Parametric analysis of target/decoy performance[1]

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ABSTRACT

As infrared sensing technology matures and the development of space-based optical sensors becomes feasible, interest has grown in protecting both ballistic and orbiting objects from detection by optical sensors. Such sensors may have high spatial resolution and sense in several wavelength bands simultaneously. This has lead to an increasing interest in the development and analysis of optical decoys that will prevent identification of high value objects in space.

This paper describes an analytical approach to the parametric analysis of target/decoy discrimination performance as a function of various controllable object characteristics.

This analysis tool can be used to answer the question, "how distinct in physical characteristics do a target and decoy have to be before they can be easily discriminated?" Three main characteristics of the objects are considered in this analysis: temperature, projected area, and rate of rotation. These characteristics are given assumed models for their statistics and described by a set of parameters including their first and second order moments.

Based upon the statistical parameters and models for the object characteristics, a set of equations are derived to compute the mean and covariance of the optical signature as seen by a sensor for the decoy and target classes. An estimate of the classification performance between the classes is made using a function of a statistical distance measure. This estimate is used as a performance measure in a parameter tradeoff analysis during an example decoy concept development process. While a purely analytical approach such as this lacks the fidelity of a sophisticated simulation model, it is computationally much simpler and is most appropriately applied during decoy concept development before the application of more rigorous simulation-based analysis.

1. INTRODUCTION

In the development of an optical decoy concept it is necessary to have a method of predicting performance based on signature characteristics of the concept. One method is the use of detailed simulations of the signatures of the target and decoy together with a classification algorithm to compute an operating characteristic. While this method is generally accurate, it may be too costly and time consuming to use in parameter tradeoff studies.

An alternative approach is presented here that uses analytic equations to relate target and decoy physical parameters to their signature characteristics and an estimate of performance. This method allows parameter tradeoff studies to be performed quickly and used as a guide in selecting parameters to be evaluated for the more computationally intensive simulation approach.

Figure 1 shows the overall configuration of the situation modeled in this paper. The sensor observes the object (target or decoy) by receiving radiation emitted by the object as well as the upwelling

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Earth radiation reflected by the object. This configuration most accurately represents the situation of an infrared sensor observing a target located over night time on the Earth.

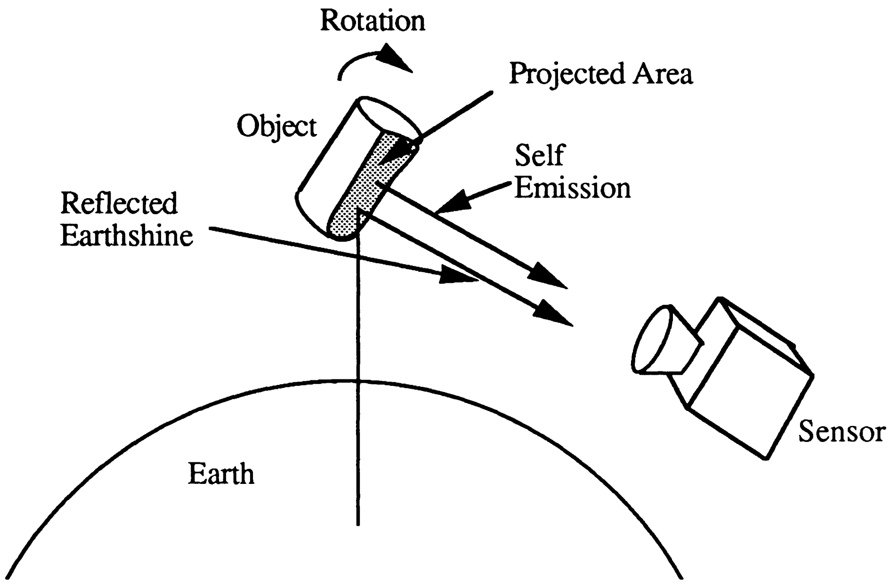

Figure 1. Configuration of sensor observation and contributions to signature.

The object physical parameters that were deemed most relevant to discrimination in this model were 1) temperature, 2) projected area, and 3) rotational motion. These parameters are considered to be random variables with assumed probability density functions within each class. Other important parameters that contribute to the optical signature such as surface reflectivity and emissivity were made constant within each class. The model converts these specified parameters into a distribution of signatures for each object class and computes an expected probability of error based upon the overlap in the feature space of the two distributions.

The model is presented and discussed in the rest of this paper as follows. Section 2 presents the model and its theoretical foundation. Section 3 discusses the use and interpretation of the model results in the context of a target/decoy discrimination problem. Section 4 presents results of applying this model to an example scenario and section 5 contains a summary and discussion of the method.

2. MODEL DESCRIPTION

2.1. Model Overview

Figure 2 shows the basic structure of the Analytical Performance Evaluation Model (APEM). Parameters of the target/decoy classes are specified along with system observational conditions and used to obtain the first and second order statistics of the optical signatures. From these statistics an estimate of the probability of error is then made.

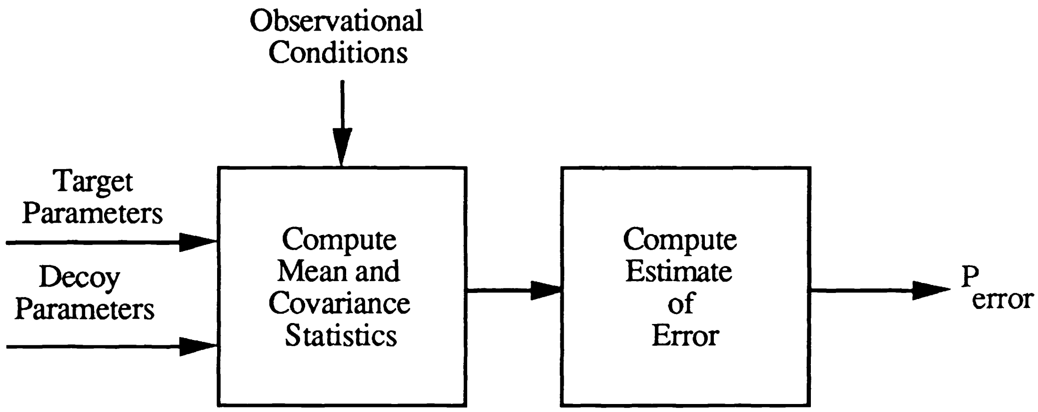

Figure 2. Block Diagram of Analytical Performance Evaluation Model (APEM).

2.2. Signature Model

The radiant intensity of the object is used as the signature feature in the model. Equation (1) shows the parameters that contribute to this feature for typical object characteristics. Boldface variables are random.

$$\mathbf{I}(t) = \mathbf{A_p} \left| \cos(\theta + \mathbf{R}\, t) \right| \left[\varepsilon L_o(\lambda_1, \lambda_2, \mathbf{T_o}) + \rho L_e(\lambda_1, \lambda_2, T_e) \right] \qquad (1)$$

Where,

$\mathbf{A_p}$ = Projected area of object (square meters). Gaussian distributed with mean $\overline{A_p}$ and variance σ_A^2 .

θ = Initial orientation angle of object (radians). Uniform $[0, 2\pi]$.

$\mathbf{R}$ = Rate of rotation of object (radians per second). Gaussian distributed with mean $\overline{R}$ and variance σ_R^2

t = Time since initial observation (seconds).

L_o = Self-radiance of object between wavelengths λ_1 and λ_2 at $\mathbf{T_o}$ (Watts/m^2-sr).

$\mathbf{T_o}$ = Temperature of object (degrees Kelvin). Gaussian distributed with mean $\overline{T_o}$ and variance σ_T^2 .

L_e = Radiance incident upon object from Earth (Watts/m^2-sr).

T_e = Temperature of Earth (degrees Kelvin).

ε = Emissivity of object.

ρ = Reflectance of object.

This formulation of the radiant intensity assumes a diffuse reflectance of the object for the Earthshine radiance and neglects the contribution from solar irradiance. Thus, it is most appropriately used for a night time condition. The addition of a solar term to the radiance component would add a relatively small contribution, especially at longer wavelengths, and would further complicate the geometric orientation factor.

The projected area factor is modeled as a random variable to allow for objects that could vary in size and shape from one sample to the next. This could be appropriate for objects (both targets and decoys) that are deployed differently or vary in configuration. Setting the variance of this factor to zero results in modeling a hard body.

The only components of the signature that depend on wavelength are L_e and L_o and for a given temperature these radiances are completely correlated for various sensor wavelength bands. Thus, the signature is considered to be from one spectral band only.

The radiance factors are computed by equation (2).

$$L(\lambda_1, \lambda_2, T) = \int_{\lambda_1}^{\lambda_2} \frac{1.191 \cdot 10^8}{\lambda^5 \left[\exp\left(\frac{14388.3}{\lambda T} \right) - 1 \right]} \, d\lambda \quad \frac{\text{watts}}{m^2 - sr} \tag{2}$$

Sequential observations of the objects lead to a possibly nonstationary sequence with correlation from hit to hit due to the rotation rate **R**. Sequential observations are thus modeled as a vector of length N, where,

$$N = \frac{\text{Total time of observation}}{\Delta T} \tag{3}$$

ΔT is the time interval between observation samples and N is rounded to the nearest integer. Thus, the feature used for signature statistics and the resulting classification estimation is a radiant intensity vector of length N. This sampling of equation (1) requires that the random rotation rate **R** be scaled and converted by ΔT into units of radians/observation. This new rate is denoted by the random variable **r** with mean $\bar{r}$ and standard deviation σ_r. Equation (4) shows the computation of each element **I**(n) of the radiant intensity feature vector.

$$\mathbf{I}(n) = \mathbf{A_p} \left| \cos(\theta + \mathbf{r}\, n) \right| \left[\varepsilon L_o(\lambda_1, \lambda_2, \mathbf{T_o}) + \rho L_e(\lambda_1, \lambda_2, T_e) \right] \tag{4}$$

2.3. Statistics of Signature

Mean Vector. The random variables in the radiant intensity model are considered to be independent; thus the mean vector becomes the product of the means of the area, rotation, and radiance factors.

The mean projected area is specified by $\overline{A_p}$ as a constant, but could be a function of observation number n to include geometrical considerations of the relative trajectories of the object and the sensor.

The mean of the absolute value of the cosine rotation term can be computed as,

$$E\{ | \cos(\theta + \mathbf{r}\, n) | \} = \frac{2}{\pi} \tag{5}$$

where $E\{\bullet\}$ is the expectation operator.

The mean of the radiance term is obtained by substituting T_o for the temperature in the object portion and evaluating the integral in equation (2) for the object and the Earth over the wavelengths within the desired sensor spectral band.

The resulting mean radiant intensity vector $\bar{I}$ is then,

$$\bar{I} = \overline{A_p}\,\frac{2}{\pi}\left[\,\varepsilon L_o(\lambda_1,\lambda_2,\overline{T_o}) + \rho L_e(\lambda_1,\lambda_2,T_e)\right]\begin{bmatrix}1\\1\\\bullet\\\bullet\\\bullet\\1\end{bmatrix}_{N\times 1} \tag{6}$$

The entries in the identity vector could be modified to reflect a change in the mean value of the parameters over the observational interval, e.g., to show a trajectory.

<u>Covariance Matrix</u>. In the one-dimensional case, the variance of a random variable can be expressed as the difference between the mean square value and the mean squared. In the multidimensional case with independent variables, the covariance matrix of the product of two random variables and a random vector is formed by the difference of the product of their mean square values and the product of their means squared, as in,

$$\Sigma_I = \left(\sigma_A^2 + \overline{A}^2\right)\left(\sigma_M^2 + \overline{M}^2\right)\begin{bmatrix} R_G(0) & R_G(1) & \dots & R_G(N\text{-}1)\\ R_G(1) & R_G(0) & \dots & R_G(N\text{-}2)\\ & \bullet & & \\ & \bullet & & \\ R_G(N\text{-}1) & R_G(N\text{-}2) & \dots & R_G(0)\end{bmatrix} - \overline{A}^2\,\overline{M}^2\left(\frac{2}{\pi}\right)^2\begin{bmatrix}1 & 1 & \dots & 1\\ 1 & 1 & \dots & 1\\ & & \cdot & \\ & & \cdot & \\ 1 & 1 & \dots & 1\end{bmatrix}_{N\times N} \tag{7}$$

The variable M is the sum of the emitted and reflected radiances from the object. That is,

$$\overline{M} = \varepsilon L_o(\lambda_1,\lambda_2,\overline{T_o}) + \rho L_e(\lambda_1,\lambda_2,T_e) \tag{8}$$

$$\sigma_M^2 = \left\{ \varepsilon \left[L_o(\lambda_1,\lambda_2,\overline{T_o} + \sigma_T) - L_o(\lambda_1,\lambda_2,\overline{T_o}) \right] \right\}^2 \tag{9}$$

This definition of the variance σ_M^2 for the sum of the radiances is used as an approximation. There is no contribution from the reflected Earthshine in this variance since the reflectivity is assumed constant for all samples within each object class.

The entries $R_G(m)$ of the correlation matrix are the correlations with lag index m for the rotational factor in equation (4). This correlation is shown in in equation (10) with the result in equation (11) where m must satisfy $m\bar{r} < \pi$.

$$R_G(m) = E\left\{ \left| \cos\left[\mathbf{r}(n+m) + \theta \right] \right| \left| \cos\left[\mathbf{r}\, n + \theta \right] \right| \right\} \tag{10}$$

$$R_G(m) = \frac{1}{2\pi} \left[(\pi - 2m\bar{r}) \cos(m\bar{r}) + 2(m^2\sigma_r^2 + 1)\sin(m\bar{r}) \right] \exp\left(-\frac{m^2\sigma_r^2}{2} \right) \tag{11}$$

While no sensor noise has be included in this model, it could be implemented as an additional term on the diagonal of the covariance matrix Σ_I.

2.4. Computing the Target/Decoy Distance Measure

The statistical distance between the target and decoy classes is computed using the Bhattacharyya distance measure assuming Gaussian distributions for the two classes. Equation (12) shows the computation of this measure with the target and decoy classes arbitrarily denoted class 1 and 2.

$$B = \frac{1}{8}\left(\overline{I_1} - \overline{I_2}\right)^T \left[\frac{\Sigma_I^1 + \Sigma_I^2}{2} \right]^{-1} \left(\overline{I_1} - \overline{I_2}\right) + \frac{1}{2} \ln \frac{\left| \frac{\Sigma_I^1 + \Sigma_I^2}{2} \right|}{\sqrt{\left|\Sigma_I^1\right|}\sqrt{\left|\Sigma_I^2\right|}} \tag{12}$$

2.5. Computing the Probability of Error

An estimate of the probability of error based upon the Bhattacharyya distance was investigated in reference [1] and found to be a tightly coupled lower bound on error. It is computed as in equation (13).

$$P_{error} = \mathrm{erfc}(\sqrt{2B}) \tag{13}$$

where,

$$\mathrm{erfc}(x) = 1 - \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \tag{14}$$

The P_{error} is similar to the Equal Probability Error (EPE) in that it equals the probability of false alarm (P_{FA}) and the probability of leakage (P_L) at the point on a decoy operating characteristic where they are equal. In the development of a decoy, the goal is to keep the error rates above a specified operating point. Therefore, the following constraint on design should be used.

$$P_{error} > \frac{P_{FA} + P_L}{2} \tag{15}$$

3. USE AND INTERPRETATION OF APEM

The model was implemented as an engineering worksheet, as well as in a Fortran program. To use, one must specify a number of general and class specific parameters.

General parameters include:

N = Total number of sequential observations
ΔT = Time interval between observations (seconds)
λ_1 = Low wavelength of sensor spectral band
λ_2 = High wavelength of sensor spectral band
T_e = Apparent temperature of the Earth (Kelvin)

Class specific parameters include:

Parameters specifying random variables of each class

$\overline{A_p}$ = Mean of projected area (square meters)
σ_A^2 = Variance of projected area
$\overline{R}$ = Mean rotational rate of object (degrees per second)
σ_R^2 = Variance of rotational rate of object
$\overline{T_o}$ = Mean temperature of object (Kelvin)
σ_T^2 = Variance of temperature of object

Parameters specifying constant values of each class

ε = Emissivity of object
ρ = Reflectivity of object

While the model is constructed to generate one number (probability of error) from a given set of target/decoy class parameters, it is most effective when applied to a range of parameter values and observing their effect on performance.

For example, one could match all but one of the parameters between the two classes and vary that one and observe its effect on probability of error. This technique can be used to set upper limits for the difference, or variation, a physical parameter may exhibit before resulting in too little classification error. Of course, in practice one cannot exactly match all observables between a target and a decoy. Thus, the combined effects of differences between the class specifications of several parameters must be investigated. The next section discusses the application of this tool to an example scenario.

4. EXAMPLE APPLICATION

4.1. Example Scenario

As an illustration of how this tool may be applied in the development of a decoy concept, the following scenario is considered. A high value target class is defined with a set of parameters as shown in Table 1. Table 2 defines the expected observational parameters of a hypothetical threat sensor.

Table 1. Target Class Parameters

Symbol	Parameter	Value
$\overline{A_p}$	Mean projected area (square meters)	1.0
σ_A	Standard deviation of projected area	0.2
$\overline{R}$	Mean rotational rate (degrees per second)	5
σ_r	Standard deviation of rotational rate	1
$\overline{T_o}$	Mean temperature (Kelvin)	300
σ_T	Standard deviation of temperature	10
ε	Emissivity	0.1
ρ	Reflectivity	0.9

Table 2. General Observation Parameters

Symbol	Parameter	Value
N	Total number of sequential observations	10
ΔT	Time interval between observations (seconds)	1.0
λ_1	Low wavelength of sensor spectral band (μm)	10
λ_2	High wavelength of sensor spectral band (μm)	15
T_e	Apparent temperature of the Earth (Kelvin)	250

The decoy design process is initiated by developing bounds on the physical parameters given a specified error performance and then exploring decoy concepts that could be implemented within these limits. APEM is appropriate for exploring these bounds and initial performance evaluation of prototypical concepts, while a more detailed simulation analysis would be required for final concept selection and evaluation.

For this example scenario the decoy concept must maintain a probability of error as shown in equation (16).

$$P_{error} > 10\ \% \tag{16}$$

4.2. Analysis Results

The model is utilized to determine how different the physical parameters of the decoy can be from those of the target while maintaining the requirement of equation (16). First, the bounds of varying individual parameters are explored while maintaining all other parameters equal for the two classes. Figure 3 shows the result for a difference in the mean projected area.

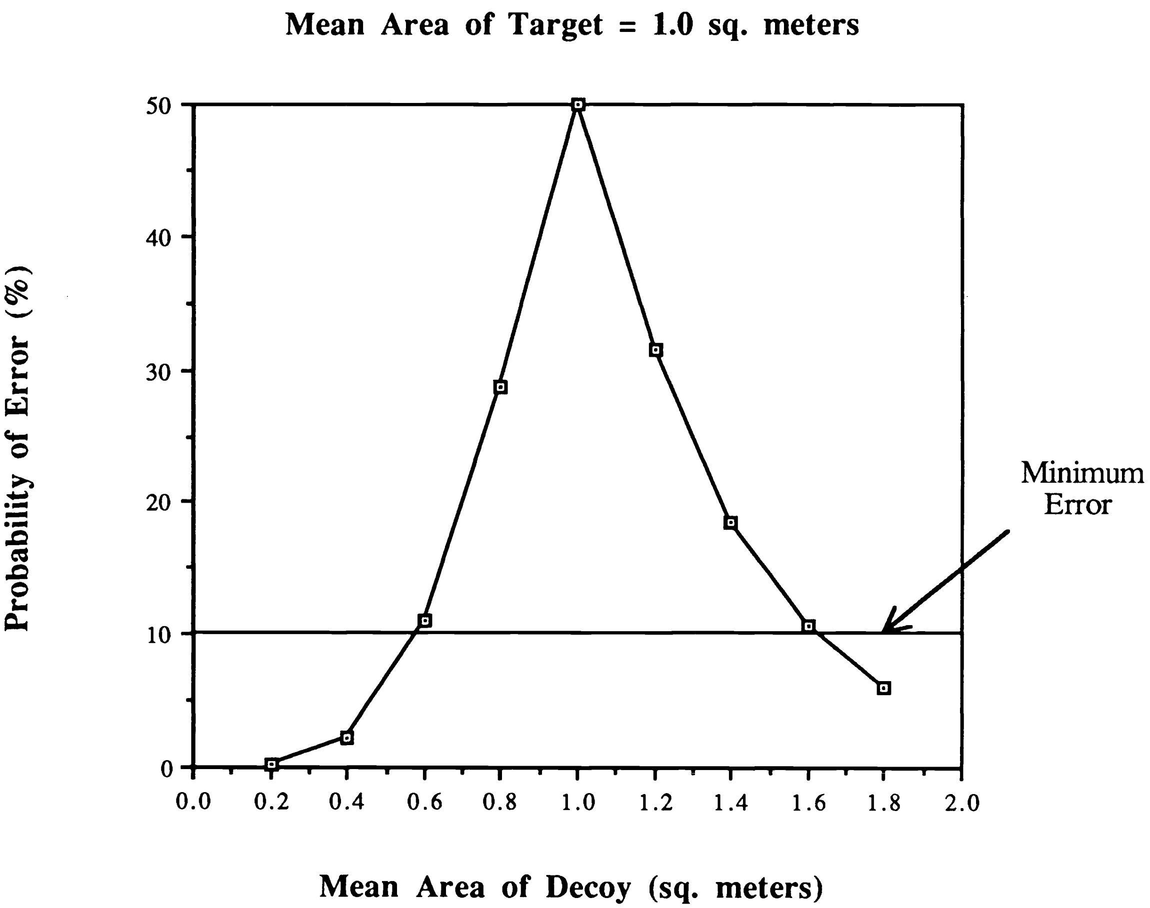

Figure 3. Error performance for difference in mean projected area only.

As can be seen, the error is 50% when all parameters are equal for the two classes, then decreases as the difference in the parameter between the two classes increases and the classes become more separable in the discrimination feature space. Table 3 summarizes the bounds for the other parameters when they are considered individually.

Table 3. Single parameter bounds for decoy parameters and specified error performance.

Symbol	Parameter	Lower Bound	Target Value	Upper Bound
$\overline{A_p}$	Mean projected area (square meters)	0.6	1.0	1.6
σ_A	Standard deviation of projected area	0.0	0.2	1.2
$\overline{R}$	Mean rotational rate (degrees per second)	3.2	5.0	7.7
σ_r	Standard deviation of rotational rate	0.0	1.0	5.0
$\overline{T_o}$	Mean temperature (Kelvin)	0.0	300	440
σ_T	Standard deviation of temperature	0.0	10	250

From the results of Table 3 it can be seen which parameters are important for the decoy concept to "match" closely to those of the target. Projected area and rotational rate are important, while the temperature of the object does not have to be as tightly controlled. This result is primarily due to the low emissivity and high reflectivity chosen for this example.

Of course, in a real situation there are differences in all of the parameters between the classes. The next step in determining bounds involves the study of the effect of having differences in more than one parameter. This can involve many combinations of parameter tradeoffs and indeed the computational simplicity of this model makes it more feasible to consider the various combinations. To illustrate the effects of such combinations an example is presented below.

The two parameters that were the most sensitive in their effect on probability of error were the mean projected area and mean rotation rate. It is appropriate to investigate how differences between the classes in these two parameters interrelate. Figure 4 shows this effect along with a variation in the length of the observation interval. All other parameters are equal for the two classes.

It is interesting to observe that when there is a difference in the mean area between the two classes, the maximum error does not occur when the mean rotational rates are equal. This offset shows the complex interdependence of variations in more than one parameter. Figure 4a shows the result when the mean area of the decoy is less than that of the target. In this case, a higher mean rotational rate results in increased overlap in the feature space and greater classification error than when the rotational rates are equal. Figure 4b shows the opposite effect when the decoy is given a higher mean area than the target. This result indicates that differences in parameters of classes do not always lead to reduced classification error and adds insight in the understanding of the implications of parameter bounds on the design of decoys.

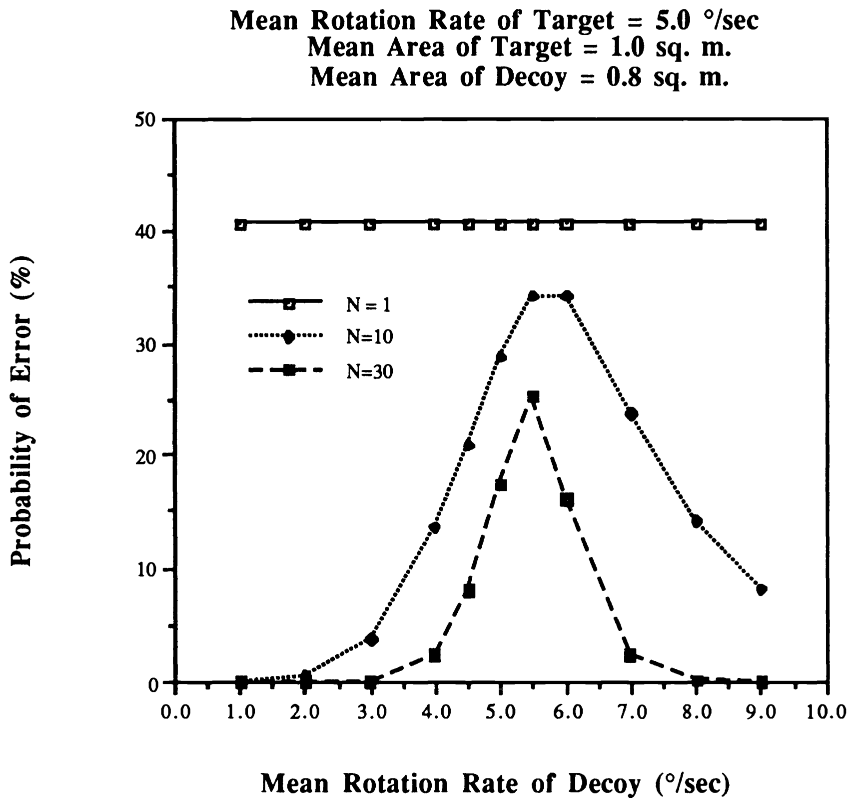

Figure 4a. Interrelationship of Smaller Decoy Mean Projected Area and Mean Rotation Rate.

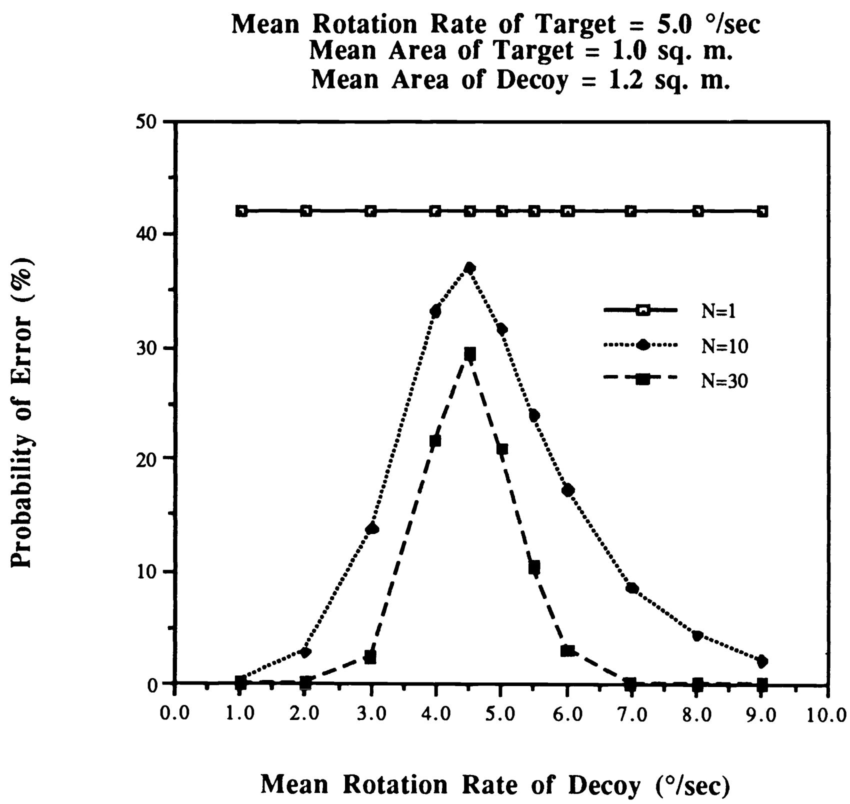

Figure 4b. Interrelationship of Larger Decoy Mean Projected Area and Mean Rotation Rate.

5. SUMMARY AND CONCLUSIONS

A computationally simple model for the evaluation of optical decoy concepts has been introduced. This model considers the probability of error based upon the statistical distance between radiant intensity vectors (sampled over time) of target and decoy classes computed from assumed stochastic models of physical processes and their parameters. The structure of this model and the equations governing the computation of signature statistics were discussed. The model was then applied to an example scenario to illustrate potential applications and uses in gaining insight to parameter interrelationships and their effect on classification error.

The model is most appropriately applied to a scenario when both the objects and the sensor are in the shadow of the Earth, as solar effects were not considered. Also, the effects of a variation in mean projected area over a trajectory or orbit and sensor noise were not considered, although their implementation would be relatively straightforward.

The primary application of this model is to the development of initial requirements for an optical decoy concept by determining bounds on physical parameters given a minimum error performance. These bounds can then be considered when evaluating various decoy concepts. For example, a tight requirement for decoy temperature may limit concepts to those that would allow strict control of its temperature. Once candidate decoy concepts have been considered, those meeting preliminary bounds should be further investigated using a more computationally intensive simulation model.

6. REFERENCE

[1] S.J. Whitsitt and D.A. Landgrebe, "Error Estimation and Separability Measures in Feature Selection for Multiclass Pattern Recognition," LARS Publication 082377, Laboratory for Applications of Remote Sensing, Purdue University, West Lafayette, IN, August 1977.