Vectors: We will for the moment deal with 1D and 2D cases.

A **scalar** is a quantity that has a **size, but no direction**. A scalar can be **positive or negative**. Scalar arithmetic is the usual stuff you learned through grade school: addition, subtraction, multiplication, division, and raising to a power. We can also take the absolute magnitude of a scalar. Normal algebraic symbols (x, y, t, m) are used for scalars. Scalar quantities in physics include mass, time, energy, charge, and temperature.

A **vector** is a quantity in which magnitude and direction are both important. Vector quantities in physics include velocity, force, momentum, and electric field. We need

(a) ways to draw vectors on a diagram,

(b) ways to quantify vectors with numbers, and

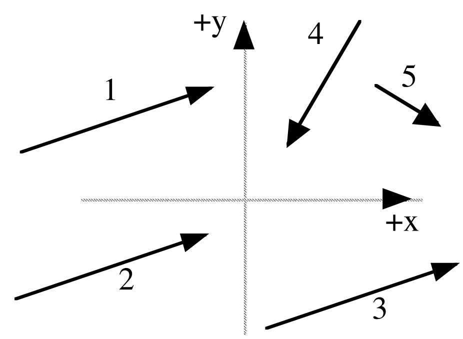
(c) methods to do arithmetic operations on vectors—these include "addition", "subtraction", and "multiplication", we use the same words as for scalars but define the operation differently.

Geometrical Representation of Vectors

It is easy to describe vectors geometrically with arrows. The magnitude of the vector is indicated by its length, and the direction is the direction on the paper. The algebraic symbol used to indicate a vector is a letter with an arrow above it. In advanced texts a vector is often indicated by making the symbol boldfaced. In our text $\vec{r}$ is used as a symbol, other texts would use ***r*.**

<u>Location of a vector is immaterial</u>—only the length and direction count. We will adopt the concepts of Quadrants from geometry, but what is important is in which quadrant a vector points, not where it is located.

Several vectors are shown to the right, along with a coordinate system.

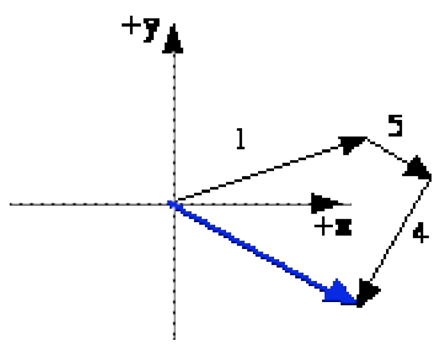


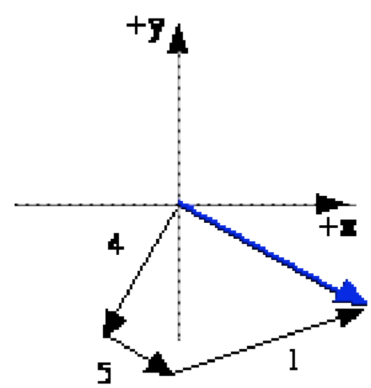
What is the quadrant for each vector?

Are any of the vectors equal—if so which?

Adding vectors geometrically—Head-to-tail

We define addition geometrically as placing vectors one after the other with the head of one attached to the tail of the next. The sum vector extends from the first tail to the last head. The order does not matter. Two representations of the addition of the same vectors are shown.





Geometrical Multiplication of a vector by a scalar.

Positive scalar: The direction is unchanged, the length is multiplied by the scalar.

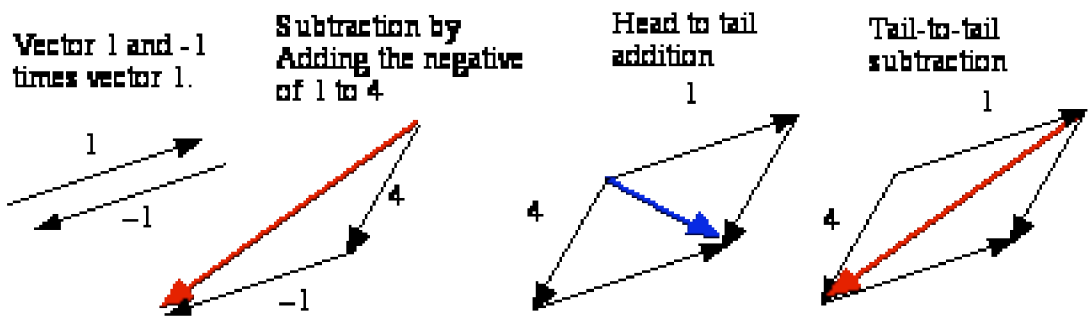
Negative scalar: The direction is reversed, the length is multiplied by the magnitude of the scalar.

Geometrical Subtraction of vectors

There are two general methods:

Head-to-tail: Multiply one of the vectors by (-1) and add the results. This is shown in the left two diagrams.

Tail-to-tail: Place the vectors tail to tail, then draw the difference starting from the tip of the one that is subtracted, and going to the tip of the other. The right hand diagrams show addition and subtraction.



Vector Components: Now we want to put numbers to vectors, again I will stick with 2D vectors.

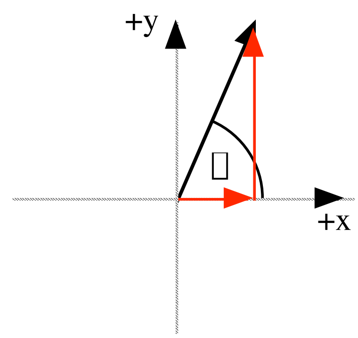
Polar representation:

$\vec{V}$ *represented by a magnitude* (*positive by definition*), $\left|\vec{V}\right|$

and a direction, measured from the positive x-*axis*, θ

or $\left(\left|\vec{V}\right|, \theta\right)$.

An example would be (15 m,30°). The usual convention is used where positive angles are measured counter-clockwise.



Cartesian Representation

A vector can be broken up into the sum of two vectors, one parallel to the x-axis, one parallel to the y-axis.

The scalar lengths of the two vectors above are given from trigonometry, in equation 3-5 of text. These are called the components of the vector, and can be positive, negative, or zero.

$a_x = |\vec{a}|\cos\theta \quad and \quad a_y = |\vec{a}|\sin\theta$ (**Polar to Cartesian** conversion)

The Cartesian representation of a two dimensional vector is the two components, (a_x, a_y)

It is more common in physics and engineering to write the Cartesian form with unit vectors,

$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

The unit vectors have magnitude 1.

Cartesian to Polar Conversion. If we know the components we get the polar form from

$|\vec{a}| = +\sqrt{a_x^2 + a_y^2}$ and $\tan\theta = \dfrac{a_y}{a_x}$. There is one remaining wrinkle: inverse tangent is a function that returns two distinct answers, the (angle) and the (angle + pi). Calculators return only one of these, it is up to you to determine the proper quadrant and pick the proper angle.

$$\vec{a} = (-3.0\,cm)\hat{i} + (-6.0\,cm)\hat{j}$$

e.g. $|\vec{a}| = 6.7\,cm, \tan\theta = 2.0, \theta = 63°$

$$But\ the\ vector\ must\ be\ quadrant\ 3\ (both\ negative),\ so\ \theta = 243°.$$

Multiplication of two vectors

Later in the course we will define the scalar product also called the dot product. This takes two vectors and produces a scalar.

At the end of the course we will define the vector product or cross product, that produces a vector from two other vectors.

Division of two vectors is not defined.