Improved Modeling of Background Distributions in an End-to-end Spectral Imaging System Model [1]

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***Abstract*— Previously, an analytical end-to-end spectral imaging system model has been developed. The model is constructed around the propagation of spectral statistics from the scene, through the sensor, and processing transformations to lead to prediction of a performance metric. In this analytical framework the description of the class statistics has been by their spectral mean vector and spectral covariance matrix (first and second order statistics). This representation is only strictly accurate when the underlying classes are Gaussian in nature. While some background classes fall into this category, many have been observed to be non-Gaussian in nature. As a work-around for this limitation, we have often formed sub-classes in the background, which when combined form a "composite" background class that can be multi-modal. However, we have observed in estimates of empirical data distributions that uni-modal backgrounds often have longer tails than those predicted by the Gaussian distribution.**

Recently, it has been demonstrated that a family of distributions, known as the elliptically contoured multivariate *t*-distributions, can provide an accurate depiction of empirically observed backgrounds. These distributions are parameterized by their multivariate mean vector and covariance matrix, but also by a degree of freedom parameter, *M*. By varying *M*, excellent fits to empirical distributions have been observed. Another key feature of these distributions is that the number of degrees of freedom has been shown to be invariant to linear transformations. Since the analytical model operates by performing a sequence of linear transformations on the statistics, the input value of *M* is preserved and can be used at any stage of the model to represent the class statistics.

This paper describes an implementation of the elliptically contoured *t*-distributions to represent background classes in the end-to-end system model. The functional form and examples of the *t*-distributions are shown. Results are presented comparing predictions of target detection performance using backgrounds modeled by multiclass Gaussian distributions with the new elliptical-t distributions.

Keywords-hyperspectral data modeling, non-gaussian distributions.

I. INTRODUCTION

Hyperspectral imaging (HSI) systems which collect high resolution imagery in hundreds of narrow, contiguous spectral channels have shown significant promise in the detection of unresolved objects, or targets [1]. However, a significant challenge for automated processing remains in the selection of appropriate detection thresholds to obtain constant false alarm rate (CFAR) performance. This threshold selection depends a great extent upon the statistical distribution of the background.

In support of hyperspectral imaging system design and analysis studies, an analytical end-to-end performance prediction tool has been developed [2]. This model is known as FASSP (Forecasting and Analysis of Spectroradiometric System Performance). One application of this tool is to predict target detection performance for a specified scenario. Through the use of statistical descriptions for the target and background, and linear transformations to model the effects of the observing system and processing, the performance can be predicted analytically, rather than through a physics-based simulation and application of a target detection algorithm. This approach runs very quickly and can support large numbers of trade studies.

However, to date, this model has assumed a Gaussian distribution for the output of a linear matched filter detector in order to calculate detection performance. Studies have shown that hyperspectral data often do not follow a Gaussian distribution [3]. Recent work has demonstrated that these data can be more accurately modeled (particularly in the tails of the distribution) by a class of distributions known as elliptically contoured multivariate t-distributions which include Gaussian as a special case [4]. Accurate modeling of the tails of the background distribution is especially important since this drives the selection of a detection threshold and specification of the false alarm rate.

This paper describes an implementation of the elliptically contoured *t*-distributions to represent background classes in the end-to-end system model. The functional form and examples of the *t*-distributions are shown. Results are presented comparing predictions of target detection performance using backgrounds modeled by multiclass Gaussian distributions with the new elliptical-t distributions.

II. ANALYTICAL END-TO-END SYSTEM MODEL

The underlying premises of the FASSP model are 1) that the various surface classes and subclasses of interest can be represented by first- and second-order spectral statistics and 2) that the effects of various processes in the end-to-end spectral imaging system can be modeled as transformations and

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functions of those statistics. The model propagates the spectral statistics through the effects of the atmosphere, the sensor, atmospheric compensation, feature extraction techniques, and then applies a detection algorithm to convert the high dimensional statistics to a scalar test statistic (matched filter output) to which a threshold can be applied and detection performance computed.

One application of the model is for unresolved, or sub-pixel target detection scenarios. Here the linear mixing model is used and the pixel of interest containing the target is assumed to be a sample from random process described by the area-weighted mixture of the target and the background classes. The rest of the analytically-described scene (no simulated "image" is generated by the model) is comprised of a number of homogeneous background classes, each covering some area fraction of the scene. The scene false alarm rate at a given threshold is then comprised of the area-weighted total of false alarms from each individual class.

As mentioned in the introduction, these class-specific false alarm rates have been computed assuming a Gaussian distribution for the output of the matched filter detector. This idealized assumption can have the effect of under-predicting the false alarm rate for a scene. By implementing the elliptical-t distributions described in this paper, the expectation is the performance predictions will be more realistic.

The actual implementation of the elliptical-t distributions is quite straightforward. They are parameterized by the mean vector and covariance matrix, as in the Gaussian case, but also by a degree of freedom parameter. The key fact here is that this degree of freedom parameter is invariant to linear transformations [5] and thus can be specified for a class in the scenario description and remain constant through the application of the transformations which model the remote sensing process.

III. BACKGROUND MODELING

The probability density function (PDF) of the multivariate elliptical t-distribution is shown in equation (1). Γ() denotes the gamma function.

$$t(\boldsymbol{x};\mu,\Sigma,M) = \frac{\Gamma\left(\frac{K+M}{2}\right)}{(\pi M)^{K/2}\,\Gamma(M/2)|\Sigma|^{1/2}} \times \left[1+\frac{1}{M}(\boldsymbol{x}-\mu)^T\Sigma^{-1}(\boldsymbol{x}-\mu)\right]^{-\frac{K+M}{2}} \qquad (1)$$

Where,

μ	mean vector,
Σ	covariance matrix,
K	dimensionality of data, and
M	degrees of freedom.

This distribution converges to two special cases for certain values of M. For M=1, it converges to the multivariate elliptical Cauchy distribution. For M=∞, it converges to the multivariate Gaussian.

The appropriateness of the elliptical-t model for hyperspectral data has been investigated and found to be a good model for the tails of the distribution [3,4]. Also, it has been found to be very flexible through the specification of the degrees of freedom parameter. Figure 1 presents plots of the probability of exceedance versus the Mahalanobis Distance between data samples and their class means, for several classes in a hyperspectral data set (solid lines), and for elliptical-t distributions with varying degrees of freedom (dashed lines). Note that many of the classes have tails that extend far to the right, as opposed to the Gaussian curve which falls nearly straight down (thick black dashed line in Figure 1.)

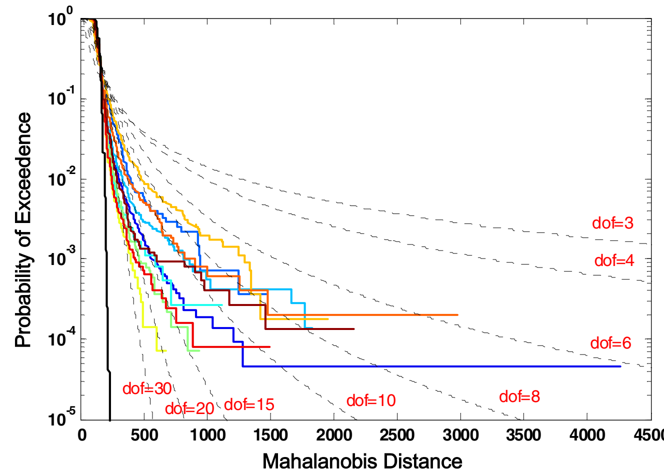

Figure 1. Empirically observed distributions (solid curves) vs. elliptical-t distributions (dashed curves) of with varying degrees of freedom.

An investigation was undertaken to compare the matched filter output distributions under the elliptical-t model to the multiclass Gaussian background composite model assumed previously within the end-to-end forecast model FASSP. A multiclass background was defined as shown in Table 1.

TABLE I. MULTICLASS SCENE COMPOSITE BACKGROUND

Class	Area of Scene
Tree type 1	24%
Tree type 2	25%
Grass type 1	20%
Grass type 2	20%
Bushes	5%
Road type 1	5%
Bare wood	1%

A man-made material was used as the target signature and a matched filter operator derived from the weighted average covariance of the composite background shown in Table 1. This operator was then applied to the individual background class statistics to generate the scalar PDFs shown in Figure 2.

Note that the matched filter does a good job of projecting the majority background classes to near zero. However, the

Bare wood class (1% of the scene) was not suppressed as well and extends far to the right where the target distribution will likely fall.

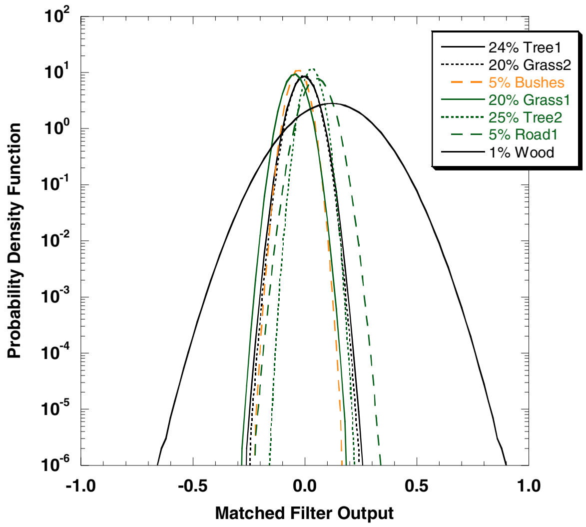

Figure 2. Probability density functions (PDFs) for the various classes modeled as Gaussian distributions.

These statistics can be related to a total false alarm rate and compared to rates derived using the elliptical-t model. Figure 3 presents this comparison. The composite curve in Figure 3 was computed by integrating from -∞ to the output value each of the individual classes in Figure 2, and the summing the results using the appropriate class area weighting. The various elliptical-t distributions were computer using the scene average mean and standard deviation output from the filter, the indicated number of degrees of freedom, and then integrating from -∞ to the output value using a scalar version of Eq 1.

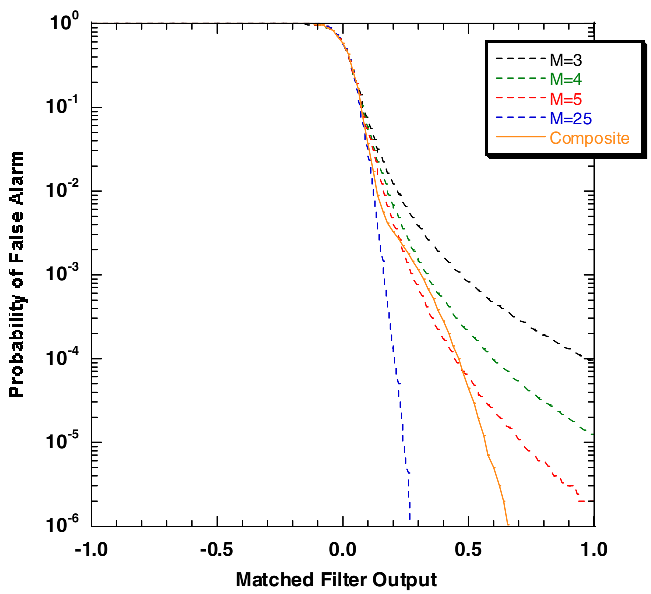

Figure 3. False alarm rates as a function of matched filter output threshold for the composite multiclass Gaussian scene (Table 1) and elliptical-t distributions with various degrees of freedom.

As can be seen, the composite multiclass Gaussian background model compares closely to the elliptical-t with M = 4 or 5 at false alarm rates greater than 10^{-3} or 10^{-4}. At false alarm rates lower than that, the two models diverge. Also, the elliptical-t model approach has the advantage of being able to specify *a priori* the "fatness" of the tails and allow the model to predict realistic false alarm rates with only a single background class.

IV. Detection Performance

While examination of the distributions of the matched filter output is interesting, the primary metric in the prediction of performance for target detection scenarios is the Receiver Operating Characteristic (ROC) curve. The FASSP model predicts this curve given the scene, the observing sensor, and the processing algorithm.

Figure 4 presents the ROC curves predicted for a given target with several different backgrounds. The solid curves predict performance for a single class background and two multiclass backgrounds, all assumed to be Gaussian. Here the background set labeled "Multiclass#2" is identical to the set shown in Table 1, while the set labeled "Multiclass#1" is the same, except the "Bare wood" class is removed and the additional 1% of the scene area allocated to "Tree type 1". As can be scene, the presence of the 1% bare wood leads to a lower detection probability at a given false alarm rate since it comprises such a small fraction of the scene and is not rejected as much as the rest of the background.

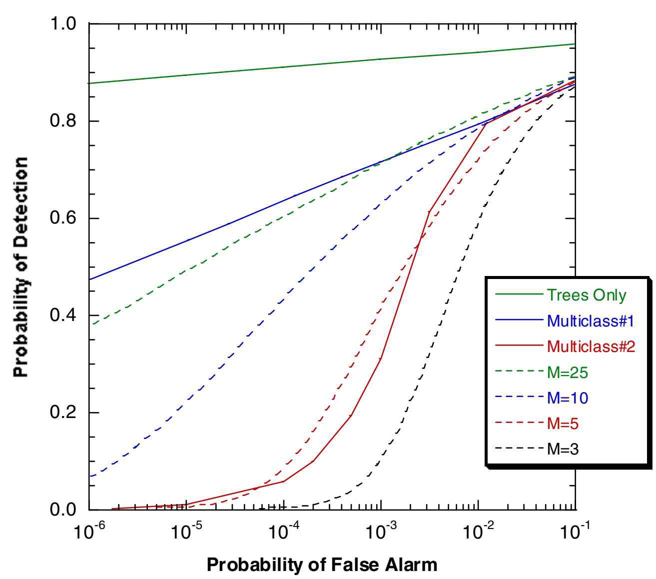

Figure 4. ROC curves for a given target in various Gaussian backgrounds (solid curves) and elliptical-t distributed backgrounds (dashed curves).

The dashed curves in Figure 4 present the performance for a single background class that has an elliptical-t distribution with the indicated number of degrees of freedom. Here we see that performance (P_D) degrades rapidly for backgrounds with a low number of degrees of freedom. However, we do see that similar performance predictions can be achieved using the two modeling approaches and appropriate scene descriptions.

Another way to compare the detection performance predictions is to look at detection performance (at a constant false alarm rate) as a function of target subpixel fraction. Figure 5 shows the results of using the model for the same variety of background types and distributions used for Figure 4.

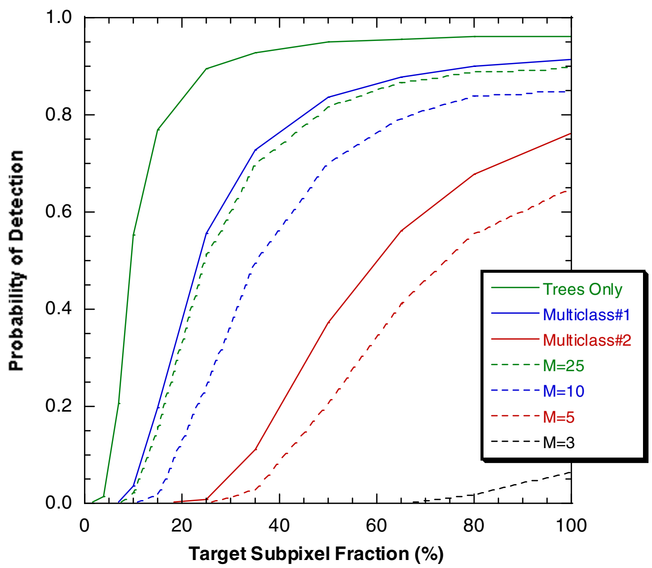

Figure 5. Probability of detection vs. target subpixel fraction curves for a given target in various Gaussian backgrounds (solid curves) and elliptical-t distributed backgrounds (dashed curves).

As can be seen, by varying the background complexity (and/or background distribution) the model predicts a wide range of subpixel fractions necessary to achieve a given P_D. With only a single Gaussian Tree background, the model predicts the target is detectable (defined as $P_D \geq 0.8$) at around 15% pixel fill fraction, while for the same target in a complex Gaussian background (Multiclass#2), this detection threshold is not quite achieved even for a full pixel target. For the simpler Gaussian multiclass background (Multiclass#1) and elliptical-t backgrounds with a number of degrees of freedom between 10 and 25, the model predicts detection at 40-60% pixel fill fractions. One item of note here is that in all cases the spectral contrast between the target and background is nearly identical; the driving factor for these performance predictions is the complexity of the background and its assumed statistical distribution.

V. Summary and Future Work

A new model for the statistical distribution of background samples has been described and its impact on performance prediction compared to the original multiclass Gaussian background approach. Elliptical-t distributions have been found to lead to more realistic performance predictions without the need to describe a complex scene. Also, since the elliptical-t distributions are described by their mean, covariance, and a single scalar degree of freedom parameter (which is invariant to linear transforms), these distributions match up well with the modeling assumptions made in the analytical performance prediction tool FASSP.

Future work involves further verification with empirical data of the suitability of the elliptical-t distributions as well as studies to accurately estimate the degree of freedom parameter for various background scene types and sensor spatial resolutions.

VI. Acknowledgements

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