1. Use the definitions:

$$f_e[x] = f_e[-x]$$
$$f_o[x] = -f_o[-x]$$

to derive the expressions for the even and odd functions.

2. Consider two spatial sinusoids with the same spatial frequency ξ_0 but arbitrary amplitudes A_1 and A_2 and arbitrary phases ϕ_1 and ϕ_2:

   (a) Prove that the sum of these two sinusoids is a sinusoid with that same frequency ξ_0.

   (b) Find the expression that relates the amplitude A and phase ϕ of the summation sinusoid.

3. For a sinusoidal functions whose phase is a power of the coordinate:

$$f[x] = \cos\left[\pi\left(\frac{x}{\alpha}\right)^n + \phi_0\right]$$

   (a) Graph the function for $\alpha = 1$, $\phi_0 = \frac{\pi}{4}$, and $n = 1$, 3, and 4.

   (b) Find the equation for the spatial frequency of $f[x]$.

   (c) Determine the dimensions of the parameter α.

4. Find the lengths of the 3-D vectors:

   (a) a. $\begin{pmatrix} +1 \\ +2 \\ -1 \end{pmatrix}$ b. $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$ c. $\begin{pmatrix} \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix}$

5. Find k so that the two vectors $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$ are orthogonal:

$$\underline{\mathbf{a}} = \begin{pmatrix} 2 \\ -5 \\ 4 \end{pmatrix}, \underline{\mathbf{b}} = \begin{pmatrix} 1 \\ k \\ -3 \end{pmatrix}$$

6. The three vectors $\underline{\mathbf{a}}, \underline{\mathbf{b}}, \underline{\mathbf{c}}$ have real-valued components and form the sides of a triangle, e.g., $\underline{\mathbf{a}} + \underline{\mathbf{b}} = \underline{\mathbf{c}}$. Prove the "law of cosines" for these vectors, *i.e.,* that:

$$|\underline{\mathbf{c}}|^2 = |\underline{\mathbf{a}}|^2 + |\underline{\mathbf{b}}|^2 - 2\underline{\mathbf{a}} \bullet \underline{\mathbf{b}}$$
$$= |\underline{\mathbf{a}}|^2 + |\underline{\mathbf{b}}|^2 - 2|\underline{\mathbf{a}}||\underline{\mathbf{b}}|\cos(\theta)$$

where "$\bullet$" represents the scalar product and θ is the angle between the vectors $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$.