SIMG-782 Digital Image Processing

Homework 6

Ex. 1 — (Circular Convolution)
Let $\mathbf{f} = [1, 3, -1, 2, 0, -3]$ and $\mathbf{h} = [-1, 3, -2]$.

(a) Calculate the convolution $\mathbf{f} * \mathbf{h}$ assuming that both $\mathbf{f}$ and $\mathbf{h}$ are zero-padded to a length of 12.

(b) Calculate the convolution using a product of discrete Fourier transforms.

(c) Plot $F_p(u)$, $G_p(u)$ and $H_p(u)$ as points in the complex plane for $0 \le u \le 11$.

(d) Calculate and interpret $\sum g_p^2(n)$ and $\sum |G_p(u)|^2$.

Ex. 2 — (2D Filtering Exercise)
An image called 'urban_image_3.png' is contained in the images directory at the course web site. Use this image as the "subject" for this problem. This image has dimensions 256×256.

(a) We have seen in the previous homework that a one-dimensional gaussian filter can be represented by $K \exp(-\pi(u/a)^2)$ where u is a suitable frequency variable, a is a parameter that controls the bandwidth and K is a scale factor. Now let u be an array of size 256×256 whose rows are all equal to the sequence $[-128, -127, -126, \ldots, 126, 127]$. Construct the function

$$H_1(u) = e^{-\pi(u/10)^2}$$

Use frequency-domain filtering to find the output image produced by filtering the input image through this filter. You may have to scale the output image for display purposes.

(b) Repeat the above with the filter

$$H_2(u) = e^{-\pi(u/50)^2}$$

Compare the results.

(c) In the previous homework we found that $\exp(-\pi u^2)$ and $\exp(-\pi x^2)$ form a Fourier transform pair. Use this fact to determine the impulse response of $H_1(u)$. Can you get the same result by taking the inverse FFT of $H_1(u)$? Note that the impulse response $h_1(x, y)$ is now a function of two variables.

(d) Produce the result of part (a) by doing convolution.

(e) Repeat for $H_2(u)$.

(f) Let $v = u^T$, so that its columns are all the sequence $[-128, -127, -126, \ldots, 126, 127]$. Construct the function

$$H_3(v) = e^{-\pi(v/10)^2}$$

Use frequency-domain filtering to find the output image produced by filtering the input image through this filter. Compare the results with those of (a).

(g) Repeat the above with the filter

$$H_4(u) = e^{-\pi(v/50)^2}$$

Compare the results.

(h) Construct the impulse response functions $h_3(x, y)$ and $h_4(x, y)$. Using them, repeat the results found from filtering in the frequency domain.

(i) A filter of the form $H(u, v) = \exp(-[(u/a)^2 + (v/b)^2])$ can be factored into something of the form $H(u, v) = S(u)T(v)$. How can this fact be exploited in building a faster filter algorithm?

Ex. 3 — (Blurred Image Restoration)
Images urban0.png and urbanblur.png are in the images directory at the course web site. Use Wiener filtering to attempt to recover the blurred image.

urban0.png

urbanblur.png