithm generates a per-pixel value that is proportional to the target energy in that pixel. If variability is important, then one would either have to generate multiple reflectance’s (if atmospherically compensated) of the target, find other image-derived instances of the target in the scene (if used on radiance data), and/or add variability due to noise effects.

The physics-based modeling approach circumvents this by naturally generating $M$-instances of how the target may appear at the sensor. We can now use the same type of OSP algorithm in conjunction with our new physically generated target “space”. The question then arises as how to proceed when you have a target space to contend with?

Defining a PBM Target Space

Prior to the PBM approach, one would take a single target and compute an (OSP-type, for example) statistic for all $N$-pixels in the image. At this point a threshold would be implemented to determine which pixels were like the target and which were not. With the PBM method, however, we have a choice on how to integrate the target space into the algorithm.

A simple definition of the target space would be to compute a mean vector $\mu_t$ from the space and use it in the projection. This would generate $N$ detection statistic values. This can be useful in the event that the target space does not vary much. However, this seems fairly simplistic given that we should take advantage of the space that describes the target. After all, we go through the entire process of physics-based modeling only
to use a single vector? If calibration error persist, we will surly miss the location or the actual target in space all together. Furthermore, if the target space variability is high or is stretched in multiple directions, unequally, the a centroid representation is clearly not the best definition to use.

A brute force approach might be to project every vector in the target space against every pixel in the image. This would generate $N \times M$ detection statistic values. This has the advantage of testing every physics-based target vector against the image. However, if there is a high degree of correlation in the target space (which there probably will be), then this method of processing is overkill.

A more elegant approach might be to take advantage of the convexity of the target space. We could use an algorithm such as MaxD to locate vertices around the target space hull. The number of vectors here would be significantly less than those of the entire space. However, the MaxD method is aimed at finding vertices or end-members around the hull. This is not exactly what we want. We would like to stay close to the mean vector if possible for it is our best guess as to what the actual target looks like.

The last, most appropriate definition, would be to choose the mean vector and a set of vectors around it. We can find the vectors around the mean by setting a statistical threshold (e.g., $1\sigma$) in the target space. For example, if we are confident in our target space description, then perhaps we only need to select vectors that are within $1\sigma$ of the mean. On the other hand, if we feel that calibration errors persist or think that the target space is not exactly correct (or shifted off) then we may want to be more liberal and select those vectors within $2\sigma$ of the mean. Furthermore, if computation time is a factor, then we could randomly sub-set our selection of vectors within the given threshold (sigma) definition.

**Question 2** How will you predict/characterize the statistics of the target space?

**Question 2** **Answer**

The previous paragraphs segued into an overall description/characterization of what the the target space should look like. We proposed a handful of descriptions settling on one that included the mean vector and a set of vectors around it. The “set” of vectors was obtained based on a user provided standard deviation. Once we have this description we can proceed with the projections on background pixels to obtain detector statistic values or abundances.
Question 3 Given these statistics how can this be incorporated into your proposed algorithmic approach?

Question 3 Answer

As previously stated, once we have a target space description we use the target vectors to project against background pixels. In the following section we describe how to use two representations of the target in the proposed algorithm.

Using a Single Mean, \( \mu_t \)

One description of the target space involved utilizing a single mean vector. Though simplistic in its description, we proceed with the utilization of this method so as to set the stage for a more generalized technique called the \( K + 1 \) approach.

Once the mean \( \mu_t \) is defined we can generate \( N \)-abundance values using

\[
a_i = \frac{\mu_t^T P_B^\perp x_i}{\mu_t^T P_B^\perp \mu_t}.
\]

(15)

for \( i = 1, 2, \ldots, N \). From this we can generate \( N \)-orthogonal \( z \) values from

\[
z_i = (e_i \beta^{-1} - a_i^2)^{1/2}.
\]

(16)

This builds up our distribution of how unlike the target the background pixels are. Assuming the distribution is normal, we can describe it with a mean and standard deviation, \( \sigma_z \).

At this point we have abundance estimates as well as \( z \) estimates, with its corresponding distribution. However, we do not have a single variance estimate for the target vector, \( \mu_t \).

This is because we selected only one target vector to project against. The original target space does have a covariance, \( \Sigma_t \), however, we can’t use it because it is band dependant.

Our algorithm has generated a single abundance for each spectral pixel, thus compacting all the spectral nature of a pixel/target combination into a scalar. What we need is to project again many target vectors so as to generate a target space standard deviation, \( \sigma_t \).

Using the \( K + 1 \) Approach

Thus far we have described the target space with a single vector being the mean. More than likely, the actual target will not be the mean but rather (hopefully) close by. We would like to include more vectors around the mean to get a better approximation of the
true target location. We can do this by including \( K \) additional vectors located within 1\( \sigma \) from the mean bringing our total to \( K + 1 \) target vectors.

We proceed with the projections, as previously defined, generating \( K \cdot N \) abundance values. That is

\[
a_{i,j} = \frac{t_j^T P_B x_i}{t_j^T P_B t_j},
\]

(17)

for \( i = 1, 2, \ldots, N \) and \( j = 1, 2, \ldots, K \). From this we can generate \( K \cdot N \)-orthogonal \( z \) values from

\[
z_{i,j} = (e_i \beta_j^{-1} - a_{i,j}^2)^{1/2}
\]

(18)

which generates a larger (possible more diverse) \( z \) distribution. Furthermore, we can generate a target distribution (\( \sigma_t \)) using \( K + 1 \) target vectors. We calculate our infeasibility metric described as

\[
INF_{i,j} = \frac{\|z_{i,j}\|}{(a_{i,j} \sigma_t + (1 - a_{i,j}) \sigma_z)^p}.
\]

(19)

At this point we will have a “cube” of abundances and corresponding infeasibility values. Next we order the abundances and perform our infeasibility check. All things being perfect, we would expect the first \( t \)-values to be target. In the event that this is not the case, we examine the FA’s and compare them to their corresponding infeasibility value. If the infeasibility value is large (a user defined standard deviation), then we eliminate that pixel from the ROC curve calculation and deem it not to be a valid false alarm. That is, the fact that the projection yields a large abundance does not necessarily mean the pixel is target-like. What we strive for is pixels with both large abundances and low infeasibility values.

We would be performing our check on a matrix of numbers. If we find a FA in the matrix with high abundance and high infeasibility, we eliminate it, as previously mentioned.

After we have “trimmed” the individual abundances, we average them across \( K \)-columns to generate a new abundance vector, \( a_n \). This new vector is then re-ordered. The re-ordering is necessary in the event that many FA’s were thrown out for a particular pixel, which would actually change the abundance value after averaging. We could then use this to generate our ROC curve.
References


