IMPORTANT INSTRUCTIONS

You must complete two (2) of the three (3) questions given for each of the core graduate classes. The answer to each question should begin on a new piece of paper. While you are free to use as much paper as you would wish to answer each question, please only write on one side of each sheet of paper that you use AND STAY INSIDE THE BOX! Be sure to write your provided identification letter, the question number, and a sequential page number for each answer in the upper right-hand corner of each sheet of paper that you use. When you hand in your exam answers, be certain to write your name on the supplied 5” x 8” paper containing your provided identification letter and place this in the small envelope, and then place this envelope along with your answer sheets in the large envelope.

ONLY HAND IN THE ANSWERS TO THE QUESTIONS THAT YOU WOULD LIKE EVALUATED

Identification Letter: ____________

THIS EXAM QUESTION SHEET MUST BE HANDED BACK TO THE PROCTOR UPON COMPLETION OF THE EXAM PERIOD
Core 4: Radiometry, Answer TWO Questions from Questions 10-12

Several constants that you may or may not need in your solutions are provided here for your convenience:

- Planck’s Constant: $h = 6.626 \times 10^{-34} \text{m}^2\text{kg/s}$
- Speed of light: $c = 3 \times 10^8 \text{m/s}$
- Charge of an electron: $e = 1.602 \times 10^{-19} \text{coulomb}$
- Boltzmann Constant: $k = 1.381 \times 10^{-23} \text{J/K}$
- Approximate Solar Temperature: $T_{solar} = 5800 \text{K}$
- Approximate Room Temperature: $T_{room} = 294 \text{K}$
- Earth-Sun Distance: $D_{earth-sun} = 1.4960 \times 10^{11} \text{m}$.
- Radius of the Sun: $R_{sun} = 6.955 \times 10^8 \text{m}$
- Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4}$
- Absolute zero on Kelvin scale converted to Celsius: $-273.15 \degree \text{C}$
10. **Radiometry (10 points).** You are comparing two different collimated, monochromatic light beams using the same detector. In each case, the beam strikes the detector perpendicular to the face of the detector. When the first beam is operating, it has a wavelength $\lambda_1$, and at the detector, it has a radiant flux $\Phi_1$ [W]. When the second beam is operating, it has wavelength $\lambda_2$, and a radiant flux $\Phi_2$ [W] at the detector.

a) **(5 points).** Find an expression for the percentage change in electron shot noise of the detector when the second beam is impinging on the detector, as seen in the Figure below, compared to the shot noise observed when the first beam is impinging on the detector.

b) **(2 points).** Assume that the quantum efficiency of the detector is that shown in the curve with the solid line in the graph in the Figure. Let $\lambda_1 = 400 \text{ nm}$ for the first monochromatic beam and $\lambda_2 = 800 \text{ nm}$ for the second beam, and let the corresponding radiant flux of the two beams be $\Phi_1 = 0.08 \mu W$ and $\Phi_2 = 0.32 \mu W$. Use this information and the graph below to evaluate your answer to part (a) numerically.

c) **(3 points).** For this part, suppose now that the flux of the two beams described in part (b) was recorded during a time interval $t = 1.6 \mu s$. Suppose also that the only other significant source of noise is the read noise of the detector, and assume that the read noise remains the same for both beams with a constant value of $N_{\text{read}} = 620 \text{ electrons}$. Assume also that all other characteristics of the detector are the same as in part (b). With these assumptions, find the signal-to-noise ratio for each beam.

![Monochromatic Light Beam and Detector](image)

Figure for Problem 10: (Top) Light beam strikes the detector at normal incidence. (Bottom) Quantum efficiency vs wavelength (source Princeton Instruments, Inc.)
11. **Radiometry (10 Points).** An isotropic light source is suspended just below the surface of the water column in a shallow region. Assume that the light is a monochromatic point source with radiant flux $\Phi_0 [W]$ and wavelength $\lambda$. Assume that there are no other sources of light in the set-up portrayed in the Figure. As shown in the Figure below, you attach a water-proof detector, with responsivity $R [AW^{-1}]$, to a rigid pole and rod assembly so that the detector is positioned directly below the light source and looking up at the light source at a depth $d$ below the water surface. At that depth, the detector measures a signal $S(d) [A]$, which through the detector calibration, provides an estimate of the flux arriving at the detector at that depth: $\Phi(d) [W]$.

a) **(3 points).** Find a mathematical expression for the extinction coefficient of the water column, using one or more of the given variables.

b) **(1 point).** Assume that the radiant flux of your source is $\Phi_0 = 100 W$, and that at a depth $d = 1.4 m$, the detector records a current signal, which through the detector calibration indicates that the received flux at the detector is $\Phi(1.4 m) = 62.9 W$. Use this information to evaluate numerically the extinction coefficient that you found in part (a).

c) **(3 points).** If the uncertainty in the depth measurement $d$ is 1.5%, the uncertainty in your detector’s measurement of the flux underwater is 2.2%, and the uncertainty in the radiant flux of your source is 3.1%, find a mathematical expression for the uncertainty in your estimate of the extinction coefficient, assuming the information given in part (b).

d) **(3 points).** Suppose that the depth of the seafloor is 6.5 m. You now lower your detector to the seafloor below the light source, where the detector records a signal of $S(6.5 m) = 0.813 A$. Using this information and what you have determined or been given in the earlier parts of this problem, find an expression for the quantum efficiency of your detector at $\lambda = 0.4 \mu m$ (the wavelength of the monochromatic lights source) and evaluate your expression numerically.

![Figure for Problem 11: Light source just below the surface and detector suspended below the light with a set of rigid rods.](image)
12. **Radiometry (10 Points).** You are conducting an experiment in a water tank in your laboratory. Initially you fill the tank with water from nearby coastal waters; the extinction coefficient of this water is $\beta \ [m^{-1}]$. After filling the tank, you place an isotropic light source at one end of the tank, and you suspend a screen across the middle of the tank blocking all light except for that light which passes through an embedded large Lambertian diffuser in the middle of the screen as shown in the Figure below. The light source has radiant intensity $I_s \ [Wsr^{-1}]$ and is positioned at a distance $x_s$ from the left end of the tank and at a depth $h_s$. You may assume that the light source is a point source, and you may also assume that there are no other sources of light in the set-up shown. The screen with the embedded diffuser is at a distance $x_{diff}$ from the left end of the tank, and the diffuser is at a depth of $h_{diff}$. The diffuser has transmittance $\tau_{diff}$. The camera is in close proximity to the diffuser, is located at a distance $x_c$ from the left end of the tank, and faces the center of the large Lambertian diffuser at the same depth, $h_{diff}$, that is $h_c = h_{diff}$, as shown in the Figure below. The optics of the camera have transmittance $\tau_{cam}$, and the camera has f-number $f_{#}^{cam}$ with detector elements that are square with dimensions $l_{pixel}$ on each side. The noise equivalent power of the camera detector elements is $NEP_{cam}$.

**a) (4 points).** Find a mathematical expression for the minimum change in radiant intensity of the light source that could be detected in terms of one or more of the given variables.

**b) (2 points).** Evaluate the minimum change in radiant intensity that you found in part a if $\tau_{diff} = 0.83$, $\beta = 0.17 \ m^{-1}$, $x_{diff} = 2.75 \ m$, $x_s = 1.1 \ m$, $h_c = h_{diff} = 1.5 \ m$, $h_s = 0.35 \ m$, $x_c = 3.25 \ m$, $\tau_{cam} = 0.89$, $f_{#}^{cam} = 2.8$, $l_{pixel} = 20 \ \mu m$, $NEP_{cam} = 1.45 \times 10^{-11}W$.

**c) (3 points).** If the radiant intensity changes by a factor of $m$, what is the corresponding percentage change in the shot noise.

**d) (1 point).** Evaluate your answer to part (c) numerically if $m = 1.1$. 

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Figure for Problem 12: water tank with isotropic light source, screen with embedded Lambertian diffuser, and camera in close proximity to the diffuser.
13. Image Processing and Computer Vision: Galaxy Morphology Classification (10 points).

Astronomers classify galaxy morphology based on their visual appearance. The simplest scheme has three classes: spiral galaxies, elliptical galaxies, and irregular galaxies. You have decided to compete in a galaxy classification challenge with a $50,000 prize for the best results on the test set. The dataset has been pre-processed so that all galaxies are individually cropped. The numbers of images in the training, validation, and test sets are given in the Table.

<table>
<thead>
<tr>
<th>Galaxy Type</th>
<th>Total Images</th>
<th>Training Set</th>
<th>Validation Set</th>
<th>Test Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elliptical</td>
<td>617</td>
<td>370</td>
<td>117</td>
<td>130</td>
</tr>
<tr>
<td>Spiral</td>
<td>513</td>
<td>308</td>
<td>97</td>
<td>108</td>
</tr>
<tr>
<td>Irregular</td>
<td>91</td>
<td>55</td>
<td>11</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 1: Galaxy morphology dataset size.

a) (1 point) Explain why simple accuracy would be a bad metric for this dataset.

b) (1 point) Explain the typical purpose of the validation set and how to use it.

c) (2 points) You decide to use a convolutional neural network (CNN) to do this classification problem. What is the danger with using a deep CNN with 19 layers for this dataset? How might this danger be overcome?

d) (3 points) One property that may be beneficial to galaxy classification is rotation invariance; however, typical CNNs do not have this built into their architecture. Explain why this issue is usually ignored for largescale datasets, but might not be an issue that cannot be ignored here.

e) (3 points) Devise a method to make your CNN robust to changes in rotation in your CNN-based system. Give the steps to your algorithm and explain how it works.
14. **Image Processing and Computer Vision: Clustering and Segmentation (10 Points).**

This problem discusses clustering and using clustering to implement image segmentation algorithms.

Consider partitioning a set of \( T \) data points \( D = \{ x_1, x_2, \ldots, x_T \} \), where \( x_t \in \mathbb{R}^d \).

a) (3 points) Assume that you are given an initial set of \( K \) clusters, i.e., \( c_1, c_2, \ldots, c_K \), where \( c_k \in \mathbb{R}^d \). Give pseudo-code for the standard \( k \)-means algorithm, i.e., Lloyd’s method.

b) (1 point) What is the objective/loss/cost function to be minimized in \( k \)-means clustering? Assume Euclidean distance.

c) (2 point) Explain how \( k \)-means++ differs from \( k \)-means. What does it improve?

d) (1 point) If we initialize the \( k \)-means clustering algorithm with the same number of clusters but different starting positions for the centers, the algorithm will always converge to the same solution. Why or why not?

e) (1 point) You are using \( k \)-means clustering in some color space to segment an image. However, you notice that although pixels of similar color are indeed clustered together into the same clusters, there are many discontiguous regions because these pixels are often not directly next to each other. Describe a method to overcome this problem in the \( k \)-means framework.

f) (2 points) Compare the \( k \)-means and mean-shift clustering algorithms for segmentation, i.e., dimensionality, cluster shape, choosing the number of clusters, and robustness.
You have been hired to build a cell counting algorithm. To develop a system for this problem, you decide to use classic image processing techniques as a baseline. In this dataset, the cell nuclei are stained blue and the membranes are stained red.

a) (1 point) First, you need to convert your sRGB images to a monochrome representation. You need to preserve the information you need for cell counting, while suppressing information that is not useful. You recall HSV color space and decide to use Value to do this conversion, i.e., \( V = \max(R, G, B) \). Explain why this was actually not the best idea and suggest an approach likely to work better.

b) (1 point) Could two monochrome images of identical sizes have identical histograms and can this happen if the images are not identical? If so, explain the general conditions in which this could occur. Assume the pixel value range is 0 to 255, and that there are 256 histogram bins centered at every possible integer.

c) (5 points) When transforming a monochrome image into a binary one using thresholding, we must choose a threshold. You decide to threshold the monochrome image using the Basic Global Thresholding Algorithm, also known as Balanced Histogram Thresholding method. Give pseudocode for this algorithm. The input should be a monochrome image \( I \) and the convergence criteria \( v \geq 0 \), which determines when to stop running the algorithm because it has converged. The output is the threshold value \( S \).

Initialize the starting threshold value \( T \) to the mean pixel value in \( I \).

d) (3 points) Now that you have a good binary image, you need to use the connected components algorithm to identify all of the cells in the image. Explain how connected components works in English or give pseudocode.
16. **Graduate Laboratory.** The project this year has been the design, development and testing of an intelligent baby monitor. These questions relate to the specific aspects of the project, and all must be answered.

There are 4 basic components to the project:

- Measurement of breathing rate
- Measurement of the heart rate (pulse)
- Measurement of the body temperature
- Detection of baby noises and their classification (crying, etc.)

**Question 16, Part 1.** For the breathing rate portion of the project:

a) What device(s) were used to capture the signal from which the breathing rate was extracted?

b) Describe in detail how the breathing rate was determined

c) What limited the ability to extract a good measurement of breathing rate?

**Question 16, Part 2.** For the heart rate (pulse):

a) What device(s) were used to capture the signal from which the heart rate was extracted?

b) Describe in detail how the heart rate was determined

c) What limited the ability to extract a good measurement of heart rate?

d) Which was easier to detect from the data, the breathing rate or the heart rate?

**Question 16, Part 3.** For the body temperature:

a) What device(s) were used to measure the body temperature?

b) What limited the ability to extract a good measurement of body temperature?

**Question 4, Part 4.** For the detection of baby noises:

a) What device(s) were used to capture the baby noises?

b) Describe in detail two ways in which the system could determine laughter, crying, and type of crying.

c) For each of the 2 ways described in 4b, explain the advantages and disadvantages of each approach.