Multi-spectral Image Acquisition and Spectral Reconstruction using a Trichromatic Digital Camera System associated with absorption filters
Part II Technical Approach

Francisco H. Imai
Munsell Color Science Laboratory, Rochester Institute of Technology

Abstract

This report summarizes the technical approach using linear and non-linear iterative models in order to predict the spectral reflectance from the digital counts of a trichromatic Digital Camera System associated with absorption filters.

Introduction

In the previous technical report[1] the digitizing system using a trichromatic IBM PRO\3000 Digital Camera System[2,3] and a set of Kodak Wratten[4] absorption filters shown in Figure 1 were fully characterized and some preliminary experiments showed the feasibility of using this method to reconstruct spectral reflectance from a multiple-of-three set of digital counts.

Figure 1. IBM PRO\3000 Digital Camera System head and Kodak Wratten absorption filter with the filter holder.

As a resulting of the imaging using this system we have a set of images as shown in Figure 2.

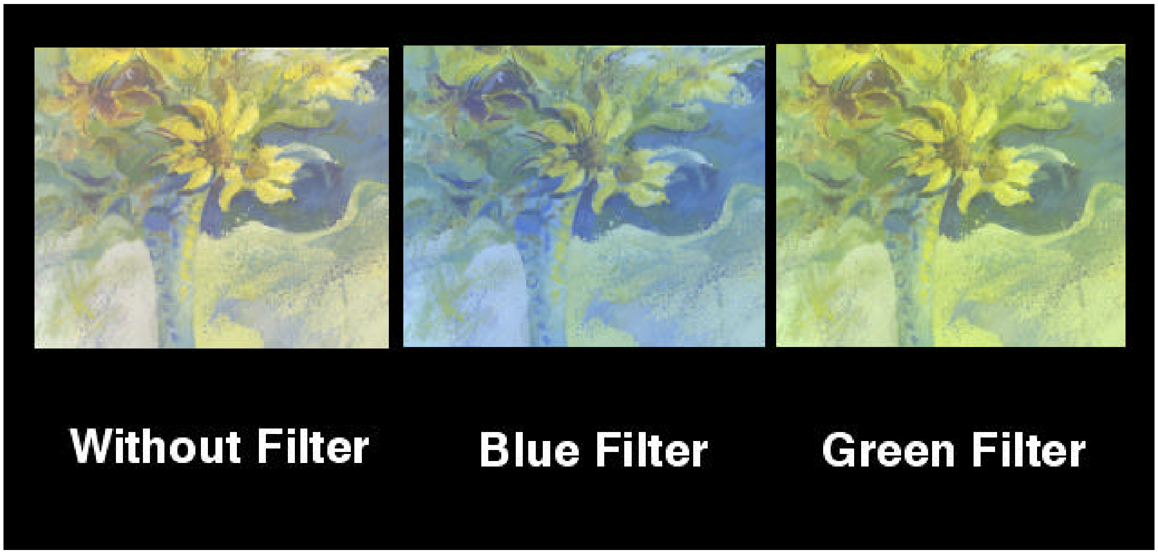

Figure 2. Image of painting digitized by a trichromatic camera and a set of two filters.

In order to relate the digital counts to spectral reflectance, a linear method based on camera modeling, was applied with elements shown in Figure 3. The spectral radiance, **S,** of the illuminant, as well as the spectral sensitivities, **D,** of the camera, the transmittances, **F,** of the filters and the spectral reflectance, **r,** of color patches are measured and the digital counts, **Dc,** were extracted from the imaged patches.

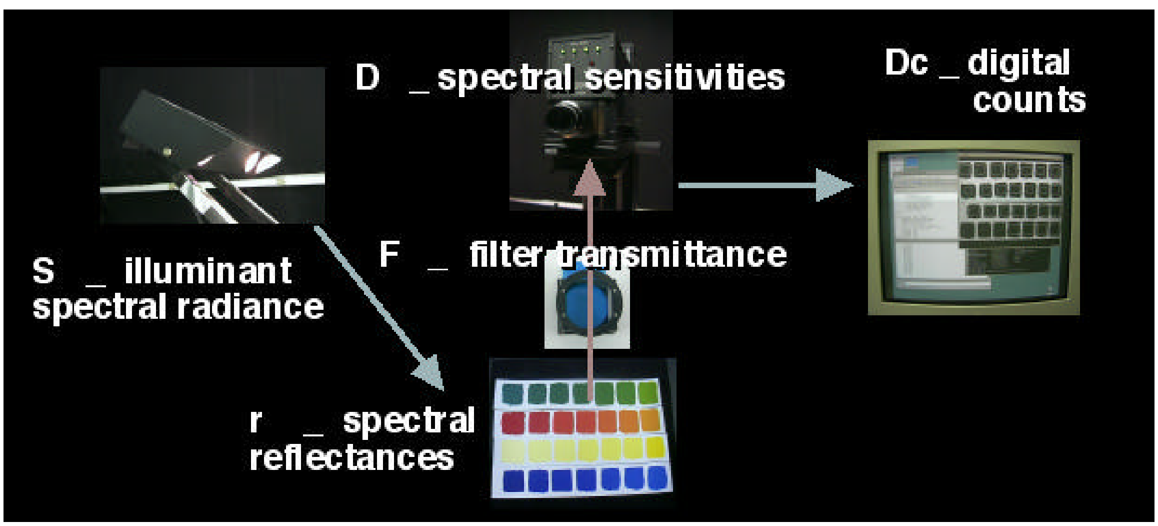

Figure 3. Schematic diagram showing the elements of camera modeling used in the experiments.

The spectral reflectance of each pixel of a painting could be estimated using *a priori* spectral analysis with direct measurement and imaging of color patches to establish a relationship between the digital counts and spectral reflectance as shown in Figure 4.

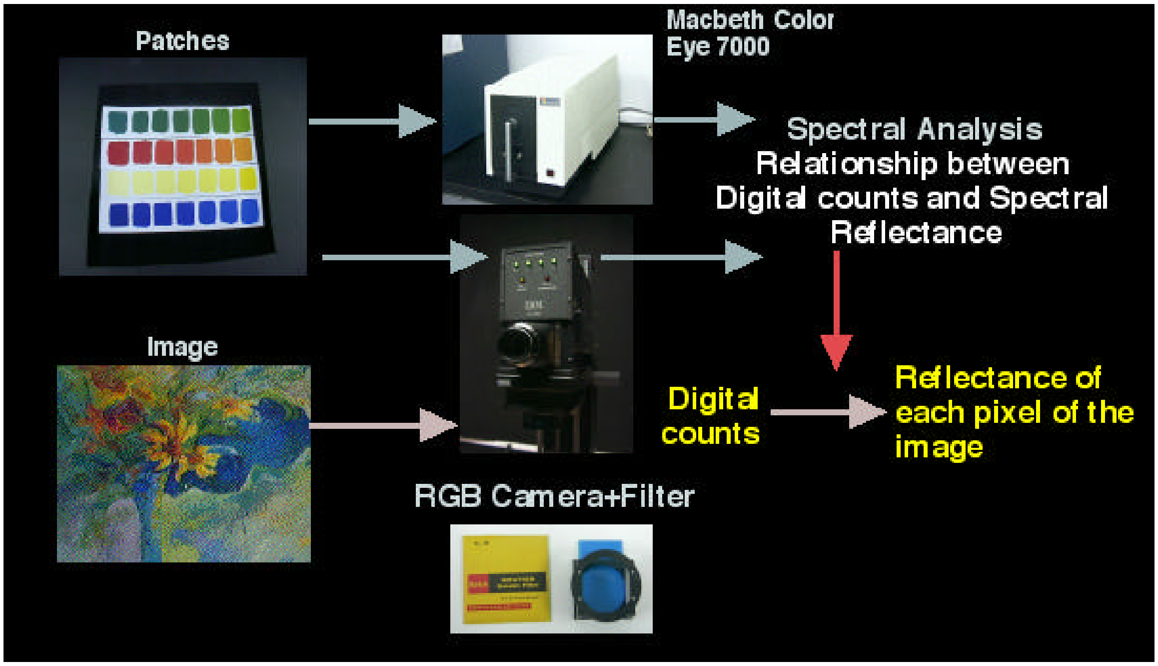

Figure 4. Schematic diagram of the method used to estimate the spectral reflectance of each pixel of an image using a trichromatic camera and a set of absorption filters.

The previous technical report showed that when this system applies the linear method with six channels (three R, G, and B channels without filter plus three R, G, and B channels with a light blue filter) to estimate the spectral reflectance of Macbeth Checker, the spectral factor RMS error was 0.2. and the mean ΔE^*_{ab} was 6.9. This result was improved adding a light green-blue filter to give three additional signals yielding nine channels. In this case, the spectral factor RMS error was 0.049 and the mean ΔE^*_{ab} was 2.2 for the Macbeth Checker.

In this technical report, the effectiveness of the linear method to estimate spectral reflectances from the digital counts is analyzed and the performance of the linear method will be compared with non-linear iterative methods.

Linear method

One can model multi-spectral image acquisition using matrix-vector notation.[5] Expressing the sampled illumination spectral power distribution as

$$\mathbf{S} = \begin{pmatrix} \mathbf{s}_1 & & & 0 \\ & \mathbf{s}_2 & & \\ & & \ddots & \\ 0 & & & \mathbf{s}_n \end{pmatrix}, \qquad (1)$$

and the object spectral reflectance as $\mathbf{r}=(r_1, r_2, \ldots r_n)^T$, where the index indicates the set of n wavelengths over the visible range and ^{T} the transpose matrix, representing the transmittance characteristics of the m filters as columns of **F**

$$\mathbf{F} = \begin{pmatrix} \mathbf{f}_{1,1} & \mathbf{f}_{1,2} & \cdots & \mathbf{f}_{1,\mathbf{m}} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{f}_{\mathbf{n},1} & \mathbf{f}_{\mathbf{n},2} & \cdots & \mathbf{f}_{\mathbf{n},\mathbf{m}} \end{pmatrix} \tag{2}$$

and the spectral sensitivity of the detector as

$$\mathbf{D} = \begin{pmatrix} \mathbf{d}_1 & & & 0 \\ & \mathbf{d}_2 & & \\ & & \ddots & \\ 0 & & & \mathbf{d}_\mathbf{n} \end{pmatrix}, \tag{3}$$

then the captured image is given by $\mathbf{D_c}=(\mathbf{DF})^T\mathbf{Sr}$**,** where $\mathbf{D_c}$ represents the digital counts, and the color vector can be represented as $\mathbf{c}=\mathbf{At}=(X, Y, Z)^T$ where X, Y, Z are the CIE tristimulus values. The CIELAB L^*, a^*, b^* are given by the non-linear transformation ξ, where $\xi(X, Y, Z) = L^*, a^*, b^*$.

If the spectral reflectance is sampled in the range of 400 nm to 700 nm wavelength in 10 nm intervals we have 31 samples. Ideally we should have 31 signals to reconstruct the spectral reflectance. However, it is possible to decrease the dimensionality of the problem by performing principal component analysis on the spectral samples. Given a sample population of spectral reflectances, it is possible to identify a small set of underlying basis functions whose linear combinations can be used to approximate and reconstruct members of the populations. Then the reconstructed sample $\hat{r}_i$ is given by $\hat{r}_i = \Phi\alpha_i$, where $\Phi = (\mathbf{e}_1 \quad \mathbf{e}_2 \quad \dots \quad \mathbf{e}_\mathbf{p})$ are the set of the eigenvectors (principal components) used for the estimation and the coefficients (eigenvalues) associated with the eigenvectors are $\alpha_\mathbf{i} = (\mathbf{a}_1 \quad \mathbf{a}_2 \quad \dots \quad \mathbf{a}_\mathbf{p})^\mathbf{T}$ where the index $p \le n$, and where n is the number of samples used to perform *a priori* principal component analysis. When the eigenvalues are arranged in descending order the fraction of variance explained by the first corresponding p vectors is

$$\mathbf{v}_\mathbf{p} = \frac{\sum_{\mathbf{i}=1}^{\mathbf{p}} \mathbf{a}_\mathbf{i}}{\sum_{\mathbf{i}=1}^{\mathbf{n}} \mathbf{a}_\mathbf{i}}. \tag{4}$$

In this linear method, a set of spectral reflectances $\mathbf{r}$ is measured and then a set Φ of eigenvectors, who explain typically more than 99.9% of the original sample, is calculated by principal component analysis. Then, the set of eigenvalues, α, is calculated by $\alpha=\Phi^T\mathbf{r}$**,** where ^{T} denotes the transpose of the matrix. We know that the set of digital counts corresponding to the spectral samples can be calculated by the equation $\mathbf{D_c}=(\mathbf{DF})^T\mathbf{Sr}$**.** A relationship between digital counts and eigenvalues can be established by the equation

$$\mathbf{A} = \alpha\mathbf{Dc}^\mathbf{T}[\mathbf{DcDc}^\mathbf{T}]^{-1} \tag{5}$$

The matrix **A** can be used to calculate the eigenvalues α_i from digital counts to reconstruct the spectral reflectance. Here, it is important to notice that the number of channels should equals the number of eigenvectors used in the system.

Non-linear iterative method

The linear method based on principal component analysis described in the previous section assumes that both digital counts and eigenvalues (and consequently eigenvectors) should be contained in the same space. A possible measure of the linear relationship of two subspaces represented by the matrices **A** and **B** can be the angle between the subspaces given by $\mathrm{acos}(A^T B)$. If the angle between the subspaces described by the eigenvectors and the digital counts are small a linear method can be used. Otherwise a non-linear method should be used. It is possible to add non-linear terms introducing products between signals in the linear regression as shown below

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \vdots \\ \alpha_{\mathbf{n}} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \mathbf{Dc}_1 \\ \vdots \\ \mathbf{Dc_n} \\ \vdots \\ \mathbf{Dc}_1\mathbf{Dc}_2 \\ \vdots \\ \mathbf{Dc_{n-1}Dc_n} \\ \mathbf{Dc}_1^2 \\ \vdots \\ \mathbf{Dc_n}^2 \\ \vdots \\ \mathbf{Dc}_1\mathbf{Dc}_2\ldots\mathbf{Dc_n} \\ 1 \end{pmatrix} \tag{6}$$

Equation 6 shows that for three signals (typically R, G, and B) we have a matrix **A** with dimensions 3 by 11. However, in more general cases we need to consider 6 or more signals. For 6 signals we have a matrix **A** with dimensions 6 by 70 that is more complex to calculate and could have convergence problems.

Instead of using linear regression using covariance terms such as shown in equation 6, it is also possible to use an approach based on iterative methods. An iterative method has been applied successfully by Rodriguez and Stockham[6] to convert scanner densities to colorimetric quantities. In their method, shown in Figure 5, the spectral characterization of a scanner and colorants are used to calculate the spectra. First, the scanner densities are converted to colorant amounts. Using Beer's law the total density spectrum can be calculated using the colorant amount as scalars and the negative log relationship between transmittance and density gives reconstruction of the full color spectrum of the pixel. A scanner model is used to simulate scanner densities. The key aspect of the method is accomplished with an iterative loop where estimated colorant amounts are processed with the scanner characterizations to simulate scanner densities.

The errors between the simulated densities and the actual scanner densities provide corrections to improve the estimates. When the error converges to zero we get accurate spectral estimation. A similar idea also has been implemented by Berns and Shyu.[7]

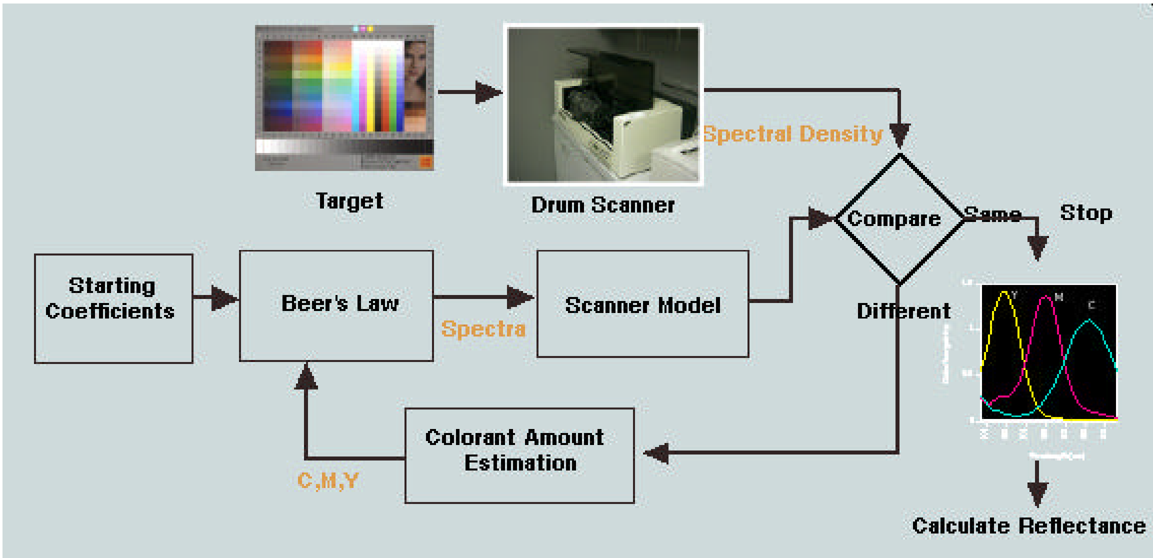

Figure 5. Schematic diagram of the iterative method used by Rodriguez and Stockham to estimate spectral reflectance using spectral density and spectral characterization of the scanner.

This method, conceived for scanners, can be adapted to the problem of estimating coefficients (eigenvalues) from digital count provided by the camera system. A schematic diagram of the iterative method is shown in Figure 6.

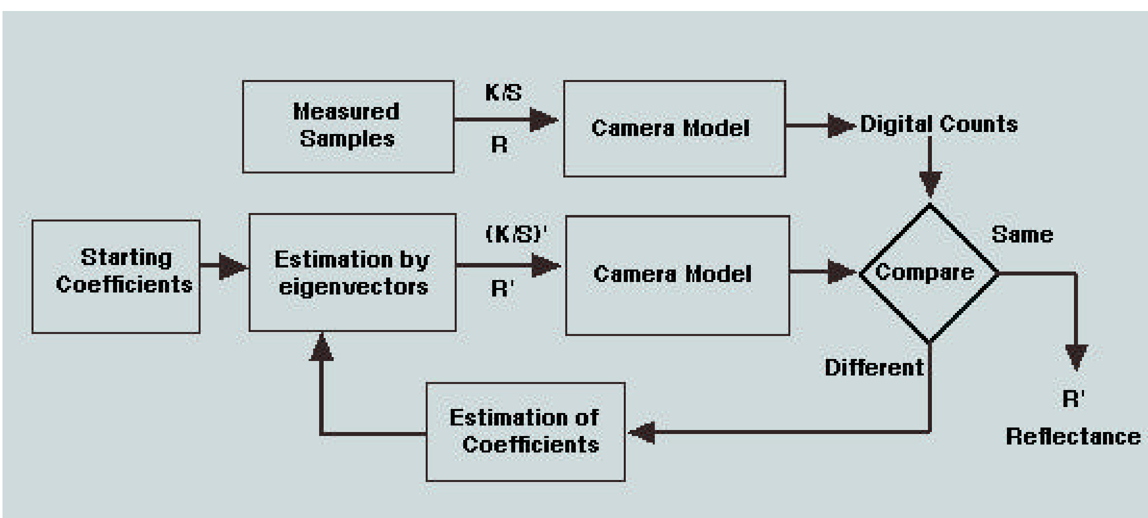

Figure 6. Schematic diagram of the proposed method to estimate spectral reflectance based on an iterative estimation of coefficients (eigenvalues) converted to digital counts using spectral estimation and camera model.

In this method, the spectral characterization of the digital camera system and the information about the eigenvectors of the samples are used. The key aspect of the method is accomplished with an iterative loop where estimated coefficients of the eigenvectors are used to estimate either the Kubelka-Munk K/S or reflectance in order to simulate digital counts by the camera model. The errors between the simulated digital counts and the measured digital counts provide corrections to improve the estimation of coefficients. When the error converges to zero we get accurate spectral estimation. As a preliminary approach, calculated digital counts from measured samples using the camera model are used instead of the actual digital counts to minimize noise due to the camera in the iterative optimization. Calculation in both K/S and reflectance spaces should be compared to evaluate the performance of the optimization in these spaces. Coefficients estimated by the linear method will be used as starting coefficients in order to analyze the influence of the choice of starting values on the speed of the optimization.

Tzeng and Berns[8] have proposed a new empirical equation that gives a near-normal and reduced dimensionality space for subtractive opaque processes, giving a good alternative for Kubelka-Munk transformation. This empirical equation is also considered in this research.

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