ROCHESTER INSTITUTE OF TECHNOLOGY
Rochester, New York

COLLEGE of SCIENCE
CHESTER F. CARLSON CENTER for IMAGING SCIENCE

New Course: LINEAR MATHEMATICS FOR IMAGING - SIMG xxx

1.0 TITLE: LINEAR MATHEMATICS FOR IMAGING
DATE: 6 April 2004
CREDIT HOURS: 4
PREREQUISITE(S): 1016-305

COREQUISITE(S): none
COURSE PROPOSED BY: Roger L. Easton, Jr.

2.0 COURSE INFORMATION:

	Contact Hours	Maximum Students / Section
Classroom	4	25
Lab	n/a	n/a
Studio	n/a	n/a
Other	n/a	n/a

QUARTER(S) OFFERED:
___Fall **X** Winter ___Spring ___Summer

STUDENTS REQUIRED TO TAKE THIS COURSE:
Imaging Science, 2nd year

STUDENTS WHO MIGHT ELECT TO TAKE THE COURSE:
Applied Mathematics, Physics, Imaging and Photographic Technology

3.0 GOALS OF THE COURSE:

The goals of this course are: (1) to develop the concepts of complex numbers, vectors, and matrices for use as representations of discrete models of linear imaging systems, and (2) to derive the alternative representation of a discrete imaging system obtained via the discrete Fourier transform. The concepts introduced in this course will be applied in the subsequent courses in the imaging science curriculum.

4.0 COURSE DESCRIPTION:

This course develops the concepts of complex numbers and linear algebra for describing imaging systems.
(1016-305) Class 4, Credit 4 (S)

5.0 POSSIBLE RESOURCES:

- 5.1 *Linear Mathematics and its Application to Imaging*, R. L. Easton, Jr.
- 5.2 *Linear Algebra and its Applications*, Gilbert Strang.

6.0 TOPICS:

- 6.1 Mathematical Representations of Imaging Tasks: $g[x,y]=\mathbf{O}\{f[x,y]\}$
  - 6.1.1 "Direct" Imaging Problem: given **O**, $f[x,y]$, find $g[x,y]$
  - 6.1.2 "Inverse" Problem: given **O**, $g[x,y]$, find $f[x,y]$
  - 6.1.3 "System Analysis": given $f[x,y]$ and $g[x,y]$, find **O**
  - 6.1.4 Representations of objects, images, and imaging systems as arrays and operations.
- 6.2 Review of complex numbers and their geometric interpretation
  - 6.2.1 arithmetic of complex numbers
  - 6.2.2 Euler relation
  - 6.2.3 Argand diagram
  - 6.2.4 phasors
- 6.3 Representations of images as vectors
  - 6.3.1 lexicographic ordering of a 2-D sampled image to create a 1-D vector
- 6.4 Projections of image vectors onto reference image vectors, basis vectors.
  - 6.4.1 scalar product of two vectors with real-valued components
  - 6.4.2 definition of the length of a vector as scalar product
  - 6.4.3 projections of vectors onto other vectors
  - 6.4.4 representation of an image vector via projections onto sets of basis vectors
  - 6.4.5 representation of set of basis vectors as matrix
  - 6.4.6 matrix-vector multiplication as set of projections
  - 6.4.7 matrix-matrix multiplication
  - 6.4.8 identity matrix
  - 6.4.9 matrix transpose
  - 6.4.10 matrix inverse (solution of "inverse" imaging problem)
- 6.5 Rotations of image vectors
  - 6.5.1 rotation matrices
- 6.6 Representations of imaging systems as matrices
  - 6.6.1 linear shift-variant systems, output image as matrix-vector product
  - 6.6.2 linear shift invariant systems, convolution, circulant matrices
- 6.7 Eigenvectors and eigenvalues
  - 6.7.1 action of matrix on eigenvector as length change
  - 6.7.2 evaluation of eigenvectors of a matrix
  - 6.7.3 diagonalization of matrices
  - 6.7.4 representation of a vector as projections onto eigenvectors
- 6.8 Diagonalization of linear shift-invariant (LSI) imaging systems (circulant matrices)
  - 6.8.1 discrete Fourier transform (DFT), Fourier Analysis
  - 6.8.2 Inverse DFT, Fourier synthesis
  - 6.8.3 Solution of the inverse imaging task for LSI systems
  - 6.8.4 Pseudoinverses of imaging systems, "null" input images
  - 6.8.5 System Analysis of an LSI (circulant) matrix
- 6.9 Generalization of discrete case to continuous coordinates

6.9.1 Continuous form of the Fourier transform
6.9.2 Continuous form of the inverse Fourier transform

7.0 INTENDED LEARNING OUTCOMES AND ASSOCIATED ASSESSMENT METHODS OF THOSE OUTCOMES:

7.1 Understanding of the application and value of the alternative representation in the frequency domain of images represented as vectors and of imaging systems represented as matrices that is obtained via the discrete Fourier transform. (HOMEWORK ASSIGNMENTS / EXAMS)

7.2 The student will be prepared for the upper-level required and elective courses in imaging science that require understanding of these concepts.

8.0 PROGRAM OR GENERAL EDUCATION GOALS SUPPORTED BY THIS COURSE:

8.1 The student will have the basic set of mathematical tools for deriving and interpreting the frequency-domain representation of images and imaging systems.

9.0 OTHER RELEVANT INFORMATION:

9.1 Course must be scheduled in a classroom equipped with a high-resolution projector (1280×1024) for classroom instruction

10.0 SUPPLEMENTAL INFORMATION:

none