Evaluation of Spectral Issues in Sparse Aperture Imaging Systems

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Overview

• Theory
• Approach
• Results
• Summary
Sparse Aperture Systems

Goal: Synthesize a larger effective collection aperture with a configuration of smaller phased subapertures

- Includes multiple aperture & common secondary, sparse & dilute telescope concepts
- Emphasize extended scene, remote sensing applications

Image courtesy of Fiete
Image courtesy of Boucher
Image courtesy of Harvey
Research Objectives

• Develop understanding of Sparse Aperture phenomenology
  – System-level diffraction (OTF/PSF) effects
  – Photon, dark current & read noise effects
  – Image restoration techniques

• Develop theoretical basis for modeling spectrally diverse Sparse Aperture system performance
  – Target source spectral radiance
  – Polychromatic system OTF
  – Spectrally diverse noise

• Implement strawman model for spectrally diverse Sparse Aperture collection systems
  – Scene spectral radiance
  – Polychromatic system OTF
  – Spectrally diverse system noise
Signal Model

**Imaging System (Space Domain)**

\[
g_{\text{incoh}}[x, y, \lambda] = f_{\text{obj}}[x, y, \lambda] \ast h[x, y, \lambda] + n[x, y, \lambda]
\]

**Imaging System (Frequency Domain)**

\[
G_{\text{incoh}}[\xi, \eta, \lambda] = F_{\text{obj}}[\xi, \eta, \lambda] \cdot H[\xi, \eta, \lambda] + N[\xi, \eta, \lambda]
\]

where:

\[
\text{PSF} \equiv h[x, y, \lambda]
\]

\[
\text{OTF} \equiv H[\xi, \eta, \lambda] = \mathcal{F}\{h[x,y,\lambda]\}
\]

**Signal Frequency Spectrum**

\[
S_{\text{freq}}[\xi, \eta] = F_{\text{obj}}[\xi, \eta, \lambda] \cdot H[\xi, \eta, \lambda] = \frac{\pi A_{\text{det}} T_{\text{int}} F_{\text{fill}}}{4(f\#)^2 h c} \int_0^{\infty} H[\xi, \eta, \lambda] L_{\text{source,FT}}[\xi, \eta, \lambda] f_{\text{opt}}(\lambda) \eta(\lambda) \lambda \text{d}\lambda
\]

**Noise Model**

\[
n[x, y, \lambda] \approx \sqrt{f_{\text{obj}}[x, y, \lambda] \ast h[x, y, \lambda] n_1[x, y] + \sigma_{\text{dc}}(T_{\text{int}}) n_2[x, y] + \sigma_{\text{read}} n_3[x, y]}
\]

Quantization

\[
I_{\text{counts}} = \frac{G_{\text{conv}} G_{\text{elec}} 2^n}{S_{\text{ADC}}} \cdot g_{\text{incoh}}
\]
**Image Restoration**

**Transfer Function**

\[ MTF = |H[\xi, \eta]| \]

**Inverse Filtering**

\[ W_{\text{inverse}}[\xi, \eta] = \frac{1}{H[\xi, \eta]} \]

**Wiener Filtering**

\[ W_{\text{wiener}}[\xi, \eta] = \frac{H^*[\xi, \eta]}{\frac{1}{H[\xi, \eta]^2} + \frac{S_n[\xi, \eta]}{S_f[\xi, \eta]}} \]

\[ W_{\text{pseudo}}[\xi, \eta] = \begin{cases} 
\frac{1}{H[\xi, \eta]} & \text{when } |H[\xi, \eta]| \geq \text{threshold} \\
0 & \text{when } |H[\xi, \eta]| < \text{threshold} 
\end{cases} \]

- Inverse Filters tend to over-boost the noise or be non-optimum at high frequencies
- Optimum solution from a mean-square error viewpoint is the Wiener-Helstrom Filter
Modeling Approach Overview

Object Radiance \( \mathcal{F}\{ \} \) × \( \mathcal{F}\{ \} \) × \( \mathcal{F}\{ \} \) \( \mathcal{F}\{-1\}\{ \} \) + \( \sum \) \( \mathcal{F}\{ \} \) Raw Image \( \mathcal{F}\{ \} \) \( \mathcal{F}\{-1\}\{ \} \) Restored Image

DIRSIG MODTRAN

Object Estimate Wavefront Error Estimate Unaberrated Optics OTF Restoration Filter

Aberrated PSF

Phase Errors Pupil Function \( |\mathcal{F}\{\}^2| \) Unaberrated PSF \( \mathcal{F}\{-1\}\{ \} \)
Strawman Collection Scenario

**Panchromatic (polychromatic world)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scenario #1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectral passband</td>
<td>0.4-0.7 µm</td>
</tr>
<tr>
<td>System f-number (#)</td>
<td>18.0</td>
</tr>
<tr>
<td>Optical sampling (Q=λf/pD)</td>
<td>2.0 (Nyquist)</td>
</tr>
<tr>
<td>Ground Sample Distance (GSD)</td>
<td>0.4572 m (18 in)</td>
</tr>
<tr>
<td>Optical Transmission (τ_{opt})</td>
<td>0.9</td>
</tr>
<tr>
<td>Secondary Obscuration (ε_{sub})</td>
<td>0.24</td>
</tr>
<tr>
<td>Focal Plane Array (FPA)</td>
<td>Staring Frame CCD</td>
</tr>
<tr>
<td>Read Noise</td>
<td>50 rms electrons</td>
</tr>
<tr>
<td>Dynamic Range</td>
<td>11 bits</td>
</tr>
<tr>
<td>Image Smear</td>
<td>0.25 pixels</td>
</tr>
<tr>
<td>rms Wavefront Error</td>
<td>0.12 waves (λ = 0.65 µm)</td>
</tr>
<tr>
<td>Atmosphere</td>
<td>MODTRAN 4.0 mid-latitude summer</td>
</tr>
<tr>
<td>Visibility</td>
<td>17.0 km</td>
</tr>
<tr>
<td>Simulation Time</td>
<td>~1700 GMT (~79° sun angle)</td>
</tr>
<tr>
<td>Target Location</td>
<td>43.2° N Lat, 77.6° W Lon</td>
</tr>
</tbody>
</table>

Sparse Aperture Configurations:
Annulus, Tri-arm, Golay-6

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Orbital Analysis

Orbit design and collection geometry evaluation performed through use of STK

Access Summary Report

Start Time (UTC)  Stop Time (UTC)
-------------------- --------------------
1 Jun 2003 16:33:33.66  1 Jun 2003 17:41:17.55

Duration (sec)  4063.893
Performed arbitrary simulated nadir ascending pass at ~1300 local time on 1 Jun 03

STK enables detailed evaluation of vehicle pointing (az, el) and position (lat, lon, alt) required for DIRSIG
Suburban Scene Radiometry

Rochester Megascene

Scene Attributes:

- Complex scene representative of typical remotely sensed targets
- Greater Rochester suburban area modeled with good fidelity
- Advertised to be accurate at 1-m (39.4-in) Ground Sample Distance
- Underlying texture map sampled at 6-in GSD
- Features within scene modeled well below 6 inches

Simulations were performed using collection geometry acquired from STK and oversampled by 3:1 (6 in to support 18-in GSD in sparse aperture simulation)

RGB spectral radiance image: 0.65\(\mu\)m (R), 0.55\(\mu\)m (G), 0.45\(\mu\)m (B)
Simulations were performed using collection geometry acquired from STK and oversampled between 3:1 (6 in) and 18:1 (1 in)
## Alternative Scene Options

### Potential Aerial Object Scenes

<table>
<thead>
<tr>
<th>Camera Type</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hymap Hyperspectral Sensor</strong></td>
<td>512 pixel scanner</td>
</tr>
<tr>
<td></td>
<td>0.45 – 2.5 μm</td>
</tr>
<tr>
<td></td>
<td>144 hyperspectral bands</td>
</tr>
<tr>
<td></td>
<td>~20 ft GSD</td>
</tr>
<tr>
<td></td>
<td>16 bits</td>
</tr>
<tr>
<td><strong>CitiPix Aerocolor Film Camera</strong></td>
<td>9.5in film</td>
</tr>
<tr>
<td></td>
<td>0.38 – 0.7 μm</td>
</tr>
<tr>
<td></td>
<td>RGB</td>
</tr>
<tr>
<td></td>
<td>~6 in GSD</td>
</tr>
<tr>
<td></td>
<td>8 bits (digitized)</td>
</tr>
<tr>
<td><strong>TerraPix Digital Aerial Camera</strong></td>
<td>4080 x 4080 pixel frame</td>
</tr>
<tr>
<td></td>
<td>0.4 – 0.9 μm</td>
</tr>
<tr>
<td></td>
<td>RGB</td>
</tr>
<tr>
<td></td>
<td>~7 in GSD</td>
</tr>
<tr>
<td></td>
<td>12 bits</td>
</tr>
</tbody>
</table>
Complex Exit Pupil

Conventional Pupil Function:

\[ \mathcal{M}(x, y) = p(x, y) \exp \left( \frac{2\pi i}{\lambda} w(x, y) \right) \]

Sparse Aperture Pupil Function:

\[ \mathcal{M}(x, y) = \sum_{i=1}^{N} p_i(x - x_i, y - y_i) \exp \left( \frac{2\pi i}{\lambda} w_i(x - x_i, y - y_i) \right) \]

Aberration (wavefront error) function:

\[ w(x, y) \equiv w(x, y, x_0) = a_1(x^2 + y^2) + a_2 xx_0 + a_3 x_0^2 \]

spherical   coma   astigmatism

+ b_1(x^2 + y^2)^2 + b_2 xx_0(x^2 + y^2) + b_3 x^2 x_0^2

field curvature   distortion

+ b_4 x_0^2(x^2 + y^2) + b_5 xx_0^3 + b_6 x_0^4

+ higher order terms
Imaging System OTF

System Transfer Function developed by cascading individual component OTF:

\[
\text{OTF}_{\text{sys}} = \prod_{i=1}^{N} \text{OTF}_i[\xi, \eta, \lambda]
\]

Optical Transfer Function

\[
\text{MTF}_{\text{sys}} = \prod_{i=1}^{N} \text{MTF}_i[\xi, \eta, \lambda]
\]

Modulation Transfer Function

\[
\text{OTF}_{\text{sys}}[\xi, \eta, \lambda] = \text{OTF}_{\text{ap}}[\xi, \eta, \lambda] \cdot \text{MTF}_{\text{det}}[\xi, \eta] \cdot \text{MTF}_{\text{smear}}[\xi, \eta] \cdot \text{MTF}_{\text{jitter}}[\xi, \eta] \cdot \text{MTF}_{\text{diff}}[\xi, \eta, \lambda]
\]

Complex Aperture OTF:

\[
\text{OTF}_{\text{ap}}[\xi, \eta, \lambda] = \rho[\lambda z_2 \xi, \lambda z_2 \eta] \int \int |p[x, y]| \, dx \, dy
\]

Note: normalized autocorrelation of the scaled pupil function

Data courtesy of Boucher
Sparse Aperture Autocorrelation

- Unaberrated System Pupil:
  \[ p[x, y] = s[x, y] \ast \sum_{i=1}^{N} \delta[x-x_i, y-y_i] = \sum_{i=1}^{N} s[x-x_i, y-y_i] \]

- Optical Transfer Function (OTF):
  \[ \mathcal{H}[\xi, \eta] \propto p[-\lambda z_2 \xi, -\lambda z_2 \eta] \ast p^*[\lambda z_2 \xi, -\lambda z_2 \eta] \]

\[ \mathcal{H}[\xi, \eta] \propto (s[-x,-y] \ast s^*[-x,-y]) \ast \sum_{j=1}^{N} \sum_{k=1}^{N} \delta[x+x_j-x_k, y+y_j-y_k] \delta_{x=\lambda z_2 \xi} \delta_{y=\lambda z_2 \eta} \]
Diffraction-Limited MTF Comparison

Filled
Fill Factor: 1.000

Cassegrain
Fill Factor: 0.942

Annular
Fill Factor: 0.190

Tri-arm
Fill Factor: 0.170

Golay-6
Fill Factor: 0.170
Polychromatic OTF Issues

- Traditional approaches resample high-resolution panchromatic imagery
- Spectrally weighted polychromatic MTF computed to apply to gray-world scene
- This technique fails to accurately account for scene spectral content and MTF spectral artifacts
- Direct spectral integration of radiance frequency spectrum and system OTF desirable

\[ S_{\text{out}}[\xi, \eta] = \frac{G_{\text{conv}} G_{\text{elec}} 2^n}{4(f\#)^2 hc} \int_0^\infty \text{MTF} \left[ \xi, \eta, \lambda \right] L_{\text{source,FT}} \left[ \xi, \eta, \lambda \right] \gamma_{\text{opt}}(\lambda) \eta(\lambda) \lambda \lambda d\lambda \]

\[ S_{\text{out}}[\xi, \eta] = \frac{G_{\text{conv}} G_{\text{elec}} 2^n}{4(f\#)^2 hc} \int_0^\infty \text{MTF} \left[ \xi, \eta, \lambda \right] L_{\text{source}}(\lambda) \gamma_{\text{opt}}(\lambda) \eta(\lambda) \lambda \lambda d\lambda \]

\[ \text{MTF}_{\text{poly}}[\xi, \eta] = \frac{\int L_{\text{source}}(\lambda) \gamma_{\text{opt}}(\lambda) \eta(\lambda) \lambda \lambda d\lambda}{\int L_{\text{source}}(\lambda) \gamma_{\text{opt}}(\lambda) \eta(\lambda) \lambda \lambda d\lambda} \]
Comparison of Filled versus Sparse Aperture MTFs

- Oscillatory behavior of sparse aperture MTF versus wavelength creates concern over spectral artifact image quality issues.
Simulation Overview

Top-level simulation methodology:
- Develop geometric pupil function
- Add phase via Zernike polynomials to incorporate aberrations
- Evaluate aberrated system OTF
- Perform Fourier Optical prediction of noise-free degraded image
- Add uncorrelated noise
- Restore imagery via conventional Wiener-Helstrom filter
MTF Evaluation
(Effects of Aberrations)

Sparse Aperture w/Piston Error
Diffraction-Limited MTF
0.07-wave rms MTF

System #1 Uncorrected MTF (Contrast Adjusted)
System #1 MTF (Contrast Adjusted)
System #1 MTF (Contrast Adjusted)

0.10-wave rms MTF
0.14-wave rms MTF
0.20-wave rms MTF
Modulation Transfer Function

Comparison of Filled versus Sparse Aperture MTFs

Aberrated Sparse Aperture MTF: 0.20-wave rms
Point Spread Function

Comparison of Filled versus Sparse Aperture PSFs

Filled Aperture PSF: Unaberrated
Sparse Aperture PSF: Unaberrated
Sparse Aperture PSF: Aberrated 0.20-waves rms
Quality Impacts

Image Quality Comparison of Filled versus Sparse Aperture

**Filled Aperture:** Unaberrated

**Sparse Aperture:**
- Unaberrated
- Aberrated 0.20-waves rms
**Wiener Filter Example**

- **Fill Factor:** 0.173
- **WFE:** 0.20-waves rms
- **SNR:** 18.5 db

**Aperture Comparison**
- **Sparse Aperture:**
  - Noisy Degraded Image
  - WFE: 0.20-waves rms
  - SNR: 18.5 db

- **Filled Aperture:**
  - Noisy Degraded Image
  - SNR: 6.0 db

**Original Object**
- **Wiener Filtered Image**
Image Restoration Example (1)

Fill Factor: 0.173  Wavefront Error: 0.10-waves rms PTT  SNR: 25.3 db

Sparse Aperture w/Piston Tip/Tilt Errors

Aperture Phase Profile

Aberrated MTF

Original Object

Predicted RGB Image

Restored RGB Image
Image Restoration Example (2)

Fill Factor: 0.173  Wavefront Error: 0.20-waves rms PTT  SNR: 25.3 db

Sparse Aperture w/Piston Tip/Tilt Errors  Aperture Phase Profile  Aberrated MTF

Original Object  Predicted RGB Image  Restored RGB Image

Rochester Institute of Technology
Image Restoration Example (3)

Fill Factor: 0.173

Wavefront Error: 0.25-waves rms PTT

SNR: 25.3 db

Sparse Aperture w/Piston Tip/Tilt Errors

Aperture Phase Profile

Aberrated MTF

Original Object

Predicted RGB Image

Restored RGB Image
Closer Examination of Restoration
Image Restoration Example (1)

Fill Factor: 0.173  Wavefront Error: 0.10-waves rms PTT  SNR: 25.3 db

Note correlated noise amplification & presence of color artifacts due to Wiener filter frequency boost mismatch

Individual spectral bands restored via central wavelength OTF to simulate physics associated with wideband collection

Precise knowledge of phase errors assumed
Image Restoration Example (2)

Fill Factor: 0.173

Wavefront Error: 0.20-waves rms PTT

SNR: 25.3 db

Note correlated noise amplification & presence of color artifacts due to Wiener filter frequency boost mismatch

Individual spectral bands restored via central wavelength OTF to simulate physics associated with wideband collection

Precise knowledge of phase errors assumed
Image Restoration Example (3)

Fill Factor: 0.173  Wavefront Error: 0.25-waves rms PTT  SNR: 25.3 db

Note correlated noise amplification & presence of color artifacts due to Wiener filter frequency boost mismatch

Individual spectral bands restored via central wavelength OTF to simulate physics associated with wideband collection

Precise knowledge of phase errors assumed
Image Restoration Example (4)

Fill Factor: 1.000
Wavefront Error: 0.30-waves rms defocus
SNR: 16.5 db

Filled Aperture w/Focus Errors

Aperture Phase Profile

Aberrated MTF

Original Object
Predicted RGB Image
Restored RGB Image
Criticality of Phase Knowledge

Fill Factor: 0.173

Wavefront Error: 0.25-wave rms PTT

SNR: 25.3 db

Sparse Aperture
w/Piston Tip/Tilt Errors

Aperture Phase Profile

Original Object

Restored RGB Image
(RGB OTF - perfect phase knowledge)

Restored RGB Image
(central OTF - perfect phase knowledge)

Restored RGB Image
(RGB OTF - no phase knowledge)

Restored RGB Image
(RGB OTF - perfect phase knowledge)

Restored RGB Image
(RGB OTF - no phase knowledge)
Summary

• Strawman spectra-radiometric sparse aperture model demonstrated

• Includes effects of spectrally-dependent OTF, phase errors, noise, spectral radiance

• Moderately aberrated, low-fill factor sparse aperture systems appear to demonstrate spectral issues