

Trichromatic color matching

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Trichromatic color matching is based on the principle that the color of a test light, with any spectral power distribution, can be matched by an additive mixture of three primary lights, usually red, green and blue. This is sometimes referred to as Grassmann's first law. In a typical colorimeter used to verify this statement, one half of the visual field that the observer examines is uniformly illuminated with an essentially monochromatic spectral light. This has a constant intensity and a known wavelength. The other half of the field is simultaneously illuminated by overlapping the beams of each of the red, blue and green primary lights. The intensities of these can be independently varied (See Figure 1).

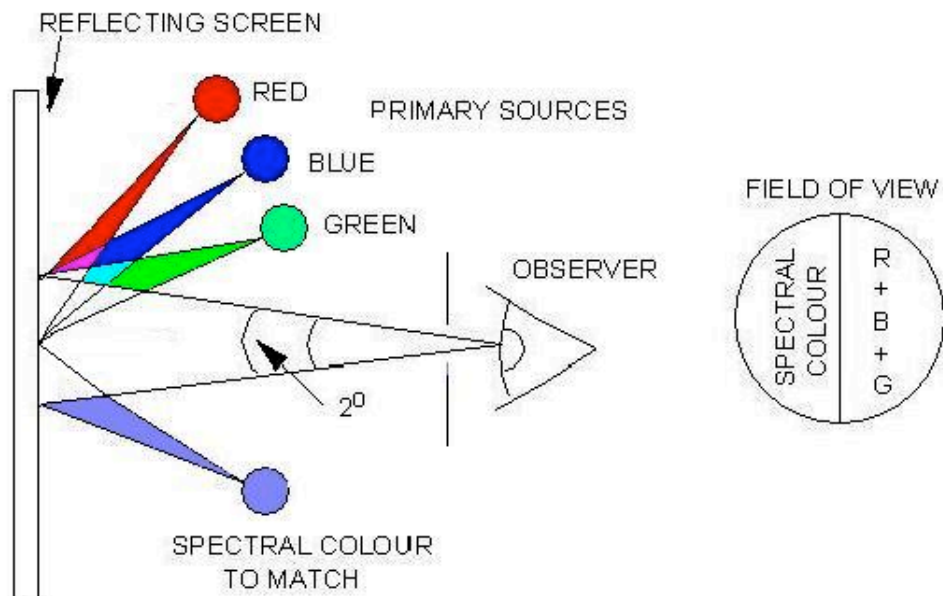


Figure 1. Simplified schematic of a visual colorimeter for trichromatic color matching.

There are a number of important experimental requirements for validation of Grassmann's first law:

1. The three primaries are chosen so that no mixture of any two can match the third one.
2. The level of illumination is relatively high to ensure exclusive cone vision (typical of daylight or indoor light).
3. The angular size of the viewing field is limited to about 2° to ensure that light falls on the foveal region of the retina where the cone cells predominate.
4. The observer has normal color vision and is not fatigued.

The amounts of the three primaries required for matching a given spectral light can be expressed in units of radiant power (Watts per steradian) or the equivalent luminous flux (lumen per steradian or candela), derived by appropriate calibration of the scales of the apertures or optical wedges that control these amounts. For example, if the amounts are recorded in terms of luminous intensity (candela), the matching condition can be expressed by a colorimetric equation such as

$$L C(\lambda) \equiv L_1 [R] + L_2 [G] + L_3 [B] \quad (1)$$

where $C(\lambda)$ is the test light of wavelength λ and luminous intensity L , $[R]$, $[G]$ and $[B]$ represent unit luminous intensity of the respective the primary (red, green and blue) and L_1 , L_2 and L_3 are scalar quantities called tristimulus values giving the amounts of the appropriate primary. A colorimetric equation is not an algebraic equation. The equivalence sign simply signifies that the test color C is visually identical to that obtained by mixing the given amounts, L_1 , L_2 and L_3 , of the three respective primaries. When the additive mixture of the three primaries exactly matches the test color, the luminous intensity of the latter is thus given by $L = L_1 + L_2 + L_3$.

If the test color's luminous intensity is changed by a factor n , the three tristimulus values all change by the same factor, so that

$$nL C(\lambda) \equiv nL_1 [R] + nL_2 [G] + nL_3 [B]. \quad (2)$$

This is Grassmann's second law.

For two different test colors $C(\lambda)$ and $C'(\lambda)$ described by

$$L C(\lambda) \equiv L_1 [R] + L_2 [G] + L_3 [B] \quad \text{and} \quad L' C'(\lambda) \equiv L'_1 [R] + L'_2 [G] + L'_3 [B] \quad (3)$$

Grassmann's third law of additivity states that for an additive mixture of the two test lights

$$L C(\lambda) + L' C'(\lambda) \equiv (L_1 + L'_1) [R] + (L_2 + L'_2) [G] + (L_3 + L'_3) [B]. \quad (4)$$

This principle of additivity is key to all colorimetry.

In actual practice, universal adherence to Grassmann's first law for exact matching of the color of any spectral light, or any other highly saturated color, requires that one of the tristimulus values be negative. This means that an exact match is only possible if the test light is somewhat desaturated by mixing it with a small amount of light from one of the primaries and the resulting color is then matched using a mixture of the other two primaries illuminating the adjacent field. A typical result for matching a blue-green spectral light might thus usually be of the type

$$L C(\lambda) + L_1 [R] \equiv L_2 [G] + L_3 [B] \quad \text{or} \quad L C(\lambda) \equiv -L_1 [R] + L_2 [G] + L_3 [B] \quad (5)$$

and a negative value is recorded for the tristimulus value for the red primary.

For a series of colors C, C_1, C_2 , etc., that differ only in total luminous intensity,

$$L C \equiv L_1 [R] + L_2 [G] + L_3 [B] \quad (6)$$

$$n_1 L C_1 \equiv n_1 L_1 [R] + n_1 L_2 [G] + n_1 L_3 [B] \quad (7)$$

$$n_2 L C_2 \equiv n_2 L_1 [R] + n_2 L_2 [G] + n_2 L_3 [B] \quad (8)$$

They form a series of stimuli with the same relative spectral power distribution. At any given wavelength, the radiant powers for these test colors have the same constant ratio. Such a series might include a bright-reddish yellow, an orange, a duller orange and a brown. These colors are all of the same quality and are said to have the same chromaticity. Even though the color sensations they provoke are different, there are definite advantages in considering them as having the same chromaticity. It is usual to specify such colors of constant chromaticity by their chromaticity co-ordinates.

$$C_i \equiv r [R] + g [G] + b [B] \quad (9)$$

where

$$r = L_1 / (L_1 + L_2 + L_3), \quad g = L_2 / (L_1 + L_2 + L_3) \quad \text{and} \quad b = L_3 / (L_1 + L_2 + L_3). \quad (10)$$

Note that colors C_1 and C_2 have the same values of r, g and b , because in the normalization the values of n_i cancel. This normalization of the values of the scalar coefficients gives chromaticity co-ordinates whose sum is equal to unity. This separates the quality of the color, its chromaticity, from its luminous intensity. Equation 10 is called a unit trichromatic equation. One advantage of using chromaticity co-ordinates is that two of the values can be represented by a point on a rectangular graph, the third value being known by difference.

The CIE system for the numerical specification of the color of a light or surface is based on a description of the average human response to the different colors of the visible spectrum as determined by trichromatic color matching using a visual colorimeter. For each monochromatic test light the observer adjusts the relative amounts of the red, blue and green primary lights until the two halves of the visual field match exactly. This type of measurement is repeated for an entire series of spectral lights of varying wavelength throughout the visible spectrum. The varying amounts of the primaries required to match each of the spectral lights of given wavelength describe that particular observer's visual response to different colored lights. The measurements are performed with a number of different observers of normal color vision. At any particular wavelength, there are usually only small differences in the three tristimulus values or chromaticity co-ordinates obtained by the different observers and the values are thus averaged.

These types of measurements must be performed under controlled conditions to give reproducible results. In particular, in the initial studies on which the CIE 1931 standard observer is based, the visual field subtended an angle of 2° at the observer's pupil. Adjustable apertures or optical wedges allow variation of the intensities of the three primary lights. It is usual, however, to establish the units for the primaries so that a mixture of equal amounts of the three match the color of a standard white. This result is expressed by the colorimetric equation

$$\text{White} \equiv 0.333 [\text{R}] + 0.333 [\text{G}] + 0.333 [\text{B}], \quad (11)$$

where [R], [G] and [B] represent, respectively, one trichromatic unit of each of the primaries (red, green and blue). Note that the equal amounts of the primaries are normalized so that their sum is unity. Using the units established in this way, the result of matching a spectral color C of wavelength λ is thus given by

$$C(\lambda) \equiv R(\lambda) [\text{R}] + G(\lambda) [\text{G}] + B(\lambda) [\text{B}] \quad (12)$$

where $R(\lambda)$, $G(\lambda)$ and $B(\lambda)$ are scalar tristimulus values and represent the number of units of the appropriate primary. Their values are wavelength dependent. Again, these values are usually normalized to give

$$\begin{aligned}
r(\lambda) &= \frac{R(\lambda)}{R(\lambda) + G(\lambda) + B(\lambda)} & g(\lambda) &= \frac{G(\lambda)}{R(\lambda) + G(\lambda) + B(\lambda)} \\
b(\lambda) &= \frac{B(\lambda)}{R(\lambda) + G(\lambda) + B(\lambda)}
\end{aligned}
\tag{13}$$

so that $r(\lambda) + g(\lambda) + b(\lambda) = 1$. These three coefficients are the spectral chromaticity co-ordinates. This normalization procedure separates the quality of a spectral color, its chromaticity described by $r(\lambda)$, $g(\lambda)$ and $b(\lambda)$, from its luminance or radiance.

Wright¹ gives a simple example illustrating how this is done. If matching the standard white light requires readings from the calibrated aperture scales such that

$$\text{White} = 30 [\text{R}] + 15 [\text{G}] + 60 [\text{B}], \tag{14}$$

then, for a system of units such that equal amounts match the standard white, the red reading must be multiplied by 2 and the green reading by 4. These factors are also then applied to the scale readings for matching the spectral test lights. Thus, if a spectral light is matched when the scale readings are 40, 10 and 20 for the red, green and blue primaries respectively, then the match corresponds to (2×40) , (4×10) and 20 units of the respective primaries. The chromaticity co-ordinates based on this system of units are thus

$$\begin{aligned}
r(\lambda) &= \frac{80}{80 + 40 + 20} = 0.571 & g(\lambda) &= \frac{40}{80 + 40 + 20} = 0.286 \\
b(\lambda) &= \frac{20}{80 + 40 + 20} = 0.143
\end{aligned}
\tag{15}$$

We are now ready to examine some experimental data. Return to the file “1931CIE_RGB.doc”.

1. Wright W.D. A re-determination of the trichromatic coefficients of the spectral colours. *Trans. Opt. Soc. London*, **30**, 141-164 (1928-29).