This presentation will consist of two parts, each on an area of optics in which the geometric construction known as the Poincaré sphere can be employed: polarization and beam intensity structure. The first part relates to the measurement of polarization through the use of an optical element displaying a continuously varying birefringence distribution caused by a trigonal stress pattern. This element serves to convert a beam with uniform arbitrary polarization (represented by a point in the polarization Poincaré sphere) onto a beam that is a combination of two simple modes (represented by the same point in the intensity-structure Poincaré sphere). When inserted into the pupil plane of an imaging system, this element induces a point spread function (PSF) that codifies the polarization information, so that the imaging system becomes a single-shot imaging polarimeter (when applied to sparse objects for which the PSFs can be resolved). The second part of the talk will give a generalization of the use of the Poincaré sphere to characterize high-order cavity modes, including Hermite-, Laguerre-, and Ince-Gaussians amongst many others. A ray-optical description will be discussed, in which any beam of this type is represented by a closed loop on the surface of the Poincaré sphere. It will be shown that the caustic structure of the modes is related to the abstract representation over the Poincaré sphere through simple geometry. Other types of beams, such as Bessel, Airy, and Mathieu beams can be considered as limiting situations of the cases discussed here.