R·I·T

2008 Imaging Science Ph.D. Comprehensive Examination
June 13, 2008
9:00AM to 12:00PM

IMPORTANT INSTRUCTIONS

You must complete six (6) of the eight (8) questions on this portion of the exam. The answer to each question should begin on a new piece of paper. While you are free to use as much paper as you would wish to answer each question, please only write on one side of each sheet of paper that you use. Be sure to write your provided identification letter (located below), the question number, and the page number for each answer in the upper right-hand corner of each sheet of paper that you use. When you hand in your exam answers, be certain to write your name and your provided identification letter on the supplied 3" x 5" note card and place this in the envelope located with the proctor.

ONLY HAND IN THE ANSWERS TO THE SIX (6) QUESTIONS THAT YOU WOULD LIKE EVALUATED

Identification Letter: ________________

THIS EXAM QUESTION SHEET MUST BE HANDED BACK TO THE PROCTOR UPON COMPLETION OF THE EXAM PERIOD

1. A 2-dimensional sinusoid of period 2 is oriented at an angle of 0.3π radians measured from the x axis. It is sampled by a $COMB$ function with unit spacing oriented parallel to the x and y axes.

   a. How many samples per period are obtained along the x and y axes?

   b. What will be recovered from the samples by ideal 2-dimensional interpolation?

2. The image classification techniques known as spectral angle mapper (SAM) relies on only one descriptive statistic regarding the image spectral classes into which the population of pixels composing an image are to be assigned, namely, the multivariate class mean vector for each class. Individual pixel class assignments are made by finding the smallest multidimensional angle between the unknown pixel and each of the multivariate class mean vectors. Using this approach, but in a different context, describe how you could develop a character recognition algorithm to identify all of the characters on a binary, scanned image of a page of text. Be sure to spell out all the caveats that are necessary for your approach to function properly and indicate any problems that may arise in the acquisition and processing phases of your process.

3. The input to an optical system is composed of collimated visible light. Your task is to design a diffraction spectrograph to measure the spectrum. The grating is a rectangle of width B and D along the x and y directions, respectively. The grating has alternating transmitting and opaque regions of width a and b, respectively. The first-order spectrum is to be dispersed at an angle of $\theta = 5°$ for light with wavelength $\lambda_0 = 500nm$ and a lens diameter d and focal length $f = 400mm$.

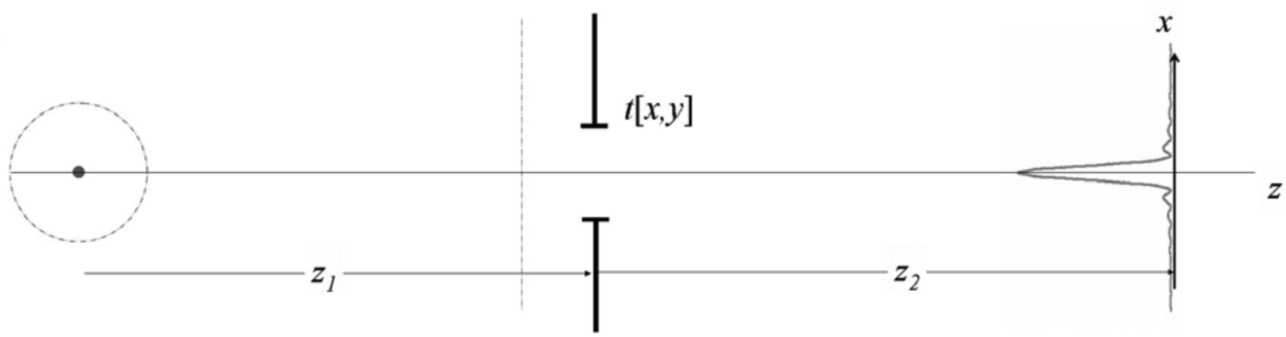
   a. Design the grating such that all diffraction orders that a multiples of the integer value n are "missing" from the spectrum ($n \geq 2$).

   b. Specify the prescription for the grating if every third order is missing.

   c. If using the grating in part (b), determine the linear extent of the grating required for the first-order spectrum to resolve the sodium doublet with $\lambda = 589.0nm$ and $\lambda = 589.6nm$.

4. The JPEG image compression methodology relies on the discrete cosine transform (DCT) applied to individual 8x8 neighborhoods in an image for energy compaction while JPEG2000 utilizes either an integer (5,3) filter-bank or a floating point Daubechies (9,7) filter bank applied to an adjustably sized window to achieve compaction. For a binary, line-art image, describe how you would utilize each of these methodologies to achieve the highest quality image possible. What artifacts, if any, would be present in the decompressed images from each approach?

5. Consider a particularly simple imaging "system:"



The system may be modeled in three stages:

i. Propagation from an "object plane" located at the origin of coordinates into the Fraunhofer diffration region over the distance z_1*;*

ii. multiplication by the (possibly complex-valued) transmittance function of an aperture (or pupil); the form of the transmittance function is $a[x,y] = t\left[\frac{x}{b}, \frac{y}{d}\right]$*, where* b*,* d *are positive real values and* $t[x,y]$ *is some function with compact support;*

iii. a second propagation to an observation plane located in the Fraunhofer diffraction region at a distance z_2 *from the pupil;*

iv. square-law detection at the observation plane.

a. Find an expression for the amplitude at the observation plane if the object consists of a single point source located at the origin of coordinates that emits on wavelength λ_0.

b. If the object function is a pair of monochromatic Dirac delta functions:

$$f[x,y] = \delta[x,y] + \delta[x + x_0, y]$$

and the aperture function is:

$$t[x,y] = RECT\left[\frac{2x - \frac{b}{2}}{b}, \frac{y}{b}\right] - RECT\left[\frac{2x + \frac{b}{2}}{b}, \frac{y}{b}\right]$$

Evaluate and sketch the impulse response of the system.

c. Find a relationship between the values x_0, b, and d such that the two objects are resolved using some reasonable resolution criterion (specify).

6. Consider a system with input functions $f_n(x)$ listed below. The functions are translated by an arbitrary and unknown distance x_0, so the actual input function is $f_n(x - x_0)$. Determine the impulse response $m_n(x)$ and/or transfer function $M_n(\xi)$ the defines the matched filter that locates x_0 . In the ideal case, we can construct a filter such that

$$f_n(x - x_0) * m_n(x) = \delta(x - x_0)$$

You may use reasonable approximations where appropriate and sketches may help.

a. $f_1(x) = GAUS(x)$

b. $f_2(x) = e^{i\pi(1 - RECT(x))}$

7. Given the discriminant function utilized in Gaussian maximum likelihood (GML) classification

$$g_i\left(\vec{x}\right) = \ln\left\{p\left(\omega_i\right)\right\} - \frac{1}{2}\ln\left\{|\vec{\Sigma}_i|\right\} - \frac{1}{2}\left(\vec{x} - \vec{\mu}_i\right)\vec{\Sigma}_i^{-1}\left(\vec{x} - \vec{\mu}_i\right)$$

begin with Bayes theorem and develop this expression. Be sure to identify all your variables and state any assumptions that are made. Once you have derived this discriminant expression, qualitatively describe each of the terms describing how they influence the decision of class assignment of an unknown vector $\vec{x}$.

8. Suppose we want to sample a 2D continuous function $f(x,y) = TRI\left(\frac{x}{b}, \frac{y}{b}\right)$ by means of a rectangular sampling scheme to obtain $\frac{1}{\Delta x} = \frac{1}{\Delta y} = \frac{3}{2}\frac{mm}{cycle}$. Let $b = 2$.

   a. Write down the expression for the sampled function $f_s(x,y)$, and its spectrum $F_s(\xi,\eta)$.

   b. Determine the impulse response $h(x,y)$ of an ideal low-pass filter.

   c. Assign a different value to the scaling factor b in order to minimize aliasing and write down the transfer function and impulse response of the new filter.

   d. Compare the percentage of the energy spectrum preserved by the filters designed in (b) and (c).

   e. In practice, can we sample $f(x,y)$ alias-free without using an anti-aliasing filter?