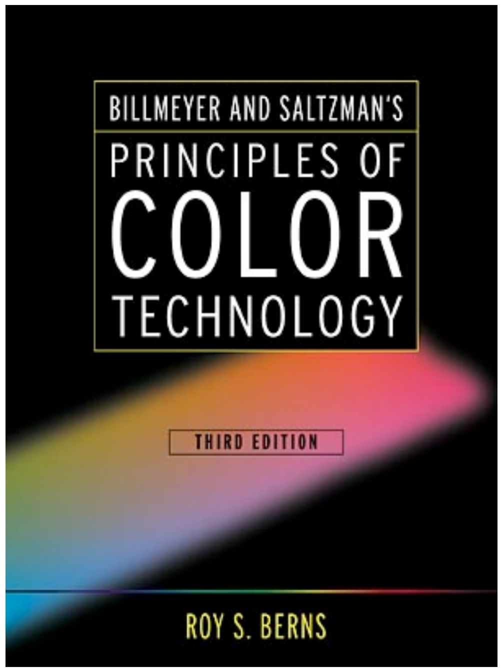
# Errata V10 and Updates

## R. S. Berns, Billmeyer and Saltzman's Principles of Color Technology, 3rd edition, John Wiley, New York, 2000

## 4th Printing




***How to tell which printing?***
You can tell by looking at the copyright page. On the bottom, you'll see numbers: 10 9 … 4. The last number lists the printing.

Thanks to all my readers for pointing out errors,
Roy Berns
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December 3, 2007

# Errata

**Page 52**
Right column, line 8. Add quotes: “Did you or did you not sell this as Selected Rhubarb?”

**Page 59**
Bottom table. The correct legend: The following table of tristimulus values for several common illuminant and observer combinations and has both CIE and ASTM values. ASTM values from E-308 are on the left. CIE values from Publication 15.2 are on the right.

The D50 2° values for Z should be 82.521 for ASTM and 82.49 for CIE.

**Page 60**
Left column, second plot. The axes should be script $\mathcal{X}$, $\mathcal{Y}$, and $\mathcal{Z}$ as they are defining the three-space axes.

**Page 65**
Left sidebar, remove the second u′ as it is redundant.

**Page 69**
The CIELAB L* equation is shown for very dark colors. For colors with $Y/Y_n$ greater than 0.008856, the equation should be:

$$L^* = 116(Y/Y_n)^{1/3} - 16$$

This was correct in the first two printings and changed incorrectly in the third printing, and not corrected in the fourth printing. In the Update section, I have written a new sidebar that can be printed, cut out, and pasted in.

**Page 70**
Right column, yellowness index equation. The correct equation should be:

$$YI = 100\frac{1.2769X - 1.0592Z}{Y}$$

**Page 103**
Right column. The equations have errors. The correct equations should be:

Bidirectional geometries:

$$E_{\substack{\text{reference}\\\text{white}}} = -2.79\Delta L^* + 1.50\Delta a^* + 2.96\Delta b^*$$

$$E_{\substack{\text{reference}\\\text{black}}} = -0.32\Delta L^* - 0.48\Delta a^* - 0.42\Delta b^*$$

$$E_{\text{wavelength}} = 0.08\Delta L^* - 0.82\Delta a^* + 0.67\Delta b^*$$

Integrating sphere, specular component included geometries

$$E_{\substack{\text{reference}\\\text{white}}} = -2.58\Delta L^* + 1.79\Delta a^* + 3.04\Delta b^*$$

$$E_{\substack{\text{reference}\\\text{black}}} = -0.36\Delta L^* - 0.62\Delta a^* - 0.54\Delta b^*$$

$$E_{\text{wavelength}} = 0.05\Delta L^* - 0.95\Delta a^* + 0.77\Delta b^*$$

**Page 202**
Right column, matrix inversion. The correct equation should be:

$$
\begin{aligned}
R &= \frac{Y_{g,\max}Z_{b,\max} - Y_{b,\max}Z_{g,\max}}{\Delta}X + \frac{X_{b,\max}Z_{g,\max} - X_{g,\max}Z_{b,\max}}{\Delta}Y \\
&\quad + \frac{X_{g,\max}Y_{b,\max} - X_{b,\max}Y_{g,\max}}{\Delta}Z \\
G &= \frac{Y_{b,\max}Z_{r,\max} - Y_{r,\max}Z_{b,\max}}{\Delta}X + \frac{X_{r,\max}Z_{b,\max} - X_{b,\max}Z_{r,\max}}{\Delta}Y \,, \\
&\quad + \frac{X_{b,\max}Y_{r,\max} - X_{r,\max}Y_{b,\max}}{\Delta}Z \\
B &= \frac{Y_{r,\max}Z_{g,\max} - Y_{g,\max}Z_{r,\max}}{\Delta}X + \frac{X_{g,\max}Z_{r,\max} - X_{r,\max}Z_{g,\max}}{\Delta}Y \\
&\quad + \frac{X_{r,\max}Y_{g,\max} - X_{g,\max}Y_{r,\max}}{\Delta}Z
\end{aligned}
\tag{A-6}
$$

**Page 208**
Right column, (K/S) of the unknown should be capital K and S and the substrate should be subtracted. The correct equation should be:

$$
\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} =
\begin{bmatrix}
\left(\frac{k}{s}\right)_{\lambda=1,1} & \left(\frac{k}{s}\right)_{\lambda=1,2} & \left(\frac{k}{s}\right)_{\lambda=1,3} \\
\left(\frac{k}{s}\right)_{\lambda=2,1} & \left(\frac{k}{s}\right)_{\lambda=2,2} & \left(\frac{k}{s}\right)_{\lambda=2,3} \\
\left(\frac{k}{s}\right)_{\lambda=3,1} & \left(\frac{k}{s}\right)_{\lambda=3,2} & \left(\frac{k}{s}\right)_{\lambda=3,3}
\end{bmatrix}^{-1}
\begin{bmatrix}
\left(\frac{K}{S}\right)_{\lambda=1,unknown} - \left(\frac{k}{s}\right)_{\lambda=1,t} \\
\left(\frac{K}{S}\right)_{\lambda=2,unknown} - \left(\frac{k}{s}\right)_{\lambda=2,t} \\
\left(\frac{K}{S}\right)_{\lambda=3,unknown} - \left(\frac{k}{s}\right)_{\lambda=3,t}
\end{bmatrix}.
\tag{E-4}
$$

**Page 214**
Left column, Eq. (F-9) should be:

$$
\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} =
\begin{pmatrix}
0.98699 & -0.14705 & 0.15996 \\
0.43231 & 0.51836 & 0.04929 \\
-0.00853 & 0.04004 & 0.96849
\end{pmatrix}
\begin{pmatrix} R_c Y \\ G_c Y \\ B_c Y \end{pmatrix}.
\tag{F-9}
$$

Right column, second to last sentence of the table description. "Eq. (49)" should be Eq. (F-10).

**Page 217**
Right column, Eq. (G-6) should be:

$$
\begin{bmatrix} R_{display} \\ G_{display} \\ B_{display} \end{bmatrix} =
\begin{bmatrix}
X_{r,\max} & X_{g,\max} & X_{b,\max} \\
Y_{r,\max} & Y_{g,\max} & Y_{b,\max} \\
Z_{r,\max} & Z_{g,\max} & Z_{b,\max}
\end{bmatrix}^{-1}
\begin{bmatrix} X_{display} \\ Y_{display} \\ Z_{display} \end{bmatrix}
\tag{G-6}
$$

**Page 241**
Left column, P.L Vora and H. J. Trussell pages are incorrect. They should be 1499-1508.

## Updates

**Page 69**
Replace side bar with:

Here are the equations for the CIE 1976 *L*a*b** (CIELAB) space. Its calculation depends on the ratio of either $X/X_n$, or $Y/Y_n$, or $Z/Z_n$ [notated *f(x)*] where $X_n$, $Y_n$, and $Z_n$ are the tristimulus values of the reference white (see page 59).

$$L^* = 116 f(Y/Y_n) - 16$$

$$a^* = 500\left[f(X/X_n) - f(Y/Y_n)\right]$$

$$b^* = 200\left[f(Y/Y_n) - f(Z/Z_n)\right]$$

$$f(x) = \begin{Bmatrix} x^{1/3} & x > 0.008856 \\ 7.787x + 16/116 & x \le 0.008856 \end{Bmatrix}$$

$$C^*_{ab} = \sqrt{a^{*2} + b^{*2}}$$

$$h_{ab} = tan^{-1}\left(b^*/a^*\right) = arctan\left(b^*/a^*\right)$$

The reverse equations are

$$X = X_n f^{-1}\left(\frac{L^*+16}{116} + \frac{a^*}{500}\right)$$

$$Y = Y_n f^{-1}\left(\frac{L^*+16}{116}\right)$$

$$Z = Z_n f^{-1}\left(\frac{L^*+16}{116} - \frac{b^*}{200}\right)$$

$$f^{-1}(x) = \begin{Bmatrix} x^3 & x > 0.206893 \\ \frac{1}{7.787}\left(x - \frac{16}{116}\right) & x \le 0.206893 \end{Bmatrix}$$

When colors are defined by $L^*$, $C^*_{ab}$, and $h_{ab}$, first transform back to *a** and *b**.

$$a^* = C^*_{ab}\cos(h_{ab})$$

$$b^* = C^*_{ab}\sin(h_{ab})$$

**Page 72**
Left column, side bar. The following equation clarifies the geometric meaning of $\Delta H^*_{ab}$ compared to what is written in the book:

$$\Delta H^*_{ab} = 2\left(C^*_{ab,standard} C^*_{ab,batch}\right)^{1/2} \sin\left(\frac{h_{ab,batch} - h_{ab,standard}}{2}\right)$$

**Page 121**
**Recent CIE Color-Difference Activities**
Since the book's publication in 2000, the CIE technical committee 1-47 has developed a new color-difference equation, CIEDE2000 or $\Delta E_{00}$. Its derivation is described in Luo, Cui, Rigg, The development of the CIE 2000 colour-difference formula: CIEDE2000, *Color Research Application*, 26:340-350 (2001) and CIE Publication 142-2001Improvement to Industrial Color-Difference Evaluation. The specific mathematics are shown below:

$$\Delta E_{00} = \left[\left(\frac{\Delta L'}{k_L S_L}\right)^2 + \left(\frac{\Delta C'_{ab}}{k_C S_C}\right)^2 + \left(\frac{\Delta H'_{ab}}{k_H S_H}\right)^2 + R_T\left(\frac{\Delta C'_{ab}}{k_C S_C}\frac{\Delta H'_{ab}}{k_H S_H}\right)\right]^{1/2}$$

$$L' = L^*$$

$$a' = a^*(1+G)$$

$$b' = b^*$$

$$G = 0.5\left(1 - \sqrt{\frac{\overline{C}^{*\,7}_{ab}}{\overline{C}^{*\,7}_{ab} + 25^7}}\right)$$

$$S_L = 1 + \frac{0.015\left(\overline{L}'-50\right)^2}{\sqrt{20+\left(\overline{L}'-50\right)^2}}$$

$$S_C = 1 + 0.045\overline{C}'$$

$$S_H = 1 + 0.015\overline{C}'T$$

$$\begin{aligned} T = 1 &- 0.17\cos(\overline{h}' - 30) \\ &+ 0.24\cos(2\overline{h}') \\ &+ 0.32\cos(3\overline{h}' + 6) \\ &- 0.20\cos(4\overline{h}' - 63) \end{aligned}$$

$$R_T = -\sin(2\Delta\Theta)R_c$$

$$\Delta\Theta = 30\exp\left(-\left(\frac{\overline{h}'-275}{25}\right)^2\right)$$

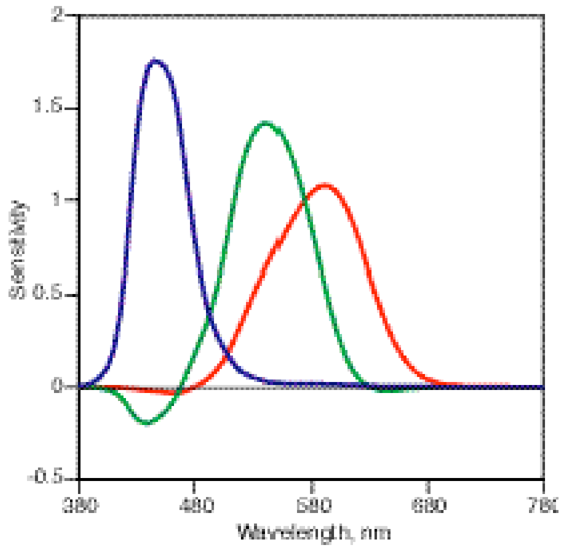
$$R_c = 2\left(\frac{\overline{C}'^7}{\overline{C}'^7+25^7}\right)^{1/2}$$

**Pages 213 – 215**
The Bradford chromatic-adaptation transformation, shown in Eqs. (F-7) – (F-9), has been replaced with the CIE chromatic-adaptation transformation that is part of CIECAM02. See: International Commission on Illumination, CIE Pub 159: 2004. A Color appearance model for color management systems. Vienna: CIE Central Bureau, 2004.

The math is the following:

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \mathbf{M}_{CAT02} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$
$$\mathbf{M}_{CAT02} = \begin{pmatrix} 0.7328 & 0.4296 & -0.1624 \\ -0.7036 & 1.6975 & 0.0061 \\ 0.0030 & 0.0136 & 0.9834 \end{pmatrix}. \quad \text{(F-7)}$$

This leads to the following set of pseudo-cone fundamentals:




The von Kries adaptation transform is the following:

$$R_c = \frac{R_{D65}}{R_n} R$$
$$G_c = \frac{G_{D65}}{G_n} G. \quad \text{(F-8)}$$
$$B_c = \frac{B_{D65}}{B_n^p} B$$

This is advantageous over the Bradford transform because it is readily invertible. The inverse matrix to (F-7) is the following:

$$\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix} = \begin{pmatrix} 1.096124 & -0.278869 & 0.182745 \\ 0.454369 & 0.473533 & 0.072098 \\ -0.009628 & -0.005698 & 1.015326 \end{pmatrix} \begin{pmatrix} R_c \\ G_c \\ B_c \end{pmatrix}. \quad \text{(F-9)}$$

The corresponding color calculation and color inconstancy index were recalculated for the vinyl.

Here are the results:

| | A, 10° | D65, 10° |
|---|---|---|
| Xn | 111.1 | 94.8 |
| Yn | 100.0 | 100.0 |
| Zn | 35.2 | 107.2 |
| | | |
| X | 36.7 | 24.8 |
| Y | 26.2 | 23.2 |
| Z | 4.3 | 11.8 |
| | | |
| L* | 58.3 | 55.3 |
| a* | 25.6 | 12.4 |
| b* | 28.7 | 27.0 |
| | | |
| Rn | 118.7 | 95.0 |
| Gn | 91.8 | 103.7 |
| Bn | 36.3 | 107.0 |
| | | |
| R | 37.5 | 26.2 |
| G | 18.7 | 22.0 |
| B | 4.7 | 12.0 |
| | | |
| | | |
| Rc | 30.0 | |
| Gc | 21.1 | |
| Bc | 13.9 | |
| | | |
| Xc | 29.5 | |
| Yc | 24.7 | |
| Zc | 13.7 | |
| | | |
| L*c | 56.7 | |
| a*c | 25.5 | |
| b*c | 24.7 | |
| | | |
| ΔL* | 1.4 | |
| ΔC* | 5.8 | |
| ΔH*ab | 12.0 | |
| | | |
| | | |
| $\Delta E^*_{94(2:2:1)}$ | 8.4 | |