

Multi-spectral Image Acquisition and Spectral Reconstruction using a Trichromatic Digital Camera System associated with absorption filters

PART VI Spectral reconstruction in Kubelka-Munk space

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Abstract

This part describes some of the significant simulations performed to reconstruct the spectral reflectance of three targets, in K/S space using several combinations of filters and trichromatic signals. It also reports the colorimetric and spectral accuracy of the reconstructions, as well as the metameric index.

I) Linear method using simulated digital counts (IBM DCS)

The idea of the linear method using simulated digital counts is summarized in the flowchart of Figure 1.

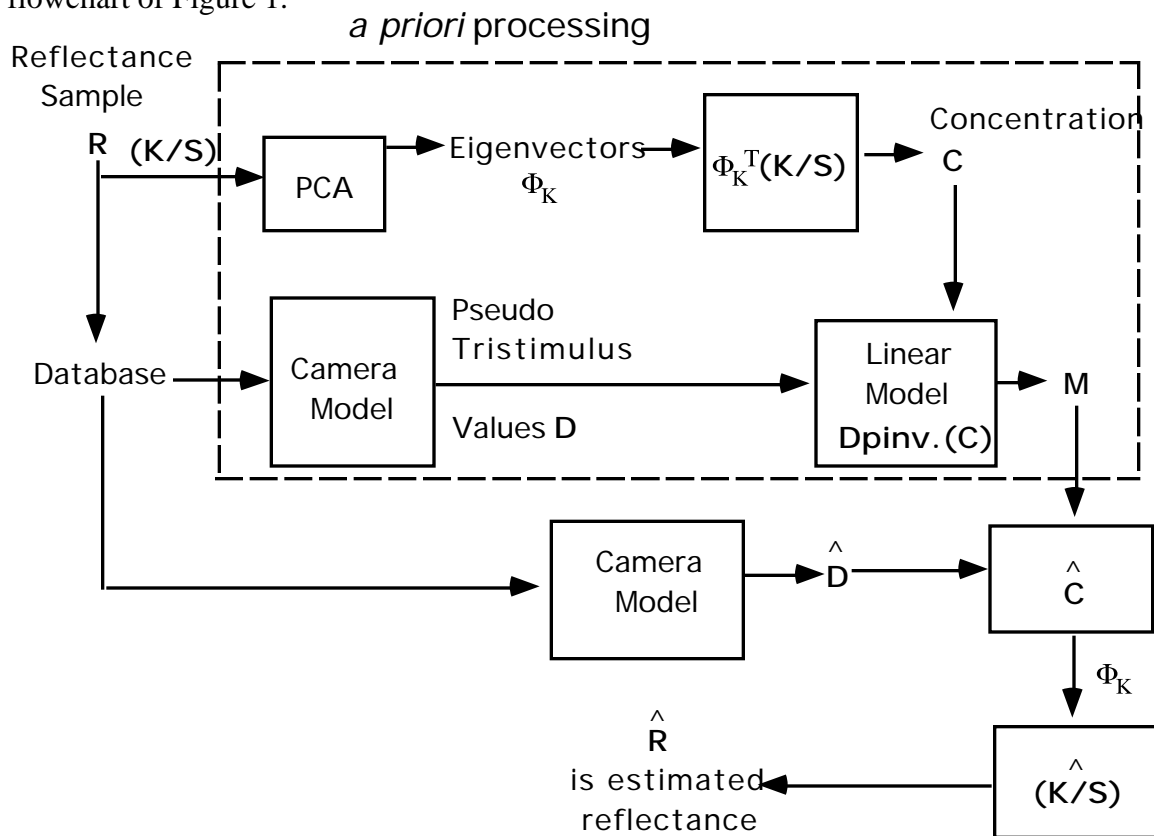


Figure 1. Flowchart of the linear method in K/S space.

At first, a camera model is built considering the spectral sensitivities of the digital camera, spectral power distribution of the illuminant, and the spectral transmittance of the filters used in the digitization. The spectral reflectances \mathbf{R} of the samples are measured and (K/S) values are calculated. Then, principal component analysis is performed to calculate the eigenvectors $\Phi_{\mathbf{K}}$ of the (K/S) values. The concentration \mathbf{C} corresponding to the (K/S) values is calculated. The camera model is used to obtain simulated digital counts from spectral reflectances \mathbf{R} . A transformation matrix \mathbf{M} from digital counts to concentration is computed. The digital counts $\hat{\mathbf{D}}$ of a subset \mathbf{R}' of the spectral reflectances is simulated using the camera model. The transformation matrix \mathbf{M} , calculated *a priori*, is used to predict the concentration corresponding to the digital counts $\hat{\mathbf{D}}$. Finally, the spectral reflectance $\hat{\mathbf{R}}$ is predicted using the concentration and eigenvectors $\Phi_{\mathbf{K}}$.

Important consideration

At first, all the simulations were performed without normalizing the digital counts between zero and one. At a second step, every digital counts were normalized dividing by 4,095. However, it still produced very inaccurate results. The digital counts do not have direct proportionality with Kubelka-Munk space. In order to warranty that both digital counts and (K/S) values are in the same space, an empirical transformation for the digital counts, given by equation 1, was defined:

$$\mathbf{D}_c = \text{pinv}(\mathbf{D}_c) + \frac{\mathbf{D}_c}{2} - 1, \quad (1)$$

where \mathbf{D}_c is the transformed digital count, \mathbf{D}_c is the normalized digital count, and pinv is the pseudo-inverse matrix operation.

Target I: Macbeth ColorChecker

The digital counts of Macbeth ColorChecker were simulated by camera modeling, combining R, G, B channels without filters and with Kodak Wratten filters number 38 and 66. The normalized digital counts were transformed according to equation (1). The results of the spectral reconstruction for 6 channels are summarized in Table I to II.

6 eigenvectors and 6 channels (R, G, B without filter and with Wratten filter number 38)

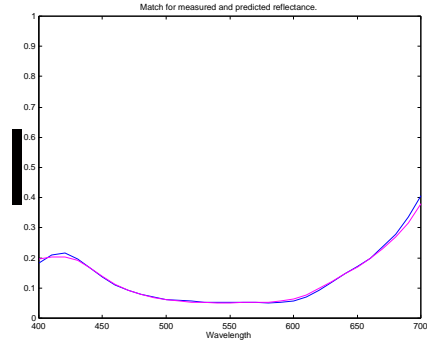
Table I. Spectral reconstruction using 6 eigenvectors; 6 signals (R, G, B without filter and R, G, B with Wratten absorption filter number 38).

Patch	ΔE^*_{94}	reflectance rms error factor	Metameric Index
Dark skin	0.31	0.0170	1.06
Light skin	5.02	0.1759	5.75
Blue sky	1.58	0.0250	0.94
Foliage	3.39	0.0123	0.69
Blue flower	1.50	0.0195	0.52
Bluish green	2.89	0.1367	6.54
Orange	1.82	0.1357	2.76
Purplish red	0.24	0.0168	0.42
Moderate red	2.24	0.1711	4.87
Purple	0.80	0.0073	0.03
Yellow green	0.42	0.0534	3.03
Orange yellow	9.61	0.1324	3.51
Blue	1.90	0.0361	1.87
Green	6.75	0.0324	0.91
Red	1.10	0.1323	1.96
Yellow	7.69	0.2615	8.51
Magenta	2.61	0.0746	0.60
Cyan	4.96	0.0292	1.25
White	18.38	0.3262	1.26
Neutral 8	15.57	0.1853	4.01
Neutral 6.5	8.33	0.0879	4.55
Neutral 5	3.41	0.0252	1.94
Neutral 3.5	0.82	0.0049	0.57
Black	0.17	0.0005	0.07
Average	4.23	0.123	2.40
Std Dev	4.78	0.030	2.27
Max	18.38	0.154	8.51
Min	0.17	0.024	0.03

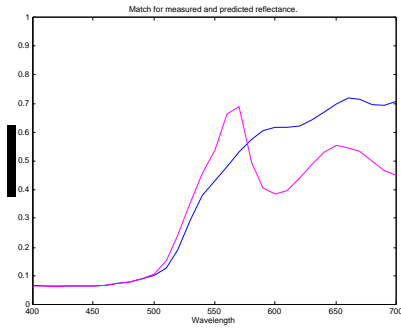
From Table I, it is possible to observe that purple patch of the Macbeth ColorChecker presented very accurate spectral match as shown in Figure 2. It is interesting because the purple patch presented the worst result for the same number of eigenvectors and the same combination of trichromatic signals when the spectral reconstruction was performed in reflectance space. In the other hand, the spectral reconstruction of patches with high spectral reflectance factor was quite inaccurate, as exemplified in Figure 2b and Figure 2c, respectively for orange yellow patch and white patch. Considering the Kubelka-Munk equations (2) and (3), (2)

$$\mathbf{R} = 1 + (\mathbf{K}/\mathbf{S}) - \sqrt{(\mathbf{K}/\mathbf{S}) \cdot ((\mathbf{K}/\mathbf{S}) + 2)} \quad (3)$$

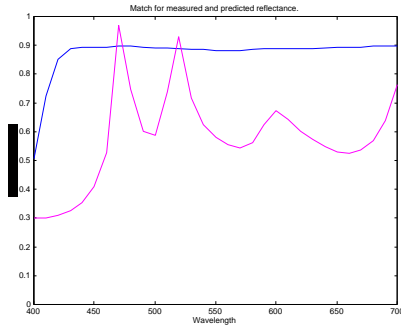
where \mathbf{R} is the reflectance vector, and (\mathbf{K}/\mathbf{S}) the absorption-scattering ratio in Kubelka-Munk space, it is possible to notice that high reflectance factors will produce very small (\mathbf{K}/\mathbf{S}) values, from equation (2). When the (\mathbf{K}/\mathbf{S}) values are predicted using principal component analysis, inaccuracies due to low dimensionality can lead to negative (\mathbf{K}/\mathbf{S}) values. When negative (\mathbf{K}/\mathbf{S}) values are introduced in equation (3), we have square root of negative values that produce complex number reflectance.



a) Purple patch



b) Orange yellow patch



c) White patch

Figure 1. Comparison of the measured and estimated spectral reflectance

Figure 3 shows the colorimetric accuracy for the spectral reconstruction using 6 eigenvectors, 6 channels (R, G, B without filter and with Wratten absorption filter number 38).

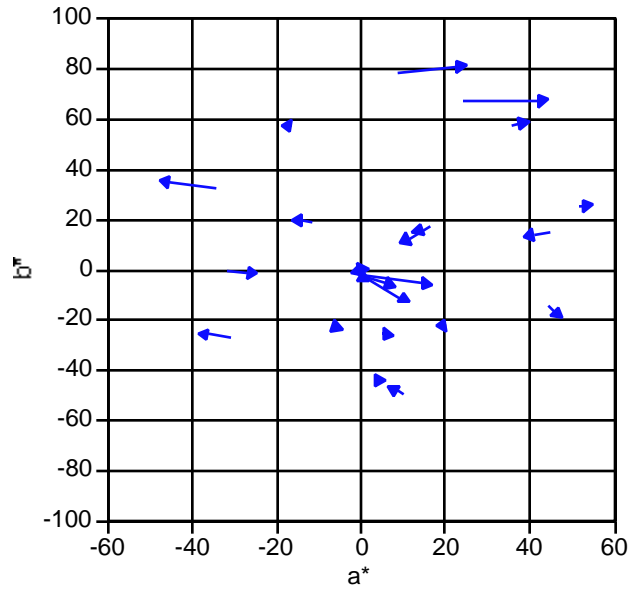


Figure 3. Colorimetric accuracy using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38; (original -> reproduction).

Figure 4 and Figure 5 show, respectively, the spectral rms error factor, and the E^*_{94} histogram for the spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38

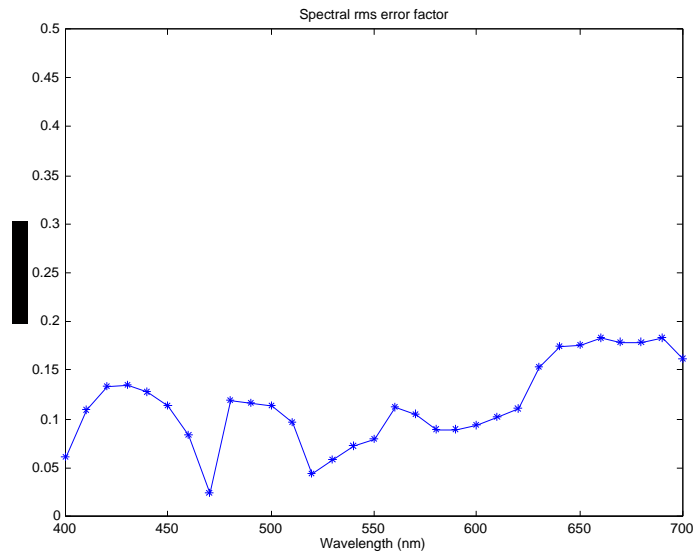


Figure 4. Spectral rms error for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

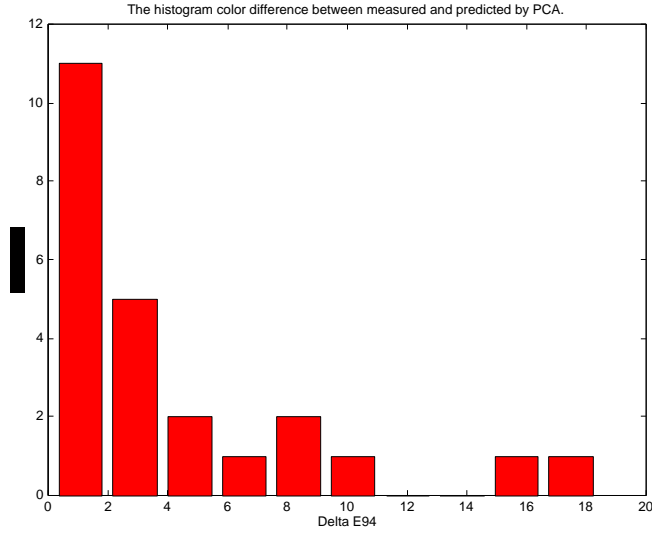


Figure 5. E^*_{94} histogram for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

6 eigenvectors; 6 channels (R, G, B without filter and with Wratten filter number 66)

Table II. Spectral reconstruction using 6 eigenvectors; 6 signals : R, G, B without filter and with Wratten absorption filter number 66.

Patch	ΔE^*_{94}	reflectance rms error factor	Metameric Index
Dark skin	2.15	0.009	0.10
Light skin	4.45	0.157	1.62
Blue sky	2.17	0.019	0.65
Foliage	3.00	0.013	0.95
Blue flower	2.68	0.033	0.66
Bluish green	11.05	0.086	1.18
Orange	5.79	0.090	1.74
Purplish red	1.48	0.021	0.13
Moderate red	1.80	0.138	3.23
Purple	2.32	0.022	0.79
Yellow green	1.50	0.041	1.52
Orange yellow	12.14	0.188	6.39
Blue	4.04	0.017	1.52
Green	6.97	0.035	0.20
Red	1.26	0.135	2.20
Yellow	10.94	0.268	8.08
Magenta	1.79	0.208	0.31
Cyan	6.37	0.131	4.79
White	17.13	0.237	2.70
Neutral 8	13.07	0.202	2.45
Neutral 6.5	6.98	0.068	0.84
Neutral 5	3.42	0.022	0.31
Neutral 3.5	1.29	0.005	0.12
Black	0.36	0.000	0.03
Average	5.17	0.089	1.77
Std Dev	4.55	0.084	2.06
Max	17.13	0.268	8.08
Min	0.36	0.000	0.03

Figure 6 shows the colorimetric accuracy for the spectral reconstruction using 6 eigenvectors, 6 channels (R, G, B without filter and with Wratten absorption filter number 66).

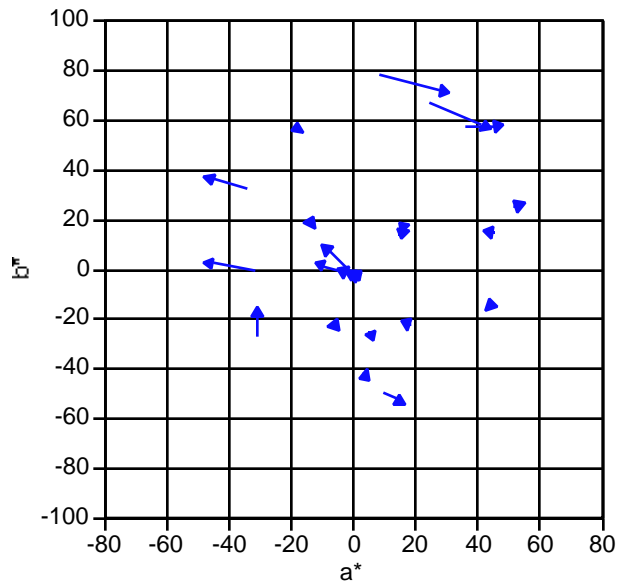


Figure 6. Colorimetric accuracy using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 66; (original -> reproduction).

Figure 7 and Figure 8 show, respectively, the spectral rms error factor, and the E^*_{94} histogram for the spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 66.

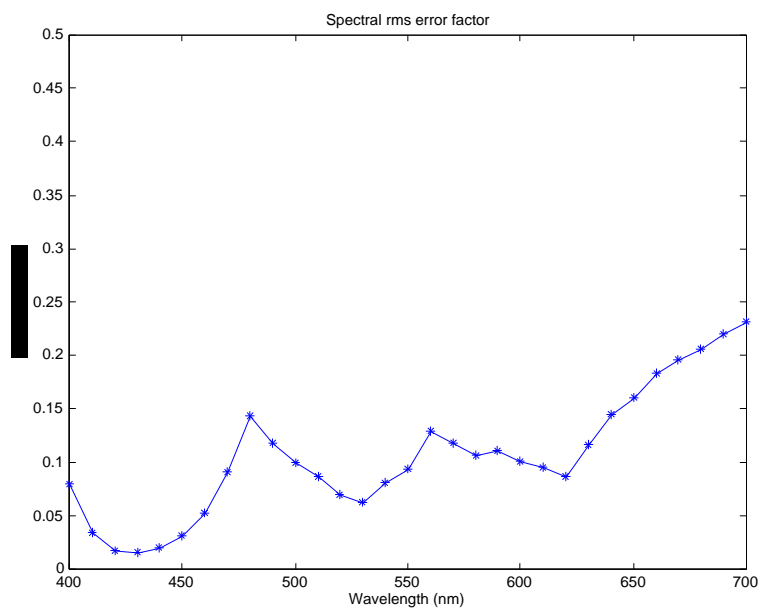


Figure 7. Spectral rms error for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and R, G, B with Wratten absorption filter number 66.

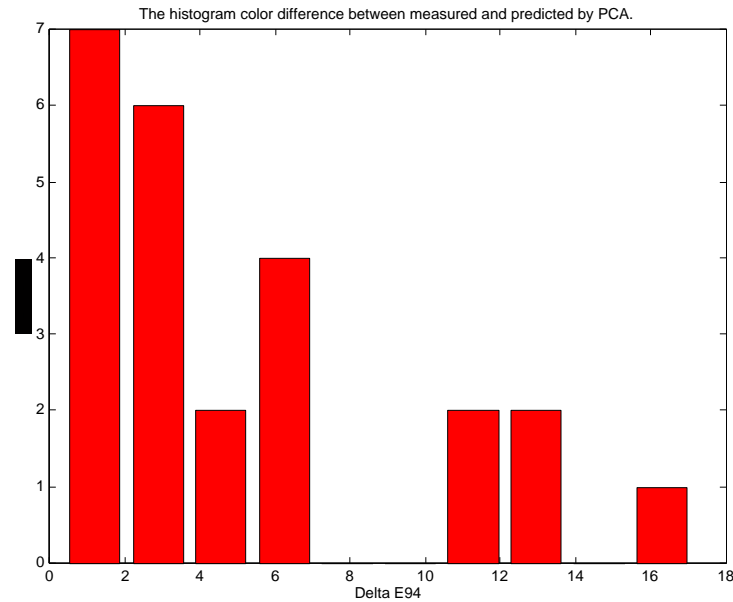


Figure 8. E^*_{94} histogram for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and R, G, B with Wratten absorption filter number 66.

Comparing Tables I and II it is possible to see that the results were quite similar further comparison of trichromatic signals combination will be omitted in this report.

Target II: Set of 147 painted patches

The digital counts of a set of 147 painted patches were imaged combining R, G, and B channels without filters and with Kodak Wratten filter number 38. The normalized digital counts were transformed according to equation (1). The results of the spectral reconstruction for 6 channels are summarized in Tables III.

6 eigenvectors; 6 channels (R, G, B without filter and with Wratten absorption filter number 38)

Table III. Spectral reconstruction using 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

Patch	ΔE^*_{94}	reflectance rms error factor	Metameric Index
Average	14.90	0.351	5.46
Std Dev	26.66	1.032	15.12
Max	135.11	5.696	95.87
Min	0.23	0.002	0.08

Figure 9 shows the colorimetric accuracy for the spectral reconstruction using 6 eigenvectors, 6 channels (R, G, B without filter and with Wratten absorption filter number 38).

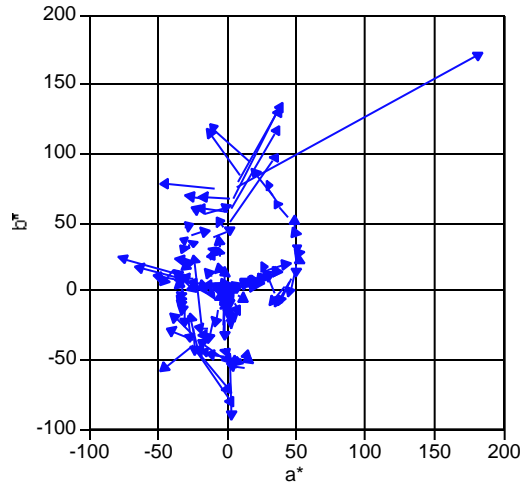


Figure 9. Colorimetric accuracy using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38; (original -> reproduction).

Figure 10 and Figure 11 show, respectively, the spectral rms error factor, and the E^*_{94} histogram for the spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

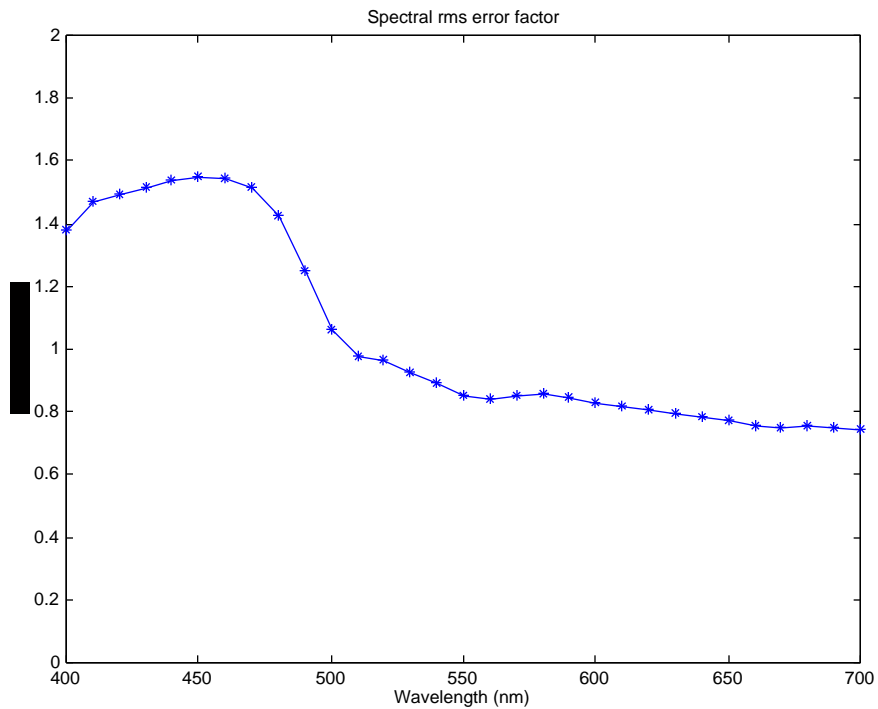


Figure 10. Spectral rms error for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

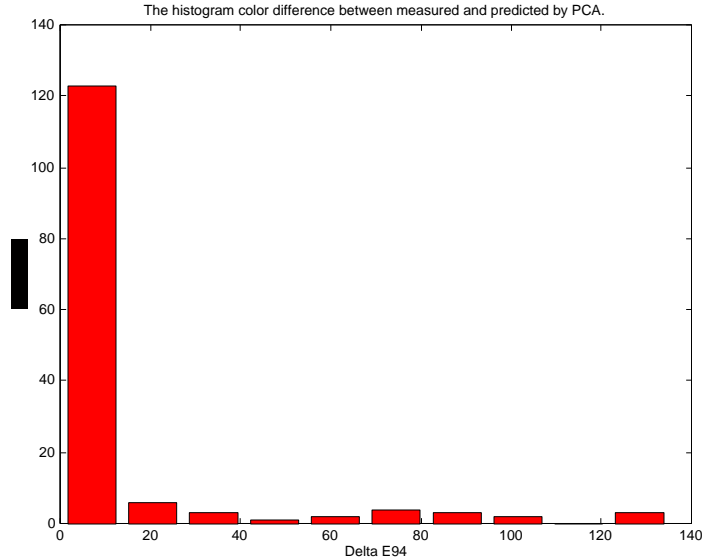


Figure 11. E^*_{94} histogram for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

From table III and figures 9 to 11 it is possible to see huge spectral mismatches and colorimetric error. It was due to the same reasons mentioned in the comments for the results of Macbeth ColorChecker spectral reconstruction.

Target III: Set of 105 painted patches

The digital counts of a set of 105 painted patches were simulated combining R, G, B channels without filters and with Kodak Wratten filter number 38. The normalized digital counts were transformed according to equation (1). The results of the spectral reconstruction for 6 channels are summarized in Table IV.

6 eigenvectors; 6 channels (R, G, B without filter and with Wratten absorption filter number 38)

Table IV. Spectral reconstruction using 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

Patch	ΔE^*_{94}	reflectance rms error factor	Metameric Index
Average	1.46	0.028	0.69
Std Dev	1.13	0.032	0.62
Max	7.20	0.129	2.81
Min	0.16	0.004	0.02

Figure 12 shows the colorimetric accuracy for the spectral reconstruction using 6 eigenvectors, 6 channels (R, G, B without filter and with Wratten absorption filter number 38).

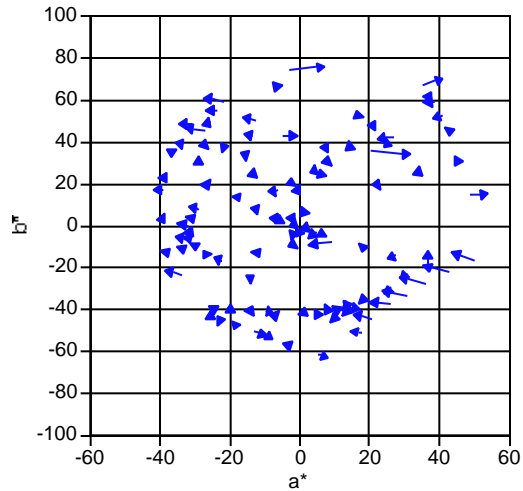


Figure 12. Colorimetric accuracy using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38; (original -> reproduction).

Figure 13 and Figure 14 show, respectively, the spectral rms error factor, and the E^*_{94} histogram for the spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

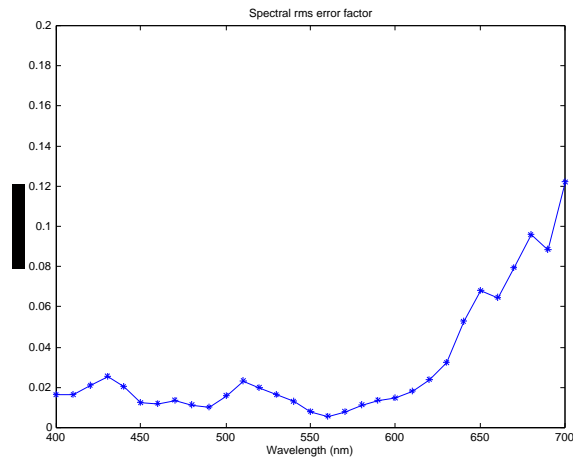


Figure 13. Spectral rms error for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

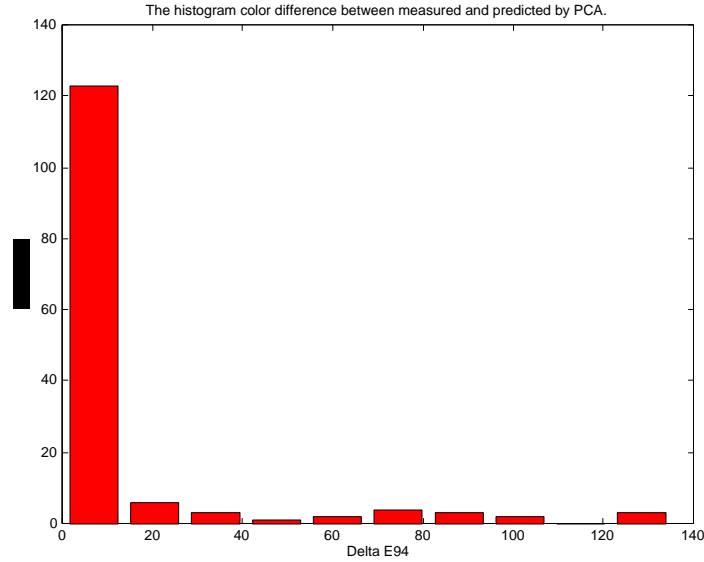


Figure 14. E^*_{94} histogram for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

6 eigenvectors; 6 channels (R, G, B without filter and with Wratten absorption filter number 38 without digital counts transformation.

The digital counts of a set of 105 painted patches were simulated combining R, G, and B channels without filters and with Kodak Wratten filter number 38. In this case, the normalized digital counts were not transformed according to equation (1). The results of the spectral reconstruction for 6 channels are summarized in Table V.

Table V. Spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38).

Patch	ΔE^*_{94}	reflectance rms error factor	Metameric Index
Average	14.52	0.167	3.90
Std Dev	17.68	0.270	7.13
Max	92.45	1.414	51.30
Min	1.24	0.011	0.02

Figure 15 shows the colorimetric accuracy for the spectral reconstruction using 6 eigenvectors, 6 channels (R, G, B without filter and with Wratten absorption filter number 38).

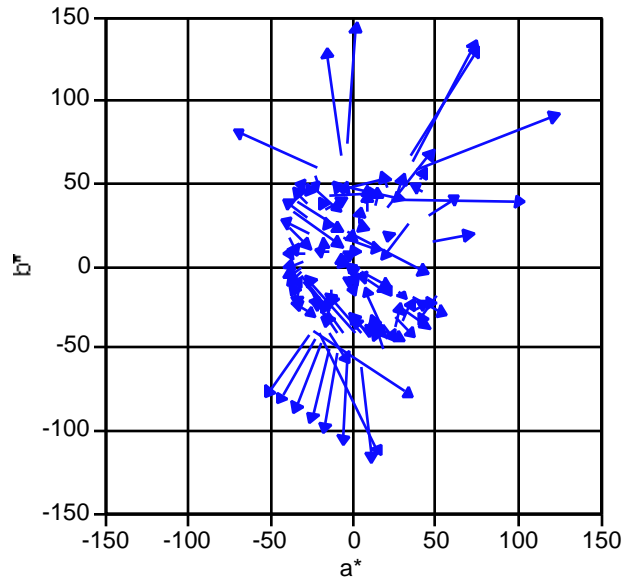


Figure 15. Colorimetric accuracy using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38; (original -> reproduction).

Figure 16 and Figure 17 show, respectively, the spectral rms error factor, and the E^*_{94} histogram for the spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

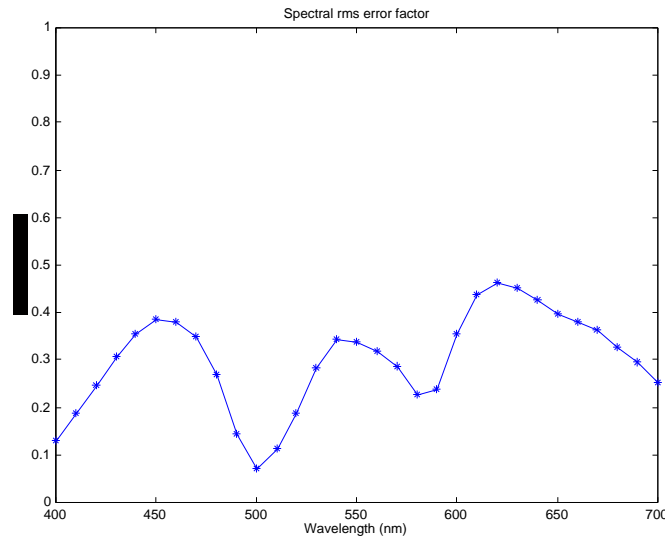


Figure 16. Spectral rms error for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

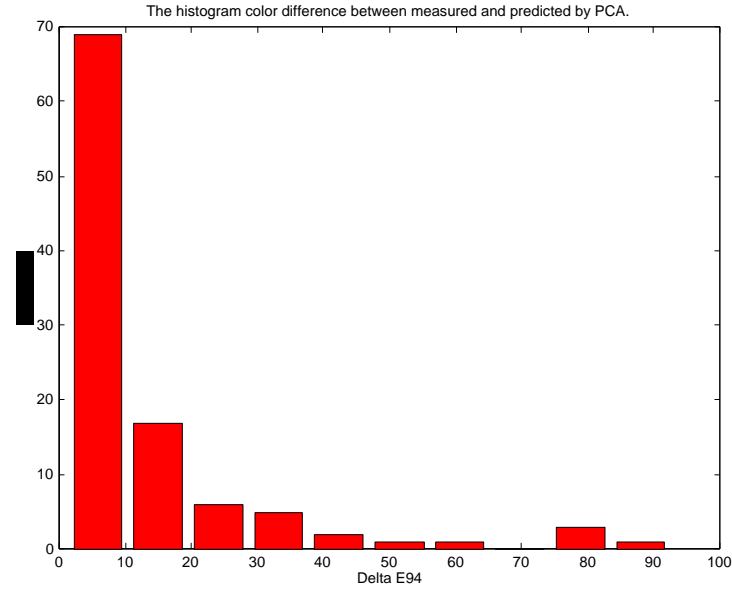


Figure 17. E^*_{94} histogram for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

Effect of changing the number of eigenvectors to 9 channels and 9 eigenvectors

The digital counts of a set of 105 painted patches were simulated combining R, G, B channels without filters and with Kodak Wratten filter number 38. The normalized digital counts were transformed using equation (1). The results of the spectral reconstruction for 6 channels are summarized in Table VI.

9 eigenvectors, 9 channels (R, G, B with without filter, with Wratten absorption filter number 38 and Wratten absorption filter number 66)

Table VI. Spectral reconstruction using 9 signals: R, G, B without filter, with Wratten absorption filter number 38, and with Wratten absorption filter number 66.

Patch	ΔE^*_{94}	reflectance rms error factor	Metameric Index
Average	1.32	0.027	0.56
Std Dev	1.55	0.035	0.87
Max	11.68	0.144	4.95
Min	0.12	0.002	0.01

Figure 18 shows the colorimetric accuracy for the spectral reconstruction using 9 eigenvectors, 9 channels: R, G, B without filter, with Wratten absorption filter number 38, and with Wratten absorption filter number 66.

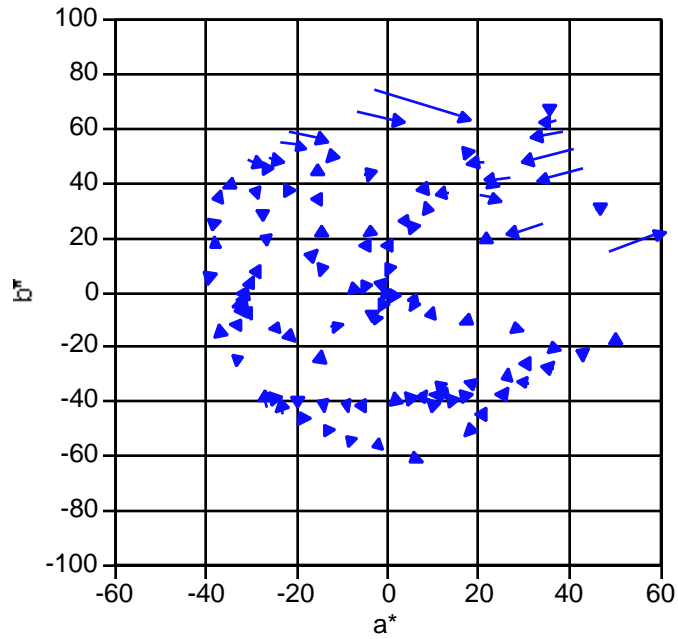


Figure 18. Colorimetric accuracy using 6 eigenvectors; 6 signals: R, G, B without filter, with Wratten absorption filter number 38, and with Wratten absorption filter number 66; (original -> reproduction).

Figure 19 and Figure 20 show, respectively, the spectral rms error factor, and the E^*_{94} histogram for the spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter, with Wratten absorption filter number 38, and with Wratten absorption filter number 66.

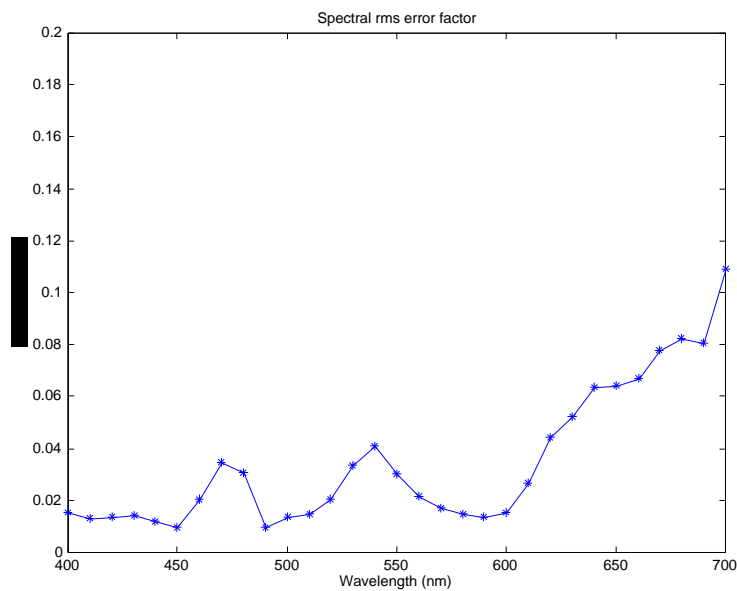


Figure 19. Spectral rms error for spectral reconstruction using 9 signals: R, G, B without filter, with Wratten absorption filter number 38, and with Wratten absorption filter number 66.

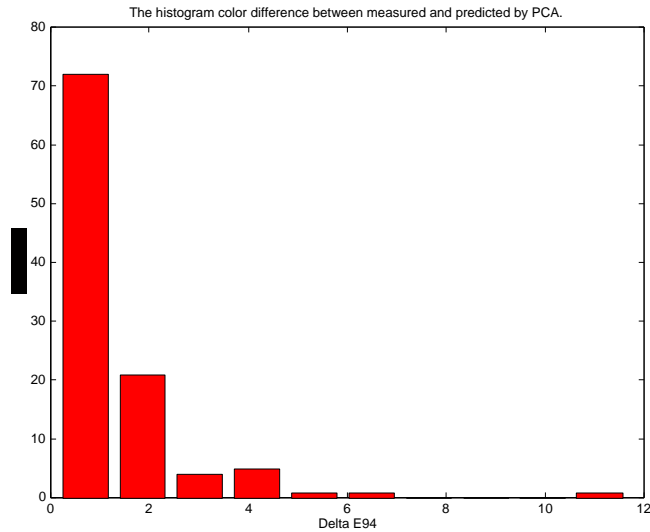


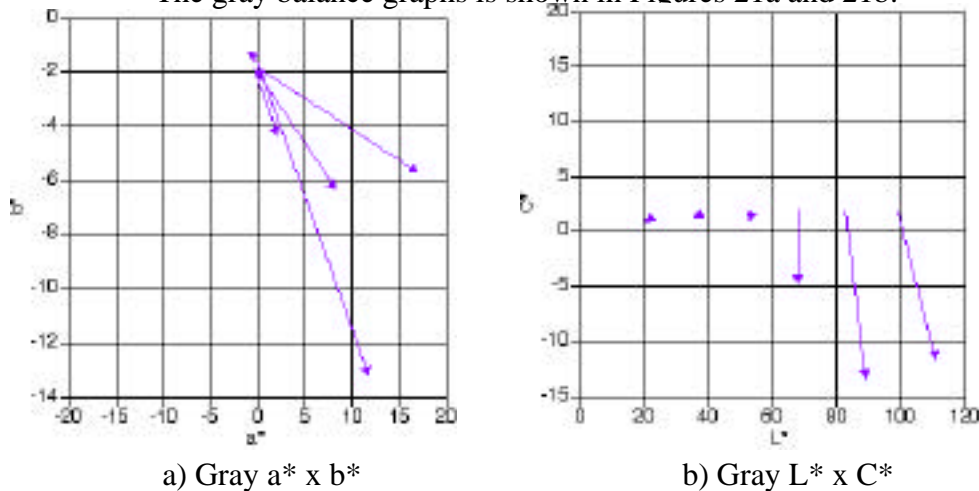
Figure 20. ΔE^*_{94} histogram for spectral reconstruction using 9 signals: R, G, B without filter, with Wratten absorption filter number 38, and with Wratten absorption filter number 66.

Comparing Tables IV and V, and figures 10 to 16, it is possible to see that the empirical transformation for digital counts proposed in equation (1) improves the colorimetric and spectral performance of the simulation. Otherwise, the comparison of Tables V and VI shows that increasing the dimension from 6 to 9 produced just a small improvement in the simulation. It happens because of the inherent problems of Kubelka-Munk equations discussed above.

GRAY BALANCE

6 eigenvectors; 6 channels (R, G, B without filter and with Wratten Filter number 38)

The gray balance graphs is shown in Figures 21a and 21b.



a) Gray $a^* \times b^*$

b) Gray $L^* \times C^*$

Figure 21. Gray balance (original \rightarrow reproduction) for 6 eigenvectors, 6 channel color reproduction (R, G, B without filter and with Wratten filter number 38).

II) Linear method using measured digital counts (IBM DCS)

This method uses basically the same idea of the linear method above, but instead of using simulated digital counts this method uses measured digital counts averaged from each patch as shown in Figure 22.

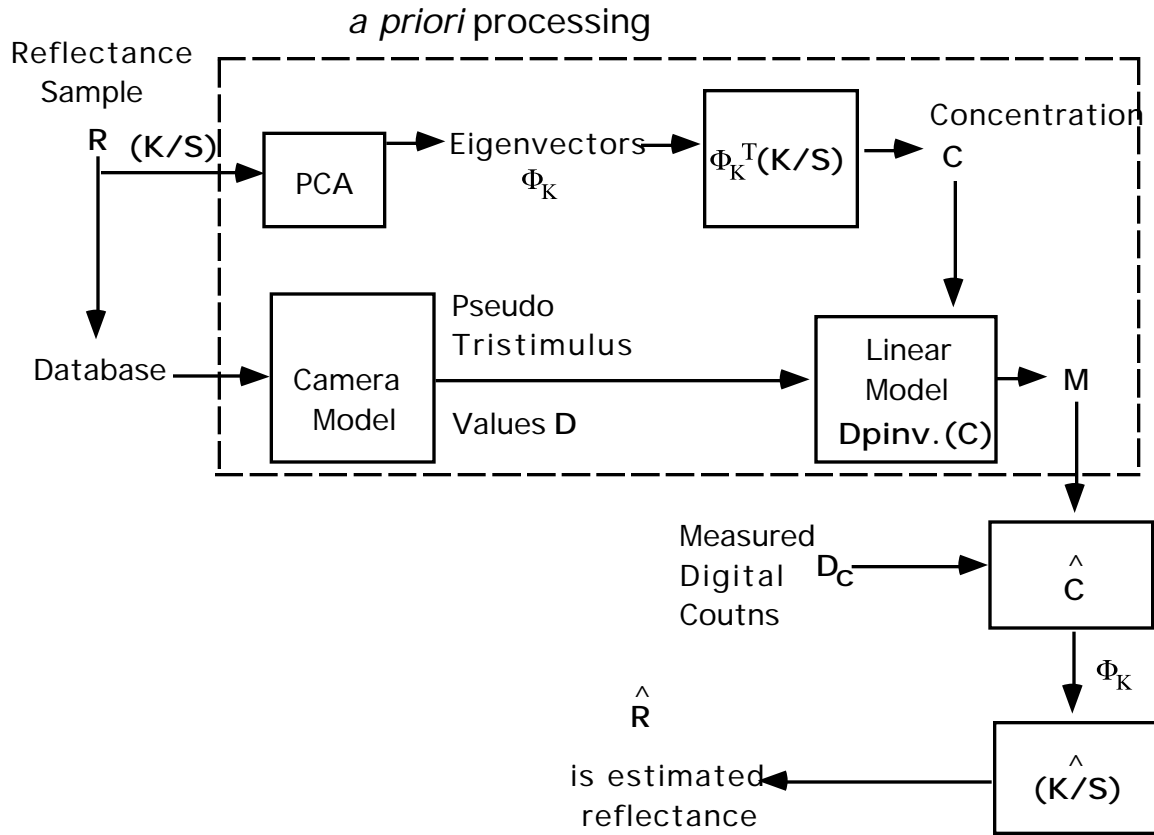


Figure 22. Flowchart of the linear method using measured digital counts

This method was applied to reproduce the colors of 6 eigenvectors and 6 channels (R, G, B without filter and with Wratten filter number 38). The results are summarized in table VII and figures 23 to 25.

Table VII. Spectral reconstruction using 6 eigenvectors; 6 signals (R, G, B without filter and R, G, B with Wratten absorption filter number 38).

Patch	ΔE^*_{94}	reflectance rms error factor	Metameric Index
Dark skin	1.22	0.007	0.32
Light skin	4.77	0.145	1.55
Blue sky	6.22	0.056	1.17
Foliage	1.67	0.016	1.09
Blue flower	6.60	0.073	2.44
Bluish green	7.07	0.115	5.77
Orange	9.46	0.157	3.68
Purplish red	3.02	0.037	0.83
Moderate red	2.76	0.103	3.19
Purple	3.42	0.044	0.78
Yellow green	11.62	0.157	4.65
Orange yellow	8.17	0.191	4.64
Blue	2.24	0.039	0.96
Green	5.06	0.037	1.00
Red	1.23	0.078	0.57
Yellow	4.83	0.132	3.25
Magenta	8.18	0.608	9.09
Cyan	10.51	0.121	5.27
White	8.37	0.183	2.03
Neutral 8	9.12	0.140	2.93
Neutral 6.5	8.66	0.087	0.31
Neutral 5	5.25	0.038	0.45
Neutral 3.5	5.09	0.021	0.79
Black	0.94	0.001	0.17
Average	5.65	0.108	2.37
Std Dev	3.16	0.121	2.23
Max	11.62	0.608	9.09
Min	0.94	0.001	0.17

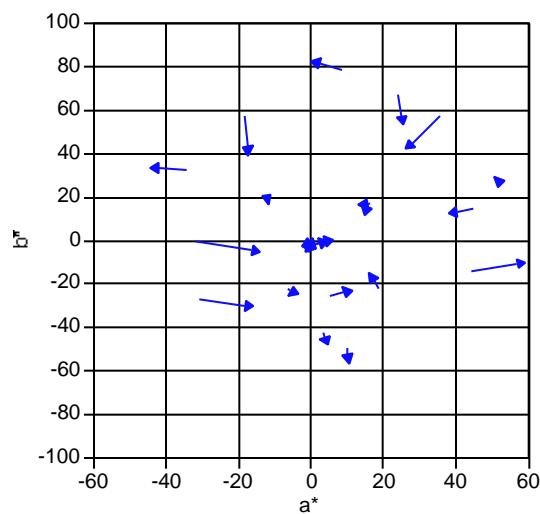


Figure 23. Colorimetric accuracy using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38; (original -> reproduction).

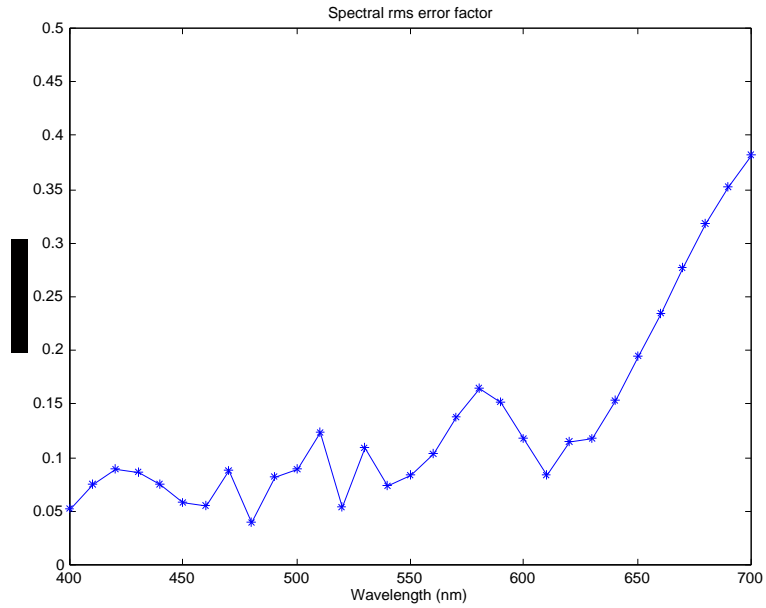


Figure 24. Spectral rms error for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

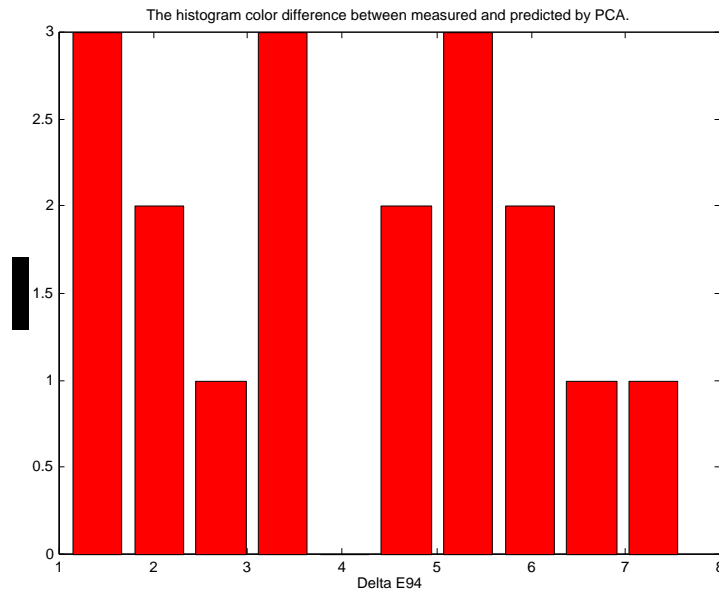


Figure 25. ΔE^*_{94} histogram for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

Linear method using measured digital counts (Sony Digital Still Camera)

In order to test the performance of the method in Kubelka-Munk space for a common commercially available digital still camera, the Sony DKC-D5PRO camera was used. Table VIII and figures 26 to 28 summarize the results.

Table VIII. Spectral reconstruction using 6 eigenvectors; 6 signals : R, G, B without filter and with Wratten absorption filter number 38.

Patch	ΔE_{94}^*	reflectance rms error factor	Metameric Index
Dark skin	6.86	0.024	0.20
Light skin	2.19	0.028	0.96
Blue sky	2.64	0.027	0.86
Foliage	6.42	0.028	1.91
Blue flower	3.91	0.052	0.26
Bluish green	3.99	0.051	2.71
Orange	3.85	0.068	1.27
Purplish red	5.40	0.045	4.46
Moderate red	1.52	0.035	1.33
Purple	1.61	0.041	2.41
Yellow green	2.52	0.020	0.39
Orange yellow	1.96	0.040	1.70
Blue	9.26	0.076	8.03
Green	5.25	0.035	2.06
Red	3.37	0.044	2.86
Yellow	2.68	0.038	1.56
Magenta	1.58	0.048	1.12
Cyan	4.94	0.038	0.37
White	1.21	0.018	0.16
Neutral 8	0.29	0.017	0.18
Neutral 6.5	3.46	0.036	0.22
Neutral 5	1.12	0.014	0.25
Neutral 3.5	1.83	0.010	0.51
Black	3.36	0.006	0.43
Average	3.38	0.035	1.51
Std Dev	2.12	0.017	1.76
Max	9.26	0.076	8.03
Min	0.29	0.006	0.16

Figure 26 shows the colorimetric accuracy for the spectral reconstruction using 6 eigenvectors, 6 channels (R, G, B without filter and with Wratten absorption filter number 66).

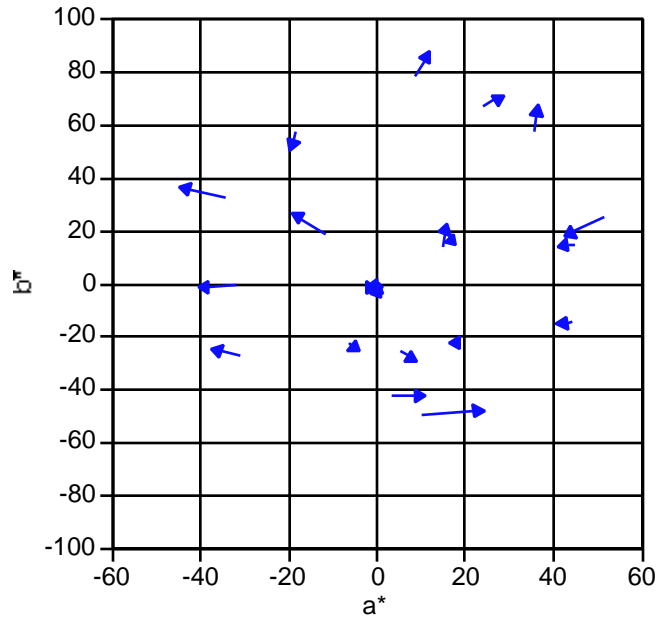


Figure 26. Colorimetric accuracy using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38; (original -> reproduction).

Figure 27 and Figure 28 show, respectively, the spectral rms error factor, and the E^*_{94} histogram for the spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and with Wratten absorption filter number 38.

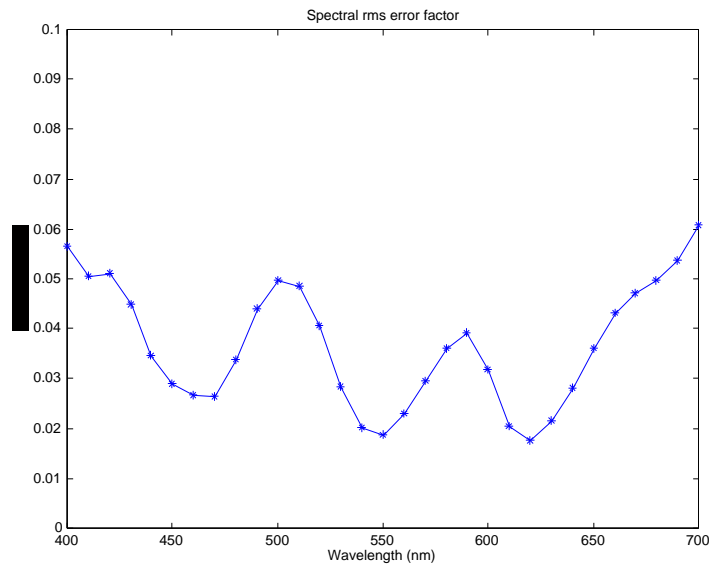


Figure 27. Spectral rms error for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and R, G, B with Wratten absorption filter number 38.

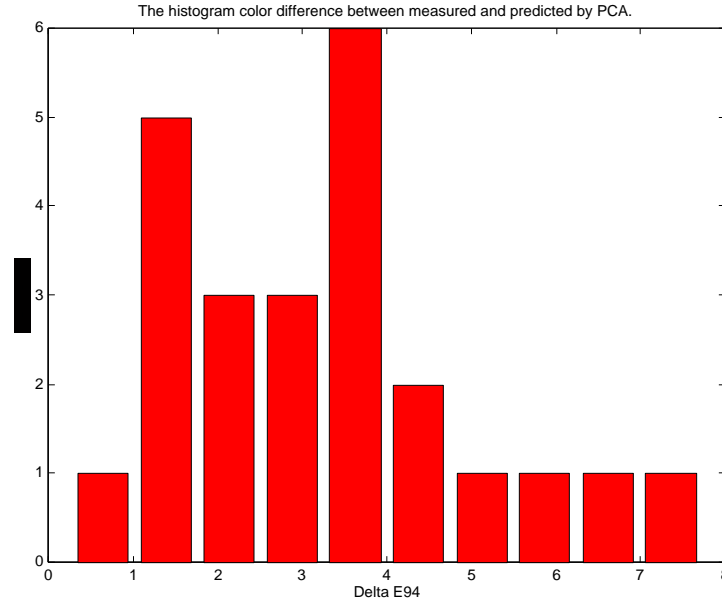


Figure 28. E^*_{94} histogram for spectral reconstruction using 6 eigenvectors; 6 signals: R, G, B without filter and R, G, B with Wratten absorption filter number 38.

DISCUSSION

It is interesting to notice from the results above that in general, as expected, performing principal component analysis in reflectance spaces worked very well for high reflectance factors, while lower reflectance factors worked better performing principal component analysis in K/S space, using 6 eigenvectors. When principal component analysis is performed in K/S space, for high reflectance factor patches, due to the error introduced by the analysis we get complex numbers as a result for reflectance calculated from the estimated absorptions. This is the mathematical limitation of Kubelka-Munk equations when used with principal component analysis.

The result of spectral reconstruction in K/S space from measured digital counts using IBM and Sony cameras presented surprisingly good results if compared with simulated digital counts for IBM digital camera for the same combination of trichromatic signals. It is possible that the inaccuracies introduced by Kubelka-Munk equation equalize the results of all performances.