R·I·T

2013 Imaging Science Ph.D. Comprehensive Examination
June 14, 2013
9:00AM to 1:00PM

IMPORTANT INSTRUCTIONS

You must complete two (2) of the three (3) questions given for each of the core graduate classes. The answer to each question should begin on a new piece of paper. While you are free to use as much paper as you would wish to answer each question, please only write on one side of each sheet of paper that you use. Be sure to write your provided identification letter (located below), the question number, and the page number for each answer in the upper right-hand corner of each sheet of paper that you use. When you hand in your exam answers, be certain to write your name on the supplied 5" x 8" note card containing your provided identification letter and place this in the small envelope, and then place this envelope along with your answer sheets in the large envelope.

ONLY HAND IN THE ANSWERS TO THE QUESTIONS THAT YOU WOULD LIKE EVALUATED

Identification Letter: ________________

THIS EXAM QUESTION SHEET MUST BE HANDED BACK TO THE PROCTOR UPON COMPLETION OF THE EXAM PERIOD

R·I·T

2013 Imaging Science Ph.D. Comprehensive Examination
June 14, 2013
2:00PM to 5:00PM

IMPORTANT INSTRUCTIONS

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ONLY HAND IN THE ANSWERS TO THE QUESTIONS THAT YOU WOULD LIKE EVALUATED

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THIS EXAM QUESTION SHEET MUST BE HANDED BACK TO THE PROCTOR UPON COMPLETION OF THE EXAM PERIOD

1. **Fourier Methods in Imaging**

   For the two two-dimensional functions:

$$
\begin{aligned}
f_1[x,y] &\equiv \exp\left[+i\pi\left(\left(\frac{x}{3}\right)^2+\left(\frac{y}{4}\right)^2\right)\right] \\
f_2[x,y] &\equiv \exp\left[+i\pi\left(\left(\frac{x}{4}\right)^2+\left(\frac{y}{3}\right)^2\right)\right]
\end{aligned}
$$

   (a) Evaluate the spatial frequencies of these functions along the x- and y-axes.

   (b) Evaluate the real part, imaginary part, magnitude, and phase of the 2-D convolution $g[x,y] = f_1[x,y] * f_2[x,y]$ and plot the lines of constant phase of $g[x,y]$ for the same values as in part (a)

2. **Fourier Methods in Imaging**

Evaluate the result of the operation

$$\exp\left[-i\pi\left(\frac{x}{\alpha_0}\right)^2\right] * \exp\left[+i\pi\left(\frac{x}{\beta_0}\right)^2\right]$$

where α_0 and β_0 are real-valued parameters. Your answer should depend on α_0 and β_0, so be sure to specify the result in the cases where $\alpha_0 = \beta_0$ and $\alpha_0 \neq \beta_0$

3. **Fourier Methods in Imaging**

Consider the 1-D and 2-D convolutions of two quadratic-phase functions with different scale factors.

(a) In the 1-D case, evaluate

$$\exp\left[+i\pi\left(\frac{x^2}{\lambda_0 z_1}\right)\right] * \exp\left[+i\pi\left(\frac{x^2}{\lambda_0 z_2}\right)\right]$$

in terms of $\exp\left[+i\pi\left(\frac{x^2}{\lambda_0\left(z_1+z_2\right)}\right)\right]$

(b) Repeat for the 2-D case, i.e., evaluate

$$\exp\left[+i\pi\left(\frac{x^2+y^2}{\lambda_0 z_1}\right)\right] * \exp\left[+i\pi\left(\frac{x^2+y^2}{\lambda_0 z_2}\right)\right]$$

in terms of $\exp\left[+i\pi\left(\frac{x^2+y^2}{\lambda_0\left(z_1+z_2\right)}\right)\right]$

Background Information for the Following Radiometry Questions

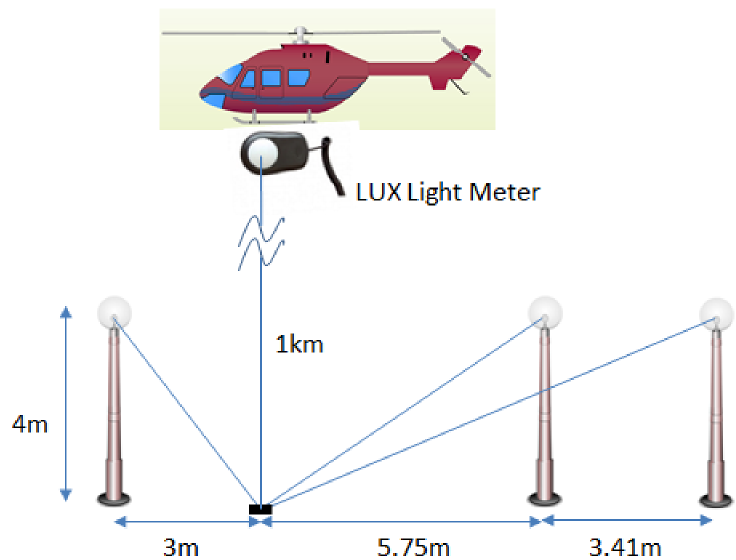
Useful Constants

$A = 2898\ [\mu m\ K]$
$\sigma = 5.67 \times 10^{-8}\ [W\ m^{-2}\ K^{-4}]$
h $= 6.624 \times 10^{-34}\ [J\ s]$
c $= 3.0 \times 10^{8}\ [m\ s^{-1}]$
e $= 1.60217 \times 10^{-19}$ [coulombs]
k $= 683\ [lm\ W^{-1}]$
k$' = 1700\ [lm\ W^{-1}]$
Sun Diameter $= 1.9 \times 10^{9}\ [m]$
Moon Diameter $= 3.475 \times 10^{6}\ [m]$
Sun-Earth Distance $= 1.49 \times 10^{11}\ [m]$

Wavelength [nm]	$\mathbf{V}(\lambda)$	$\mathbf{V}'(\lambda)$
520	0.710	0.935
530	0.862	0.811
540	0.954	0.650
550	0.995	0.481
560	0.995	0.329
570	0.952	0.207
580	0.870	0.121
590	0.757	0.065
600	0.631	0.033

4. **Radiometry**

While out galavanting at night in downtown Rochester, I notice a suspicious package in the road illuminated by 3 nearby sodium vapor street lamps (central emission at 590 nm with a bandwidth of 10 nm). From a distance, the package seems to be 50 cm in diameter. I fly over the package with my remote controlled helicopter at an altitude of 1km hoping to somehow estimate its reflectance. Attached to the underside of my helicopter is a Light Meter which produces a signal of 1.165×10^{-5} lux or lumens per meter squared. The bureau of street lighting tells me all lamps in that area draw 2 amps at 110 volts and are typically 50% efficient. To get a better model of the scenario, and ultimately better estimate of the packages reflectance, I find out that that the atmospheric make up in and around my area of interest contains molecules with a cross section of $5 \times 10^{-12}\, m^2$ and a number density of $8 \times 10^7\, m^{-3}$. What is the reflectance of this package?

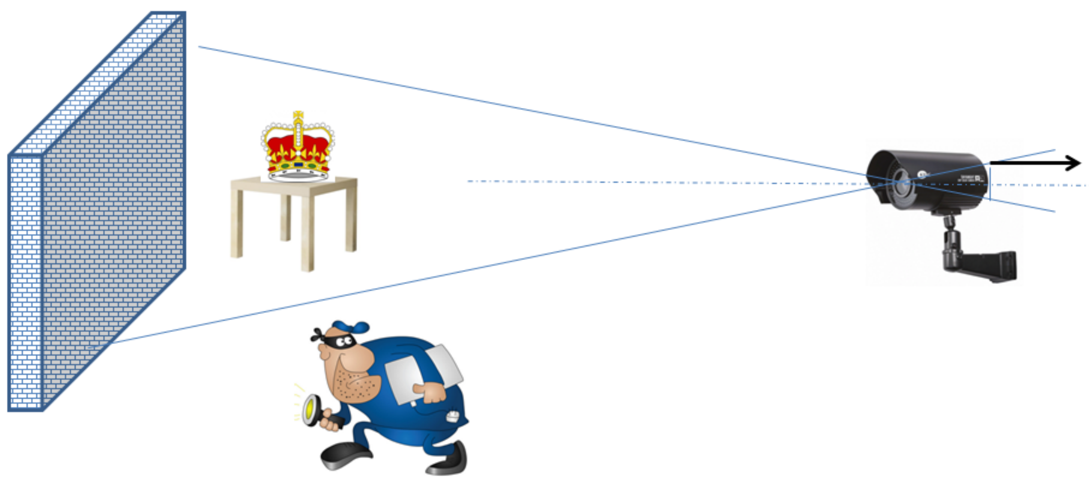


5. **Radiometry**

   You are in charge of estimating the radiance (centered around 500 nm) from the moon using data from the Japanese satellite Kaguya, which convieniently orbits the moon at a distance of 100 km. From specification sheets, you know the on board square detector is 3 cm on a side and integrates for 20 micro seconds with 100 percent quantum efficiency. From the same specification sheet, you see that the noise from the electronics around 3000 electrons. A separate on-board instrument tells you the total (aggregate) noise is 2.662×10^6 electrons. What is the moons radiance (factoring in all sources of noise) and is this system shot noise limited? Finally, what would you estimate the extinction coefficient to be?

6. **Radiometry**

I'm trying to break into the Tower of London to steal the Crown Jewels. Inside information tells me security has an Telops TEL-1000 LWIR camera viewing/monitoring the Jewels. I research the camera and find out it has 98% transmissive optics with a focal length of 40 mm and aperture diameter opening of 1 cm. Its field of view (FOV) is 30 degrees with a detector area of 1 square centimeter, noise level of 500 mA and responsivity of 1000 amps per watt. If am to successfully break in, I need to know the camera will not "see" me. I probably should determine its $NE\Delta T$ just *off-axis* as I enter the FOV, as illustrated in the graphic below. I'll have to guess at some parameters to generate an effective model. I'll use a lens-falloff value of 3.0 and assume everything in the room is at ambient temperature (i.e., 295 K). I know the only thing in the cameras FOV is a brick wall (as shown) in which I find out the emissivity is 0.60. As for me, I know my temperature is 310 K with an emissivity of 0.90. Given this information, in the absence of other noise sources, will I be able to walk away with the Jewels?



7. **Optics for Imaging**

Dielectric Thin Film

The reflection coefficient for an electric field at normal incidence of a dielectric interface is given by $r = (n_1 - n_2)/(n_1 + n_2)$, where n_1 and n_2 are the refractive indexes of the incident and transmitting materials, respectively. Likewise the transmission coefficient of the electric field is given by $t = 2n_1/(n_1 + n_2)$. The field has a wavelength $\lambda = 1\mu m$.

To achieve a greater reflection than a simple air-glass interface, a layer of thickness L of the dielectric material rutile ($n_r = 2.6$) is deposited on the glass ($n_g = 1.5$).

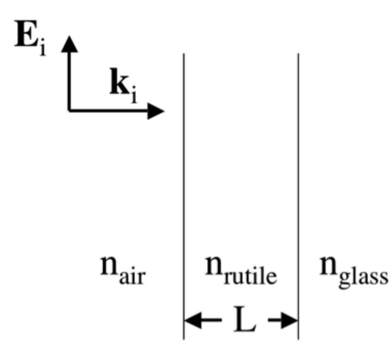
(a) Determine a full expression (in terms of parameters such as refractive indexes, thickness, wavelength) for the complex reflection coefficient (amplitude and phase) of the air-rutile-glass system. (You may assume the glass is infinitely thick.) State any assumptions you make to simplify the problem without losing essential physics.

(b) Insert the values of the refractive indexes into the above expression, paying attention to signs. Determine an expression for the magnitude of the reflection coefficient of the system.

(c) At what values of L is the magnitude of the reflection coefficient a maximum? You must mathematically justify your response.

(d) Compare the value of the reflection coefficient to that of a simple air-glass interface.

(e) What percentage of irradiance is reflected in the two cases in part (d)?



8. **Optics for Imaging**

Van Leeuwenhoek Microscope

Van Leeuwenhoek's interest in microscopes and a familiarity with glass processing led to one of the most significant, and simultaneously well-hidden, technical insights in the history of science. By placing the middle of a small rod of soda lime glass in a hot flame, Van Leeuwenhoek could pull the hot section apart to create two long whiskers of glass. Then, by reinserting the end of one whisker into the flame, he could create a very small, high-quality glass sphere. These spheres became the lenses of his microscopes, with the smallest spheres providing the highest magnifications. An experienced businessman, Leeuwenhoek realized that if his simple method for creating the critically important lens was revealed, the scientific community of his time would likely disregard or even forget his role in microscopy. He therefore allowed others to believe that he was laboriously spending most of his nights and free time grinding increasingly tiny lenses to use in microscopes, even though this belief conflicted both with his construction of hundreds of microscopes and his habit of building a new microscope whenever he chanced upon an interesting specimen that he wanted to preserve. He made about 200 microscopes with different magnification. [source: en.wikipedia.org]

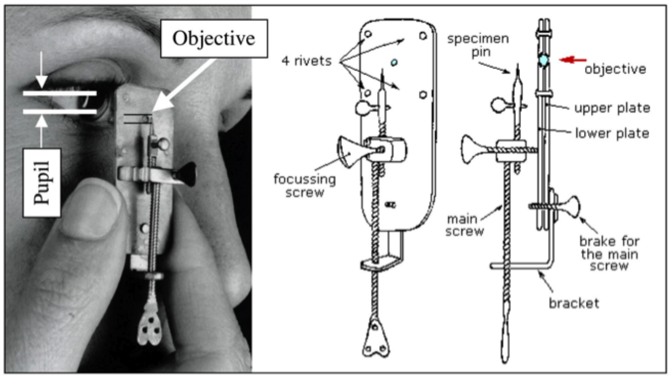
(a) Using the diagrams and photograph below, provide a ray-tracing diagram showing the formation of an image of a microscopic object on the specimen pin on the retina of the eye. Assume the lens and eye form an afocal system (i.e., they share a common focal point). You may treat the ball lens as a thin lens to simplify the ray tracing analysis.

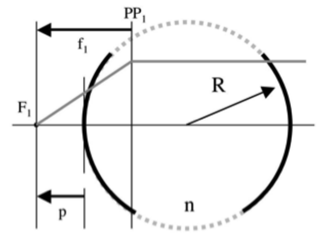
(b) Assume the diameter of the woman′s pupil is 4 mm, and the focal length of her eye is 17 mm. Assume the near point of her eye is $Z_{near} = 0.25$ m. Note that the microscope is held close to the eye. Why is that? Justify your answer with equations and references to the ray tracing diagram.

(c) From your ray tracing diagram and estimates based on the photograph, determine the magnification that can be achieved if the ball lens produces a virtual image at the near point. You must estimate the radius of the ball lens in the Van Leeuwenhoek apparatus shown in the photo. Its focal length is given by $f_1 = nR/(2n-2)$ where $n = 1.5$.

(d) Determine the distance the specimen must be placed from the focal point if the virtual image coincides with the near point of the eye. Express your answer in units of μm.

(e) Determine the magnification (the ratio of the image height on the retina and object height): h_{eye}/h.





Ball Lens

$p = R - nR / 2(n-1)$, $f_1 = - nR / 2(n-1)$

9. **Optics for Imaging**

Camera Obscura

An optical field of wavelength λ, described by the distribution $E(x, 0)$ is incident upon an aperture of radial size a. The beam propagates to the far-field region where a second aperture of radial size b exists, as shown in the diagram. The transmitted beam is finally detected in the far-field region of the second aperture (i.e, in the x'' plane).

(a) Mathematically describe the conditions for the values of a and b where the distances Z_1 and Z_2 satisfy the far-field criterion.

(b) Express the electric field in the far-field plane, x', and provide a definition for the transverse wavevector, k_x, in terms of the variable x'. The field $E(x', Z_1)$ must be written in integral form (not short-hand).

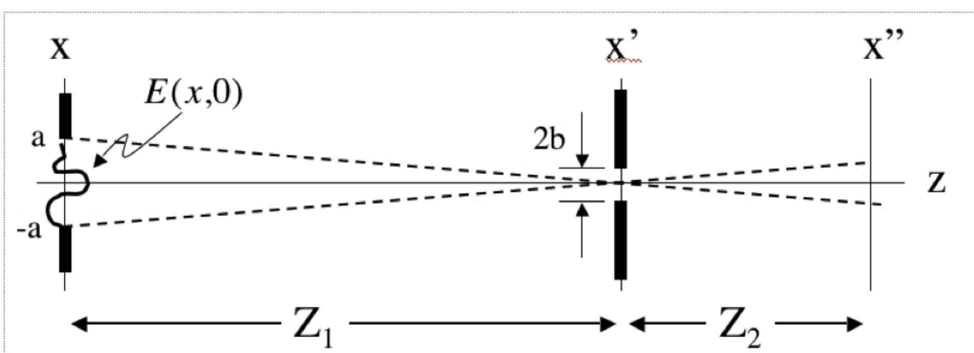
(c) Express the field in the second far-field plane, x'', and provide a definition for the transverse wavevector, k'_x, in terms of the variable x''. The field $E(x'', Z_1 + Z_2)$ must be written in integral form (not short-hand).

(d) Evaluate the integral in (c) if $E(x, 0) = \delta(x - x_1) + \delta(x - x_2)$, where x_1 and x_2 are constant values. Express your answer in terms of the sum of two sinc functions, where the variable x'' is in the argument.

(e) Simplify the expression in (d) for the case where $x_1 = -x_2 = a$ and $Z_1 = Z_2$.

(f) Define the Rayleigh resolution criterion and determine an expression for b (in terms of the other parameters of the system) that would just-resolve the two objects in the image plane, x''.

(g) It should be apparent that if b is too large, the plane x'' does not contain images. Describe the electric field in the plane x'' if b approaches ∞. Evaluate the integral and express the field in terms of trigonometric function(s) having the argument a function of x''.



10. **Probability, Noise, and System Modeling**

An inspection system must determine which of two patterns, illustrated as A and B below, are present under a sensor. The sensor produces the array X of the count for each cell. The count is a Poisson random variable whose expected value is $a = 6$ or $b = 4$, depending upon which pattern is under the sensor. The cell counts are statistically independent.

$A_1 = a$	$A_2 = b$
$A_3 = b$	$A_4 = a$

Pattern A

$B_1 = b$	$B_2 = a$
$B_3 = a$	$B_4 = b$

Pattern B

$X_1 = 4$	$X_2 = 3$
$X_3 = 4$	$X_4 = 6$

Observed X

Patterns A and B and observed array X when $a = 6, b = 4$.

Let A and B represent the events "pattern A" and "pattern B", respectively. It has been found by experience that $P(B) = 2P(A)$.

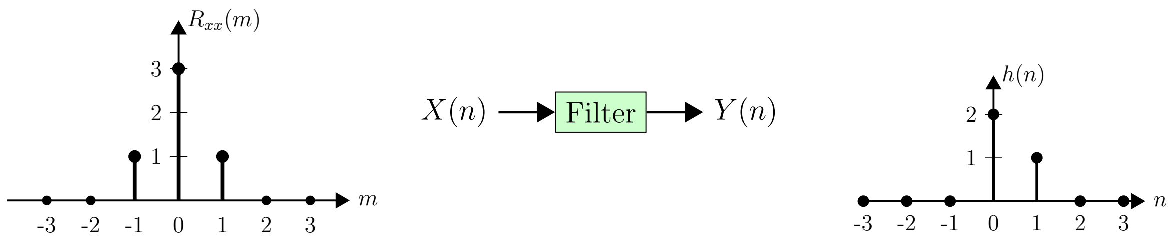
(a) Construct a decision rule that selects the most likely pattern, A or B give an observed pattern X.

(b) Determine the most likely pattern for the data given in the array X above.

11. **Probability, Noise, and System Modeling**

Let $X(n)$, $-\infty < n < \infty$, represent samples of a discrete ergodic random process with $E[X] = 0$. The autocorrelation function is $R_{xx}(m) = E[X(n)X(n+m)]$ is shown in figure (a). This random process is filtered, as indicated in figure (b), by a device with impulse response $h(n)$ shown in figure (c). The output and input are related by

$$Y(n) = 2X(n) + X(n-1)$$



(a) (b) (c)

(a) Determine the variance of X.

(b) Determine the autocorrelation function $R_{yy}(m)$ for $m = 0, 1, 2, 3$.

12. **Probability, Noise, and System Modeling**

You have a generator of independent random numbers u that are uniformly distributed over the interval 0 to 1.

(a) You want i.i.d. random numbers that have a probability density function $f(x)$. Describe a transformation $x = G(u)$ that will produce the desired distribution.

(b) Determine the function $G(u)$ to use to generate exponentially distributed random numbers

$$f_x(x) = \begin{cases} ae^{-ax}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

where a is the parameter that controls the mean value of the distribution.

13. **Digital Image Mathematics**

(a) (40 points) Given the full column rank $m \times n$ matrix $\mathbf{A}$, which contains m measurements of n independent variables, and the $m \times 1$ vector $\mathbf{b}$, which contains m measurements of the dependent variable, derive the form of the $n \times 1$ least-squares solution vector $\mathbf{x}$, which contains the n model coefficients. Begin by defining the cost function $\mathcal{J}_{LS}(\mathbf{x})$ corresponding to the sum-squared error (SSE) criterion.

(b) (30 points) Explain the geometric interpretation of the $\hat{\mathbf{x}}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{A}\hat{\mathbf{x}}$ term in the above derivation, and discuss its significance in obtaining the least-squares solution.

(c) (30 points) Find the pseudoinverse of the singular matrix

$$\mathbf{B} = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$$

14. **Digital Image Mathematics**

(a) (50 points) Given the ellipse

$$c^2 = \frac{x^2}{\sigma_{xx}} + 2\frac{xy}{\sigma_{xy}} + \frac{y^2}{\sigma_{yy}}$$

develop an expression for the ellipse in a new basis with decorrelated variables.

(b) (50 points) For an arbitrary set of observations on an arbitrary set of variables, principal component analysis (PCA) and the discrete Fourier transform (DFT) provide identical sets of *real* basis vectors. Assume that variance decreases with increasing spatial frequency. Sketch the data, the original basis, the PC basis, and the DFT basis, and provide an explanation for your solution.

15. **Digital Image Mathematics**

Given an input function $f(x, y)$, a finite-size detector $d(x, y)$ with detector size $d \times d$, a sampling function $s(x, y)$ with pixel pitch $\Delta x = \Delta y$, and a realistic interpolator $t(x, y)$ with the following forms:

$$
\begin{aligned}
f(x,y) &= SINC^2\left(5x, 10y\right) \\
d(x,y) &= \frac{1}{d^2} RECT\left(\frac{x}{d}, \frac{y}{d}\right) \\
s(x,y) &= \frac{1}{\Delta x \Delta y} COMB\left(\frac{x}{\Delta x}, \frac{y}{\Delta y}\right) \\
t(x,y) &= TRI\left(\frac{x}{\Delta x}, \frac{y}{\Delta y}\right)
\end{aligned}
$$

(a) (80 points) For $\Delta x = \Delta y = \frac{1}{10}$ and $d << \Delta x$, find an expression for the recovered spectrum $\hat{F}(\xi, \eta)$, and discuss any *degradations*. Sketch the result (cross sections are fine) in the frequency domain.

(b) (20 points) Will there be *aliasing* artifacts? Explain.

16. **Digital Image Processing**

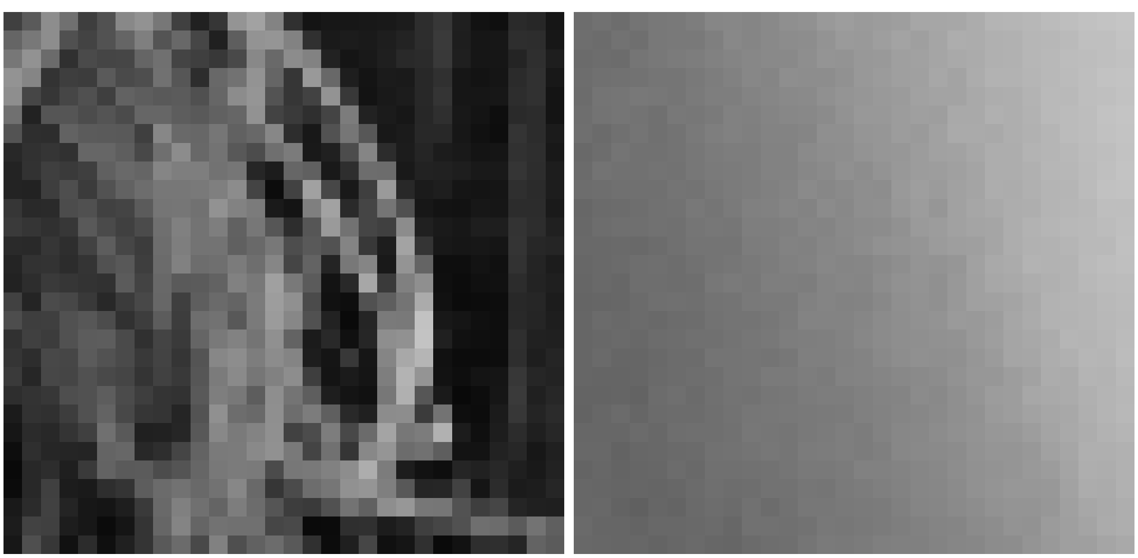
A time-lapse imaging system is set up to document the construction of a new building on the RIT campus. The system gathers an image every 15 minutes throughout the construction process, day and night, every day. The camera in the system is a 35mm DSLR. The folks who implemented the system wanted to produce the most visually pleasing sequence possible so they fixed the aperture at f/8, to maintain a constant depth of focus, and fixed the shutter speed at $\frac{1}{250}$s, to minimize any motion blur that might be present in the workers and machinery in the scene. During a test sequence with the system, they realized that as cloud cover came and went and as nighttime fell upon the scene, that the mean brightness of the image shifted significantly and distracted from the visual appeal they were looking for. So they decided to allow the camera to automatically adjust the ISO setting to maintain a constant mean grey level for each frame.

Once the photographer downloaded the digital photographs from the SD card, it was noticed that there was a dramatic change in the file size for each of the images, both throughout each day, and as time progressed through the construction process. Consider everything that is involved in this collection process and comment on all the possible influences that might cause this changing file size. Be specific.

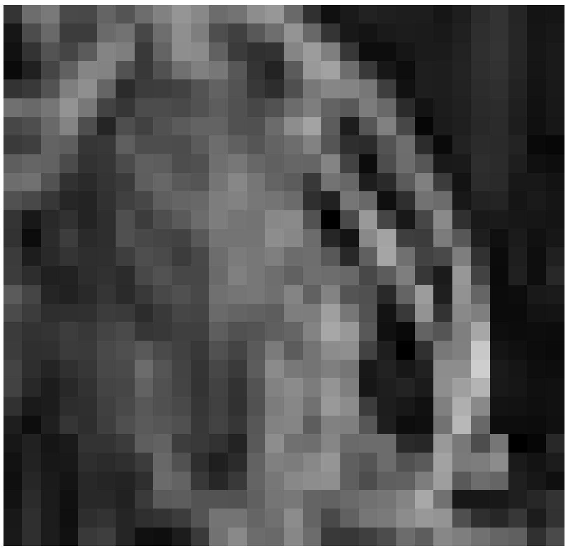
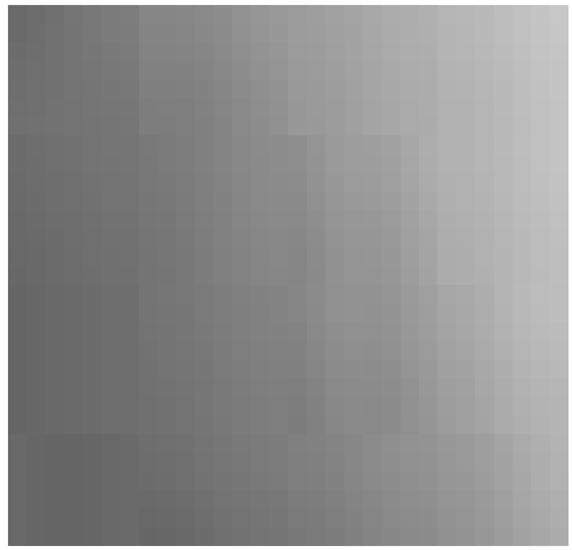
17. **Digital Image Processing**

What would a exam be like without a question involving our favorite image? [*That is not the real question, so read on ...*] There are two highlighted regions on the image of Lenna shown below.

The image was cropped two different times so that only these highlighted portions remained.



These two regions were then saved using the JPEG compression process in Adobe Photoshop with a quality factor of 3. Remember that Adobe Photoshop allows you to select a “Quality” when saving a JPEG-compressed image that ranks from “Maximum (12)” to “Low (3)”. Both of these regions are 32x32 pixels in size. The area outlined in white resulted in a file that required 33,824 bytes to save while the area outlined in black resulted in a file with 32,711 bytes in it. These JPEG files are redisplayed below.

With the information given above and the images that are displayed here for your inspection, explain 1) why the cropped images resulted in files that required varying amounts of disk space, and 2) why the redisplayed images exhibit different levels of artifacts. Be specific in your answers with regards to particular portions of the JPEG compression process as well as your knowledge of the human visual system.

18. **Digital Image Processing**

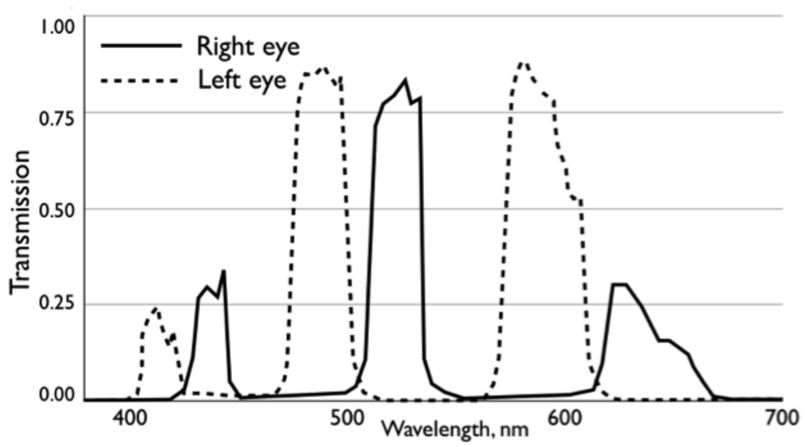
Arithmetic coding plays a critical role in the JPEG2000 compression methodology. Given the histogram for a tile extracted from a digital image,

DC	#
0	1
1	5
2	16
3	18
4	10
5	8
6	4
7	2

determine the three (3) digital counts that are encoded using the floating point value 0.24 (using the conceptual implementation of arithmetic coding described by Gonzalez, 2007).

19. **Human Visual System**

One method for displaying stereo 3D information in motion-picture theatres is to provide glasses to observers in which each eye is fitted with interference filters. Each eye passes red, green, and blue, but in non-overlapping regions. The figure below shows such a system; the left eye has band-pass notches centered at 410, 490, and 590 nm, and the right eye has band-pass notches centered at 440, 525, and 625 nm.



Images for the left and right eyes are projected in a rapid sequence; R-L-R-L$\cdots$

(a) Explain how the human visual system extracts depth information from stereo cues such as those produced by this system.

(b) The two eyes clearly receive very different spectral content. Is it plausible that they could still perceive similar colors, and allow observers to see normal colors on the screen?

(c) If this system was designed for the average human observer, how would you expect it to differ for an anomalous trichromat?

20. **Human Visual System**

It is now possible to buy 4K television monitors for the home market. With a pixel resolution of 3840×2160 pixels, the systems have 4X more pixels than current 1080p displays.

Based on the characteristics of the average human visual system:

(a) What minimum viewing distance would you recommend for a 65 inch diagonal 4K TV? (The display is 144 cm wide and 81 cm high.) Defend your answer.

(b) You work for a startup planning to deliver content for 4K TVs on double-sided, dual-layer DVDs. You still need to achieve $\sim 2\times$ more compression. Propose a conceptual compression scheme that would take advantage of some aspect of the human visual system to achieve this extra compression.

21. **Human Visual System**

Almost all cathode-ray tube (CRT), liquid-crystal display (LCD), plasma, and organic light-emitting diode (OLED) television displays use red, green, and blue (RGB) primaries to reproduce colors. Regardless of which display technology is used, moving images are updated at least 30 times per second, and any significant variation in screen luminance is modulated at a temporal frequency of at least 60 Hz.

(a) Explain why color television systems have three primaries.

(b) Explain why color television systems use RGB primaries. (Include in your explanation an alternative set of primaries, and why RGB is a better choice than your alternative.)

(c) Explain why moving images are updated at least 30 times per second.

(d) Explain why significant luminance variation is modulated at a temporal frequency of at least 60 Hz.