An Analytical Model for Optical Polarimetric Imaging Systems

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Abstract—Optical polarization has shown promising applications in passive remote sensing. However, the combined effects of the scene characteristics, the sensor configurations, and the different processing algorithm implementations on the overall system performance have not been systematically studied. To better understand the effects of various system attributes and help optimize the design and use of polarimetric imaging systems, an analytical model has been developed to predict the system performance. The model propagates the first- and second-order statistics of radiance from a scene model to a sensor model and, finally, to a processing model. Validations with data collected from a division of time polarimeter are presented. Based on the analytical model, we define a signal-to-noise ratio of the degree of linear polarization and receiver operating characteristic curves as two different system performance metrics to evaluate the polarimetric signatures of different objects, as well as the target detection performance. Several examples are presented to show the potential applications of the analytical model for system analysis.

Index Terms—Analytical model, imaging system, polarization, target detection.

I. INTRODUCTION

Polarization of light provides valuable information about scenes that cannot be obtained directly from intensity or spectral images. Rather than treating the optical field as scalar, polarization images seek to obtain the vector nature of the optical field from the scene [1], [2]. Polarized light reflected from scenes has been found to be useful in several applications, such as material classification [3], [4], computer vision [5], and target detection [6]–[10]. Recently, a study on polarimetric combined with spectral imaging has demonstrated the capability of remote sensing platforms to detect [11] and track vehicles [12], [13]. Polarimetric imaging is however a relatively new and not fully explored field for remote sensing, as pointed out in [2]. The relatively unstable polarimetric signatures that are highly dependent on the source–target–sensor geometry, as well as the lack of available polarimetric imaging data, make the utility of optical polarimetric imaging still an open question for remote sensing applications.

The system approach to analyze the remote sensing system has been addressed in [14] and [15]. The Forecasting and Analysis of Spectroradiometric System Performance (FASSP) model presented in [15] is a mathematical model for spectral imaging system analysis. This model divides the remote sensing system into three subsystems: scene, sensor, and processing. This mathematical modeling approach generates statistical representations of the scene and analytically propagates the statistics through the whole system. The FASSP model has been found to be extremely helpful in the study of different applications, such as land cover classification [16] and target detection [15], [17]. Another approach for imaging system analysis is image simulation modeling, such as the Digital Imaging and Remote Sensing Image Generation (DIRSIG) model [18], which produces synthetic images.

Many studies on optical polarimetric imaging systems have been done on different system components, such as scene simulation, polarimetric sensor design, and polarimetric data processing.

In the study on scene simulation, characterization of the optical properties of material has always been challenging. The polarimetric BRDF (pBRDF) is often used to characterize fully the polarization behavior of object surface. The acquisition of pBRDF is often based on empirical modeling or experimental measurement. A pBRDF model based on microfacet theory is given in [19]. Recently, this model has been extended by including a shadowing/masking function and a Lambertian component [20]. The pBRDF model has been implemented in the image simulation model DIRSIG [21]. A physics-based model pBRDF was also extended to cover the long-wave IR wavelengths [22]. An approach for measuring the first column of the pBDF matrix is given in [23]. A method to determine pBRDF using in-scene calibration materials is shown in [24]. Some early work of measuring the polarization of natural backgrounds, such as soil, vegetation, and snow, can be found in [25]–[27]. Several models are available to characterize the pBRDF of natural background materials. In [28] and [29], pBRDF models were developed for general vegetation. A polarized reflectance representation for soil can be found in [30] and [31]. Recently, in [32], modeling and simulation of polarimetric hyperspectral images has been presented by including skylight polarization and polarized reflectance models.

In terms of polarimetric sensor design, a number of studies have reported on designing an optimum polarimeter for Stokes parameter measurement by minimizing various condition numbers of the system measurement matrices [33]–[35]. Recently, several methods have been proposed for optimizing the polarization state of illumination in active Stokes images by maximizing the contrast function in the presence of additive Gaussian noise [36] or maximizing the Euclidean distance...
between the target and background Stokes parameters [37]. A framework of finding the optimal combination settings of analyzer rotation angles to improve the target detection performance was presented in [38].

Regarding the processing of polarimetric data, some studies have focused on reconstructing the Stokes parameters from intensity images. A maximum-likelihood blind deconvolution algorithm was proposed in [39] for the estimation of unpolarized and polarized components, as well as the angle of polarization and the point spread function. A Stokes parameter restoration algorithm was proposed in [40] based on penalized-likelihood image reconstruction techniques. Recently, band-limited reconstruction algorithm has been proposed to measure temporally varying Stokes parameters without constant scene assumption [41], [42]. Other interesting studies on polarimetric data processing are material classification and object detection. The techniques of using polarization to improve material classification or segmentation can be found in [43]–[45]. Material classification in turbulence-degraded polarimetric imagery using the blind deconvolution algorithm [39] was introduced in [4] and [46]. Man-made target detections based on RX and topological anomaly detection approaches were compared using synthetic polarimetric images generated by DIRSIG [9] and laboratory measurements [47]. An image fusion approach for object detection can be found in [48]. A maximum-likelihood-based detection and tracking algorithm using long-wave IR microgrid polarimeter data was presented in [49].

The related studies presented are very helpful for obtaining a better understanding of each subsystem of polarimetric imaging. However, the combined effects of the scene characteristics, the sensor configurations, and the different processing algorithm implementations on the overall system performance have not been systematically studied, and a convenient system approach to analyze the end-to-end system performance of polarimetric imaging systems does not exist. It is the goal of gaining a better understanding of the potential performance and limitations of polarimetric imaging technology and of heading toward more optimal designs of polarimetric imaging systems that motivates the system approach presented in this paper. The outline of this paper is as follows. In Section II, a detailed introduction of the analytical model for a polarimetric imaging system is first introduced. Then, the proposed model is validated based on real data in Section III. In Section IV, based on the system model, we define different performance metrics. In Section V, examples of system performance evaluation based on the analytical model are presented. We conclude this paper in Section VI.

II. FORMULATION OF ANALYTICAL MODEL

The framework of the analytical model is shown in Fig. 1. The passive optical polarimetric imaging system is analyzed from the perspective of scene, sensor, and processing. In the scene model, we analyze how the light interacts with the object surface and how the polarized light is reflected to the sensor aperture. In the sensor model, we consider how the light is passed through optical components and then digitized by the camera. In the image processing model, Stokes parameters are reconstructed based on polarization images, and the degree of linear polarization (DoLP) is further calculated. Finally, some system performance metrics are generated to help evaluate the end-to-end system performance and system optimization. The analytical model is introduced below in detail. Note that the model is limited to the reflective portion of the optical spectrum as thermal emission aspects are not considered.

A. Scene Model

1) Optical Properties of Object Surface: The optical properties of the object surfaces are often characterized by a bidirectional reflectance distribution function (BRDF) in spectral image analysis. When considering polarization, the BRDF should be extended to a polarimetric BRDF (pBRDF). In general, the pBRDF can be represented by a $3 \times 3$ Mueller matrix when only linear polarization is considered, which is the case in this paper. The pBRDF can be estimated based on an empirical approach or an analytical approach. For the empirical approach, the pBRDF matrix is found based on measurement. In [50], a framework for the outdoor measurement of pBRDF is introduced, but only the first column of the pBRDF matrix is considered. The full measurement of the pBRDF can be achieved based on different permutations of polarization states of the light generator and the analyzer [19], and at different scene geometries [2]. The empirical approach is therefore not very flexible and requires a large number of measurements. Alternatively, the pBRDF could be analytically estimated using physics-based models. In this paper, the pBRDF is predicted based on microfacet theory, which is commonly used in the modeling of the BRDF [51], and recently extended to the modeling of the pBRDF [19]–[22]. Generally, the pBRDF is modeled as a combination of a polarized specular component and an unpolarized diffuse component that is caused by scattering. Readers are referred to the literature for detailed introduction of the pBRDF calculation, and we only present the simplified pBRDF Muller matrix of the object class as a $3 \times 3$ matrix $F$. The incident irradiance and reflected radiance are accordingly extended to Stokes parameters and are related by the pBRDF matrix as

$$
\begin{align*}
\begin{bmatrix}
L_0 \\
L_1 \\
L_2
\end{bmatrix} =
\begin{bmatrix}
f_{00} & f_{01} & f_{02} \\
f_{10} & f_{11} & f_{12} \\
f_{20} & f_{21} & f_{22}
\end{bmatrix}
\begin{bmatrix}
E_0 \\
E_1 \\
E_2
\end{bmatrix}
\end{align*}
$$
where $[L_0 \; L_1 \; L_2]^T$ ($T$ denotes transpose) is the Stokes vector of the reflected radiance with units of W m$^{-2}$ sr$^{-1}$ (sr is steradian), $[E_0 \; E_1 \; E_2]^T$ is the Stokes vector of the incident irradiance with units of W m$^{-2}$, and $f_{ij}$ is one component of the F matrix with units of sr$^{-1}$.

2) Solar and Atmosphere Modeling: The radiance calculations are functions of the incident and reflected zenith angles $\theta_i$ and $\theta_d$, and the relative azimuth angle $\phi$ between the sun and the sensor. All radiance calculations are functions of wavelength $\lambda$, but the subscript has been dropped for clarity.

The direct solar irradiance is randomly polarized, and the solar reflected radiance can be calculated by considering only the first column of the pBRDF Mueller matrix $[f_{00} \; f_{10} \; f_{20}]^T$ and is expressed as

$$L_r = \tau \cos \theta_d E_s [f_{00} \; f_{10} \; f_{20}]^T$$

where $L_r = [L_{r0} \; L_{r1} \; L_{r2}]^T$ is the Stokes vector representation of the direct solar reflected radiance, $\tau$ is the transmittance along the surface-to-sensor path, and $E_s$ is the solar irradiance incident on the surface from direct transmission through the atmosphere.

Due to Rayleigh scattering, the downwelling radiance distributed over the entire sky is polarized. The polarization state of the skylight is dependent on the incident zenith angle $\theta_d$ and azimuth angle $\phi_d$. The object surface also shows various polarization behaviors when the light comes from different directions since the pBRDF is considered. Therefore, it is necessary to integrate the skylight reflected off the surface over the whole hemisphere, and the surface reflected downwelling radiance can be expressed as

$$L_d = \tau \int F(\theta_i, \phi) L_{\text{sky}}(\theta_d, \phi_d) \cos \theta_d d\Omega$$

where $L_d = [L_{d0} \; L_{d1} \; L_{d2}]^T$ is the Stokes vector representation of the reflected downwelling radiance, $L_{\text{sky}}(\theta_d, \phi_d)$ is the downwelling radiance originated from direction $(\theta_d, \phi_d)$, and $d\Omega = \sin \theta_d d\theta_d d\phi_d$.

The upwelling radiance is also polarized and expressed as

$$L_u = [L_{u0} \; L_{u1} \; L_{u2}]^T.$$ (4)

The solar irradiance and the magnitude of the downwelling radiance and the upwelling radiance ($L_0$ component) are often estimated using an atmospheric scattering code such as the MODerate resolution atmospheric TRANsmission (MODTRAN) [52]. A polarized version of MODTRAN (MODTRAN-P) has been developed to include the polarization influence due to Rayleigh scattering, and used to estimate the Stokes parameters of atmospheric radiance [53]. Alternatively, the atmospheric polarization effect can be estimated based on measurement. In [54] and [55], all-sky polarization images were acquired. In this paper, MODTRAN-P is used for solar and atmosphere modeling.

In Fig. 2, we present a simulation of the Stokes parameters of downwelling radiance after iteratively running MODTRAN-P 370 times. The DoLP of downwelling radiance is also computed based on Stokes parameters as

$$\text{DoLP}_d = \frac{\sqrt{L_{d1}^2 + L_{d2}^2}}{L_{d0}}.$$ (5)

The zenith angle is changed from 0° to 90° with a 10° step, and the azimuth angle is changed from 0° to 360°, also with a 10° step. The parameter settings of MODTRAN-P input are listed in Table I.

3) Statistics of Radiance at Sensor Aperture: Based on the pBRDF model, and the solar and atmosphere models, the statistics of radiance at the sensor aperture can be generated. The radiance reaching the sensor aperture is found as the combination of the solar reflected radiance, downwelling radiance, and upwelling radiance, i.e.,

$$L_S = L_r + L_d + L_u.$$ (6)

For convenience of computation, the sensor spectral response functions are convolved with the radiance in the scene model. In this paper, we are particularly interested in the polarization image captured in grayscale (panchromatic image) over visible wavelengths. The radiance received by the sensor is therefore an intensity that is no longer specified as a function of wavelengths.

The radiance is represented by the first- and second-order statistics. The mean of the radiance is also represented as Stokes vector, i.e.,

$$L_S = [L_{S0} \; L_{S1} \; L_{S2}]^T.$$ (7)
A covariance matrix due to this viewing geometry can be found accordingly as

$$\Sigma_g = E \{ (\mathbf{L}_S - \mathbf{L}_S)(\mathbf{L}_S - \mathbf{L}_S)^T \}.$$  \hfill (9)

When the field of view of the sensor is narrow or the object is not widely distributed, the angle range shown in Fig. 3 can be approximated as $\Delta \theta_i = \Delta \theta_g = 0$. The mean $\bar{L}_{Si}$ can be then simply estimated using (6), and the covariance matrix $\Sigma_v$ is omitted.

Another source of radiation variation may be due to the nonuniformity of the surface reflectance or, for example, texture variation. This kind of variation can also be characterized by introducing covariance matrix $\Sigma_v$. In our analytical model, $\Sigma_v$ is considered a diagonal matrix and could be derived from measurements. Derivations of estimating $\Sigma_v$ were presented in [38] and [56].

Finally, the covariance matrix of Stokes vector radiance at sensor’s aperture is found as

$$\Sigma_{LS} = \Sigma_g + \Sigma_v.$$ \hfill (10)

The radiance statistics will be further propagated into the sensor model to generate signal statistics.

B. Sensor Model

A polarization-sensitive optical system can be used to measure the polarization state of light. The framework of the sensor model is shown in Fig. 4. The sensor model takes the mean and covariance of the sensor reaching radiance to produce the signal mean and covariance by applying sensor effects, such as the polarizing effects of optical components, sensor spectral response, radiometric noise sources, etc. The sensor model introduced here is specified for a division of time polarimeter (DoTP) system, and the scene considered is stationary. However, it should be noted that the sensor model could be easily extended to other types of polarimeters, such as division of focal-plane polarimeter. For an ideal linear polarizing filter, which is also called an analyzer, rotated at an angle of $\psi_i$ with respect to horizontal, the transmitted radiance $L_i$ can be expressed as

$$L_i = \frac{1}{2} (L_{S0} + \cos 2\psi_i \cdot L_{S1} + \sin 2\psi_i \cdot L_{S2}) = \mathbf{m}_i^T \mathbf{L}_S$$ \hfill (11)

where $\mathbf{m}_i = 0.5[1 \cos 2\psi_i \sin 2\psi_i]^T$, and $\mathbf{L}_S$ is the incident Stokes vector radiance. With the linear polarizing filter rotated at $N$ different angles, an $N$-channel polarization-sensitive system can be constructed as

$$\begin{bmatrix} L_1 \\ \vdots \\ L_N \end{bmatrix} = \begin{bmatrix} \mathbf{m}_1^T \\ \vdots \\ \mathbf{m}_N^T \end{bmatrix} \begin{bmatrix} L_{S0} \\ L_{S1} \\ L_{S2} \end{bmatrix} = \mathbf{M} \mathbf{L}_S.$$  \hfill (12)

An $N \times 3$ system matrix can be made up by including all the vectors associated with the rotation of analyzers as

$$\mathbf{M} = [\mathbf{m}_1 \cdots \mathbf{m}_N]^T.$$ \hfill (13)

Several operation strategies can be used to set the analyzer rotation angles, for example, Pickering’s method of $(0^\circ, 45^\circ, 90^\circ)$,
Fessenkov’s method of \((0°, 60°, 120°)\), and modified Pickering’s method of \((0°, 45°, 90°, 135°)\).

The passed radiance shown in \((11)\) is an ideal case. Some radiance of the orthogonal polarization state may also pass through the analyzer. This leakage effect can be characterized by including a copolarized transmission \(\tau_\parallel\) and a cross-polarized transmission \(\tau_\perp\). The \(i\)th component of the system matrix can be updated by considering the leakage effect as

\[
m_i = \frac{1}{2} [\tau_\parallel + \tau_\perp (\tau_\parallel - \tau_\perp) \cos 2\psi_i (\tau_\parallel - \tau_\perp) \sin 2\psi_i]^T. \tag{14}
\]

The statistics of the radiance passed through the analyzers can be found by applying the system matrix to the sensor reaching radiance statistics. The mean of the radiance can be found as

\[
\bar{L} = E\{M_\perp L_S\} = M\bar{L}_S. \tag{15}
\]

The covariance matrix of the radiance can be also found accordingly as

\[
\Sigma_L = E\{(L - \bar{L})(L - \bar{L})^T\} = M\Sigma_{L,S}M^T. \tag{16}
\]

The radiance passed through the analyzers is converted to electrons and further to sensor output signals using

\[
I = \left[ \int \frac{L \cdot \tau_o \cdot \eta \cdot t \cdot \lambda \cdot A_d}{(1 + 4f \#^2) \cdot h \cdot c} R(\lambda) d\lambda + N_d \right] \cdot g_e \tag{17}
\]

where \(I\) is the intensity per channel, \(L\) is the input radiance, \(\tau_o\) is the transmittance of the optical components, \(\eta\) is the quantum efficiency of the detector, \(t\) is the integration time, \(\lambda\) is the central wavelength, \(A_d\) is the effective area of the detector, \(f \#\) is the \(f\) number, \(h\) is the Plank’s constant, \(c\) is the speed of light, \(R(\lambda)\) is the normalized sensor response, \(N_d\) is the dark current, and \(g_e\) is the sensor gain. As aforementioned, the sensor spectral response functions are convolved with the radiance in the sensor reaching radiance calculation. Radiance mean \(\bar{L}\) and covariance matrix \(\Sigma_L\) are accordingly converted to signal mean \(\bar{I}\) and covariance matrix \(\Sigma_{I,s}\). The signal mean of an \(N\)-channel system is expressed as

\[
\bar{I} = [\bar{I}_1 \cdots \bar{I}_N]^T \tag{18}
\]

and the covariance matrix \(\Sigma_{I,s}\) is an \(N \times N\) matrix.

As shown in Fig. 4, various noise sources can be observed in a sensor system. These noise sources add more uncertainties to output signal and can be characterized by including covariance matrix \(\Sigma_n\). In our model, \(\Sigma_n\) is an \(N \times N\) diagonal matrix, whose \(i\)th component in the diagonal is the variance of sensor noise in the \(i\)th channel and is computed as

\[
\sigma^2_{n,i} = g_e I_i + \sigma^2_c \tag{19}
\]

where \(\sigma^2_{n,i}\) is the variance of total sensor noise in the \(i\)th channel, \(I_i\) is the signal mean in the \(i\)th channel (representing shot noise modeled as a Poisson process), and \(\sigma^2_c\) is the variance of signal independent noise, such as read noise and quantization noise.

The covariance matrix of the digital output signal can be expressed as

\[
\Sigma_I = \Sigma_{I,s} + \Sigma_n. \tag{20}
\]

C. Processing Model

In the processing model, we consider the transformation of digitized intensity to Stokes parameters, and the reconstructed Stokes parameters are used as features for further analysis.

The mean and the covariance matrix of the reconstructed Stokes parameters can be found, respectively, as

\[
\bar{S} = [\bar{S}_0 \bar{S}_1 \bar{S}_2]^T = \bar{W}\bar{I} \tag{21}
\]

\[
\Sigma_S = \bar{W}\Sigma_{I,s}\bar{W}^T \tag{22}
\]

where \(\bar{W}\) is the inverse of the estimated or calibrated system matrix \(\bar{M}\). If \(\bar{M}\) is a non-square matrix, its pseudo-inverse can be applied using

\[
\bar{W} = (\bar{M}^T\bar{M})^{-1}\bar{M}^T. \tag{23}
\]

A DoLP value can be then calculated based on the reconstructed Stokes parameters. In this paper, the mean of DoLP is approximated based on the means of Stokes parameters, i.e.,

\[
\text{DoLP} = \sqrt{\frac{S_1^2 + S_2^2}{S_0^2}} \tag{24}
\]

and the variance of DoLP can be found as

\[
\sigma^2_{\text{DoLP}} = a_1^2\sigma^2_{S_0} + a_2\sigma^2_{S_1} + a_3\sigma^2_{S_2} + \sum \rho_{ij} a_i a_j \sigma_{S_i} \sigma_{S_j} \tag{25}
\]

where

\[
a_1 = -\frac{\sqrt{S_1^2 + S_2^2}}{S_0^2} \tag{26}
\]

\[
a_2 = \frac{S_1\sqrt{S_1^2 + S_2^2}}{S_0^2} \tag{27}
\]

\[
a_3 = \frac{S_2\sqrt{S_1^2 + S_2^2}}{S_0^2} \tag{28}
\]

\(\sigma_{S_i}\) is the standard deviation of the \(i\)th component in Stokes vector, \(\rho_{ij}\) is the correlation coefficient between variables \(S_i\) and \(S_j\), and the summation is taken over all the possible combinations of Stokes parameters. A derivation of the statistics of DoLP was presented in [56].
It should be noticed that the estimation of the DoLP statistics are affected by the intensity SNR as well as different estimators. As shown in [57], the mean and standard deviation of estimated DoLP diverge from the true values at low SNR, and different estimators such as maximum likelihood, empirical mean, and median estimator present a different behavior. To show how the estimated DoLP statistics is affected by the intensity SNR, Monte Carlo experiments are performed. The experiments are performed based on specified Stokes vector radiance received by the sensor, and are run at different SNR of $S_0$. A four-polarization sensitive imaging system with the analyzers rotated at $0^\circ$, $45^\circ$, $90^\circ$, and $135^\circ$ is considered. In the sensor model, only shot noise is considered. With an increasing SNR of $S_0$, $10^5$ realizations of the intensity are generated for each SNR. The mean and the standard deviation of DoLP are calculated at each SNR level. The analytical model predictions of DoLP statistics is compared with the results generated by Monte Carlo simulations in Fig. 5. In the simulation, the incident light is with normalized Stokes vector $S = [1 \ 0.5 \ 0.5]^T$. It is shown that the analytical-model-predicted statistics of DoLP diverge at low SNR, which is accordance with the observation presented in [57]. In this paper, all the experiments are performed with the $S_0$ SNR larger than 20 dB to guarantee the model-predicted DoLP statistics have better agreement with the true values.

III. MODEL VALIDATION

The proposed analytical model has been validated using real data. A DoTP system composed of a 12-bit Prosilica GC780 camera containing a $782 \times 582$ Sony charge-coupled device (CCD) camera, an Edmund Optics 35-mm focal length lens, a UV–IR cut filter, and a rotatable linear polarizer, was used to collect outdoor imagery. The noise characteristics of the CCD camera were calibrated and presented in our previous publication [38]. Polarization images were collected using the modified Pickering method from a rooftop on May 25, 2010. The scene was composed of a piece of black-painted panel, a piece of white-painted panel, and a piece of asphalt. Two different sun–object–sensor geometries were chosen to validate the model: One was with a solar zenith angle $\theta_i$ of $22^\circ$, and relative azimuth angle $\phi$ between the sun and the sensor of $180^\circ$, and the other was with $\theta_i$ of $43^\circ$ and $\phi$ of $90^\circ$. Both were viewed by the sensor at zenith angle $\theta_r$ of $53^\circ$. Due to the limited length of this paper, only validations for black- and white-painted panels at one of the two scene geometries are presented. (See [56] and [58] for more complete validation results.) The panchromatic and DoLP images of the black- and white-painted panels collected at one of the scene geometries are shown in Fig. 6. The input parameters of the scene model are refractive indexes, surface roughness, and directional hemispherical reflectance values of the black paint and white paint. MODTRAN-P was run to generate the solar illumination and the polarized atmospheric radiance. Because the test objects are not widely distributed, no scene geometry variation was considered in the validation, and the texture variation of the object surface was measured using the technique introduced in [56]. The sensor model parameter settings were selected based on documents provided by the manufacturer and sensor calibration. The detailed parameter settings for scene and sensor models were presented in [56].

A. Multichannel Intensity Statistics

The output of the sensor model is the intensity in digital counts. We compare the model-predicted mean and standard deviation of the intensity to the real measurement of the black
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Fig. 7. Comparison of the analytical-model-predicted intensity mean and standard deviation to the real measurements of the black paint and the white paint at scene geometry \((\theta_i, \theta_r, \phi) = (22^\circ, 53^\circ, 180^\circ)\). (a) Comparison of mean. (b) Comparison of standard deviation.

The statistics of the intensity are compared at four different channels, which were measured using the modified Pickering method with \(0^\circ, 45^\circ, 90^\circ\), and \(135^\circ\) analyzer rotations. It can be seen that the analytical model can predict the trend of the data very well, although some discrepancy between the model predictions and real measurements can be seen in magnitude.

B. Stokes Vector and DoLP Statistics

The model has been further validated based on the Stokes vector and DoLP statistics. The histograms of the normalized \(S_1\) and \(S_2\) Stokes parameters and the DoLP values as observed from the black paint and white paint are shown in Fig. 8(a). The histograms of the real data were fitted by Gaussian curves, which are shown as solid lines in the figures. It can be seen that the analytical model can predict the trend of the data very well, although some discrepancy between the estimated and actual material properties, or calibration error of the sensor.

Fig. 8. Model validation with black paint and white paint at scene geometry \((\theta_i, \theta_r, \phi) = (22^\circ, 53^\circ, 180^\circ)\). (a) Real data. (b) Model prediction.

The discrepancy between the model predictions and real measurements are possibly due to differences between the real atmosphere and the predicted atmosphere by MODTRAN-P, any diversity between the estimated and actual material properties, or calibration error of the sensor. However, the results show that the analytical model is able to predict the general polarization behavior of different materials and data trends at different scene geometries, which can be found with more details in [56].

IV. SYSTEM PERFORMANCE METRICS

Here, system performance metrics are defined from two different perspectives: strength of polarimetric signatures of objects and target detection performance. Those performance metrics can be then used to evaluate the potential performance of a polarimetric imaging system.

A. Strength of Polarimetric Signature

The polarization signatures of different objects are often defined using Stokes vector and DoLP values. The DoLP is particularly helpful since it is a normalized value and can be used to evaluate quantitatively the strength of polarimetric signature. However, the measurement of DoLP is always contaminated with sensor noise. The DoLP measurement can be varying, and the variance of DoLP due to sensor noise is closely related to the SNR of intensity measurement. Therefore, the strength of polarimetric signature is characterized dynamically by introducing DoLP SNR as

\[
\text{SNR}_{\text{DoLP}} = \frac{\overline{\text{DoLP}}}{\sigma_{\text{DoLP}}}
\]

where \(\overline{\text{DoLP}}\) is the mean of DoLP, which can be computed using (24), and \(\sigma_{\text{DoLP}}\) is the standard deviation of DoLP, which can be computed using (25). The DoLP SNR represents a normalized performance metric, as compared with the noise equivalent DoLP (NEDoLP). An expression of the NEDoLP based on modified Pickering measurement is presented in [59].

B. Target Detection Performance

Target detection algorithms that have been developed for spectral imagery have potential applications for polarimetric imagery by replacing the spectral vector with the polarimetric intensity or Stokes vector. The RX anomaly detection algorithm [60] that has been widely used for spectral image analysis is
modified and used to evaluate the target detection performance. The RX detector is expressed as

$$D_{RX}(z) = z^T \Sigma^{-1} z \geq \eta \Rightarrow H_1$$

$$\eta \Rightarrow H_0$$

(30)

where \(z = x - m\); \(x\) is the sample vector composed of multichannel intensity or Stokes parameters; \(m\) and \(\Sigma\) are the background mean and covariance matrix, respectively; \(\eta\) is the user-specified threshold to control the false alarm rate; \(H_1\) is the hypothesis that the target is present; and \(H_0\) is the hypothesis that the target is absent. The RX anomaly detection algorithm can be recognized as the estimation of the squared Mahalanobis distance of the test pixel from the mean of the background. The RX algorithm is only applied on the Stokes parameters in this paper. A closed-form expression of the statistical distribution of \(D_{RX}\) cannot be easily derived. Monte Carlo experiments are therefore performed to estimate \(D_{RX}\) of the test pixels. In each Monte Carlo experiment, \(10^5\) realizations of each Stokes parameter are generated based on its statistics estimated using (21) and (22) in the analytical model. The Stokes parameters are assumed to follow Gaussian distribution according to the observations shown in [7].

Receiver operating characteristic (ROC) curves are then generated based on the \(D_{RX}\) values to help evaluate the target detection performance. The ROC curve is a graphic plot that characterizes the probability of detection (PD) at different probability of false alarm levels by varying threshold \(\eta\).

V. EXAMPLES OF SYSTEM PERFORMANCE EVALUATION

Here, we present how the system model and defined performance metrics may be used to study potential system performance based on different parameters. In these studies, two different materials were considered: black paint used as the target and asphalt used as the background. Their optical properties were presented in [56]. MODTRAN-P was run to generate the solar illumination and the polarized atmospheric radiance, based on the 1976 U.S. standard atmospheric model, rural extinction, 23-km meteorological range, and the day of year of 145. The simulated sensor has an f number of 2, pixel pitch of 8.3 \(\mu m\), a depth of 12 bits, field of view of \(15^\circ \times 15^\circ\), and integration time of 30 \(\mu s\). The background object asphalt is considered widely distributed within the field of view. Other configurations of the sensor, such as the sensor response, can be found in [56]. In the processing model, the modified Pickering method is used to reconstruct the Stokes parameters.

A. POLARIZATION CONTRIBUTIONS OF RADIOMETRIC SOURCES

The radiance reaching sensor’s aperture has contributions from different radiometric sources, as modeled in Section II-A2. The impact of different radiometric sources on the DoLP SNR was first studied. The contributions from different radiometric sources are modeled as the reflected radiance, the upwelling radiance, and the total radiance. The reflected radiance is the combined solar and downwelling radiance reflected only from the object surface and can be represented as

$$L_{reflect} = L_r + L_d$$

(31)

where the radiance is also modeled as a Stokes vector. The reflected radiance can be thought of as the target’s polarimetric signature. The upwelling radiance is the radiance propagated to sensor’s aperture without reaching the object surface, which can be thought of as contaminating the target polarimetric signature. The total radiance can be found using (6). The DoLP SNR of a black-painted object with contributions from each radiometric source is computed at different scene geometries, and the results are compared in Fig. 9.

In Fig. 9, it is shown that the DoLP SNR calculated based on the total radiance is different from the radiance truly reflected from the object surface, which is due to the contamination from the upwelling radiance. At certain scene geometries, such as \(\theta_i = 25^\circ\) and \(\theta_r = 10^\circ\), as shown in Fig. 9(a), the upwelling radiance results in an even higher DoLP SNR than the reflected radiance at \(\Delta \phi = 0^\circ\) and \(\Delta \phi = 90^\circ\). In the same figure, it is also shown that, although the DoLP SNR of the total radiance is much higher at \(\Delta \phi = 0^\circ\), the polarization information obtained at this azimuth angle is dominated by the upwelling radiance and reflects very little information about the polarimetric signature of the object. The polarization sensed by the sensor is usually based the total radiance; therefore, a higher DoLP SNR does not necessarily mean a stronger polarimetric signature. However, these figures show the general trend that the DoLP SNR of reflected radiance increases when the azimuth angle changes from \(0^\circ\) to \(180^\circ\), and therefore stronger polarimetric signature is expected to be captured at the forward reflection direction. A higher DoLP SNR of reflected radiance is also observed at an oblique viewing angle, such as \(\theta_r = 50^\circ\) shown in Fig. 9(b). In the following performance studies, we will concentrate on the azimuth angle of \(180^\circ\), at which more accurate polarimetric signatures of the object are expected to be captured.
B. Effect of Sensor Platform Altitude

The DoLP SNR was analyzed by considering the altitude of the sensor platform. The DoLP SNRs of the black paint are estimated at platform altitudes of 3, 5, 7, and 9 km, with scene geometry at \( \theta_i = 50^\circ \), \( \theta_r = 50^\circ \), and \( \Delta \phi = 180^\circ \). The simulation results are shown in Fig. 10. The reflected radiance and total radiance were also considered separately in the calculation of the DoLP SNR. In Fig. 10(a), the DoLP SNR was calculated based on reflected radiance, and in Fig. 10(b), the DoLP SNR was calculated based on the total radiance. It is shown that the DoLP SNR of reflected radiance decreases at higher altitude and therefore results in a weaker polarimetric signature. However, this trend is not followed in Fig. 10(b) when DoLP SNR is calculated based on the total radiance. This is because, at higher altitudes, upwelling radiance is increased significantly. The observations suggest that, at higher altitudes, higher DoLP SNR does not necessarily mean a stronger polarimetric signature since the polarimetric signature may be severely contaminated by the upwelling radiance.

The effect of sensor platform altitudes on the target detection performance is further analyzed by applying the RX anomaly detector and by plotting ROC curves. The target detection performance is compared at different platform altitudes in Fig. 11. It is shown that the detection performance decreases by increasing the altitude. The observation is in accordance with the results shown in Fig. 10. At a lower platform altitude, the DoLP SNR of reflected radiance from target is much higher; therefore, stronger polarimetric signature is present.

C. Effect of Scene Geometry

In this section, we further study the effect of scene geometry on the polarization behavior of object, as well as the target detection performance.

The DoLP SNRs of black paint were estimated at different solar incident and sensor viewing angles, and the results are shown in Fig. 12. The ROC curves are plotted in Fig. 13 by performing RX detection at four different scene geometries. It is shown that, at an oblique solar incident angle \( \theta_i = 50^\circ \), the detection performance is enhanced. When \( \theta_i = 10^\circ \), the detection at an oblique sensor viewing angle \( \theta_r = 40^\circ \) outperforms the detection at \( \theta_r = 20^\circ \), and the result is in accordance with the DoLP SNR analysis shown in Fig. 12. When \( \theta_i = 50^\circ \), the detection at \( \theta_r = 20^\circ \) outperforms the detection at \( \theta_r = 40^\circ \). This result however does not follow the observation that we have in Fig. 12, where the DoLP SNR of black paint at \( \theta_r = 40^\circ \) is always higher than the DoLP SNR at \( \theta_r = 20^\circ \). This is mainly due to the fact that the DoLP SNR is calculated solely based on the target object, but the detection performance is determined by the combined factors of both the target and background. In this simulation,
the background object asphalt is not randomly polarized, and its polarization properties also determine the target detection performance.

D. Sensitivity to Sensor Noise

The effect of sensor noise is studied by defining a sensor noise factor $K_n$; using which, the total sensor noise can be scaled as

$$\sigma_N = K_n \sqrt{g_c \bar{I}_i + \sigma^2_C}$$

(32)

where $\sigma_N$ is the standard deviation of total sensor noise based on the noise factor $K_n$ and parameters $g_c$, $\bar{I}_i$, and $\sigma_C$, as defined in Section II-B.

The DoLP SNR of black paint was first studied by changing the sensor noise factor from 1 to 3.5 with 0.5 step, and the results are shown in Fig. 14. The DoLP SNR was also analyzed at different solar incident zenith angles. It is shown that, with a larger sensor noise factor, the DoLP SNR of black paint decreases. The black paint presents higher DoLP SNR at an incident zenith angle of around $40^\circ$ and $60^\circ$, showing good agreement with the results presented earlier.

The target detection performance was also studied with three different sensor noise factors $K_n = 1$, 2, and 3, and the ROC curves are compared in Fig. 15. In this simulation, the incident zenith angle was fixed at $50^\circ$. The target detection performance is affected significantly by the level of sensor noise. The PD value at a constant false alarm rate decreases when the sensor noise factor is increased.

VI. CONCLUSION

An analytical model has been presented to predict the end-to-end system performance of a passive optical polarimetric imaging system. Three subsystems, i.e., scene, sensor, and processing, were modeled, respectively. The scene model produces the first- and second-order statistics of the radiance reaching sensor's aperture. The sensor model then converts the radiance statistics to the statistics of digital output signals. The processing model extracts the features (Stokes parameters and DoLP) from the multichannel intensity signals, and the statistics of the extracted features were generated. The analytical model was validated using real data collected from a rooftop. The results based on model prediction have shown good agreement with experimental measurement. Based on the analysis, two different performance metrics were defined to evaluate the end-to-end system performance. The DoLP SNR was defined to estimate the strength of polarimetric signature, and ROC curves generated by applying an RX anomaly detector were used to evaluate target detection performance. Studies were performed to analyze the potential system performance, and several examples were presented by varying different system parameters. It has been shown that the analytical model can help us better understand the effects of various system attributes.


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